DESIGN OF REINFORCED CONCRETE STRUCTURES

Second Edition **Volume 2**

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Features

- Good theoretical background for each topic with code provisions.
- Numerous illustrations and figures for each topic.
- Reflects the very latest Egyptian Code provisions (ECP 203-2007) and includes all major changes and additions.
- Extensive examples in each chapter on design and analysis of reinforced concrete structures utilizing SI units.
- All examples are worked out step by step ranging from simple to advanced.
- Full reinforcement details for every example.
- Numerous design charts covering a wide range of cross sectional shapes and straining actions.

This volume covers the following topics

- Solid Slabs
- Hollow Blocks
- Paneled Beams
- Flat Slabs
- Stairs
- Short Columns
- Eccentric Sections
- Slender Columns
- R/C Frames

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Second Edition

2008

PREFACE

Teaching reinforced concrete design, carrying out research relevant to the behavior of reinforced concrete members, as well as designing concrete structures motivated the preparation of this book. The basic objective of this book is to furnish the reader with the basic understanding of the mechanics and design of reinforced concrete. The contents of the book conform to the latest edition of the Egyptian Code for the Design and Construction of Concrete Structures ECP-203. The authors strongly recommend that the Code be utilized as a companion publication to this book.

The book is aimed at two different groups. First, by treating the material in a logical and unified form, it is hoped that it can serve as a useful text for undergraduate and graduate student courses on reinforced concrete. Secondly, as a result of the continuing activity in the design and construction of reinforced concrete structures, it will be of value to practicing structural engineers.

Numerous illustrative examples are given, the solution of which has been supplied so as to supplement the theoretical background and to familiarize the reader with the steps involved in actual design problem solving.

In writing the book, the authors are conscious of a debt to many sources, to friends, colleagues, and co-workers in the field. Finally, this is as good a place as any for the authors to express their indebtedness to their honorable professors of Egypt, Canada and the U.S.A. Their contributions in introducing the authors to the field will always be remembered with the deepest gratitude.

This volume contains the following chapters

- Solid slabs
- Hollow block slabs
- Paneled beams
- Flat slabs
- Reinforced concrete stairs
- Short columns
- Eccentric sections
- Slender columns
- Reinforced concrete Frames

It also includes appendices containing design aids.

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SOLID SLABS

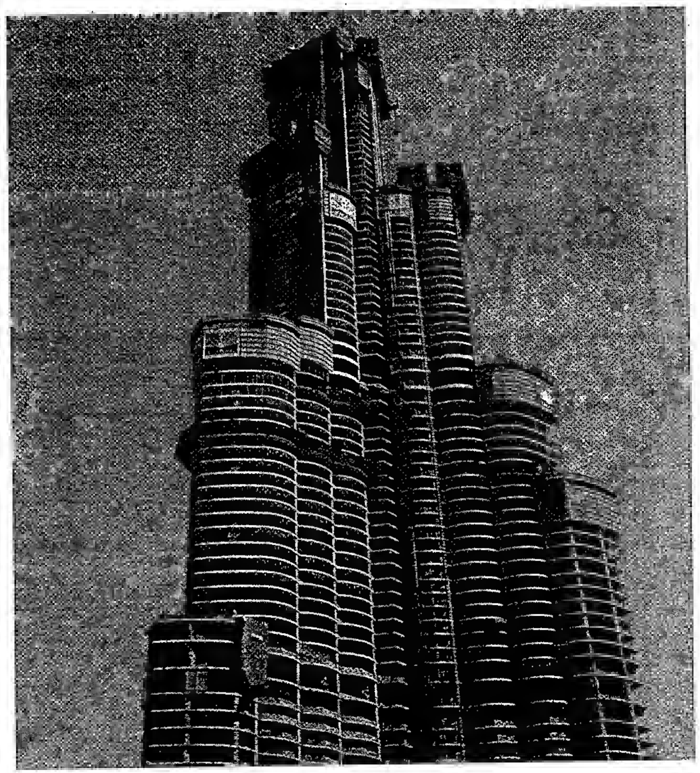


Photo 1.1 Burj Dubai during construction (2007)

1.1 Introduction

Reinforced concrete solid slabs are used in floors, roofs and as decks of bridges as shown in Photo 1.1. Slabs may span in one direction or in two directions depending on the slab dimensions and the surrounding supporting elements. Slabs spanning in one direction are referred to as one-way slabs while those spanning in two directions are referred to as two-way slabs.

1.2 One-Way Solid Slabs

1.2.1 Definition

One-way solid slabs are extensively used in buildings, especially for spans less than 4 meters. To be classified as a one-way slab, the ratio of the long side to the short side of the slab panel must exceed 2. They are referred to as one-way slabs because the bending is mainly in the short direction. Typically, a 1m wide strip of the slab is analyzed and the reinforcement required for this strip is used in all parts of the slab.

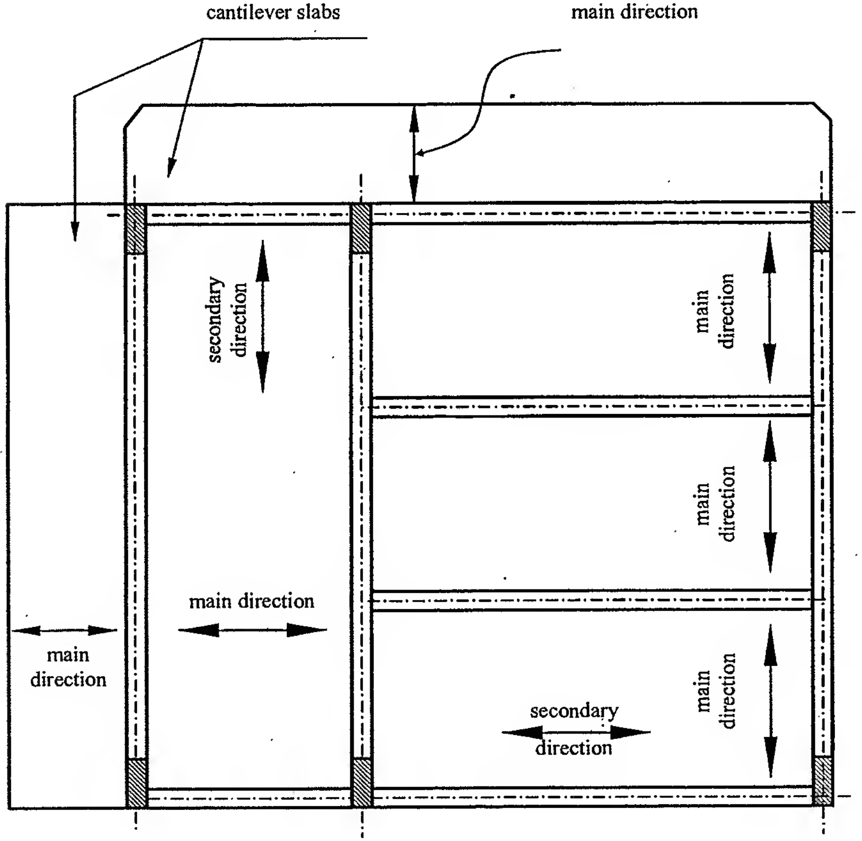


Fig. 1.1 One-way slabs

1.2.2 Structural Behavior

The direction in which the slab bends is called the main direction as shown in Fig 1.1. The main reinforcement is placed in this direction as illustrated in Fig. 1.2.

For a one-way slab supported on four beams like the one shown in Fig. 1.2, the strip near the middle acts as a one-way slab. However, at the slab parts near the edges, some of the load is transferred in the longitudinal direction producing a two-way slab action. Thus, top reinforcement should be added on each side of the girder to account for this action. If this reinforcement is ignored, wide cracks may develop on the top of the slab along edges (A-B) and (A′-B′)

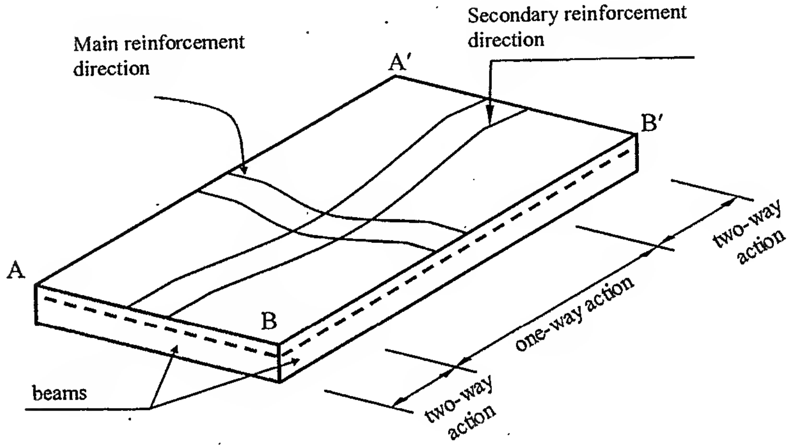
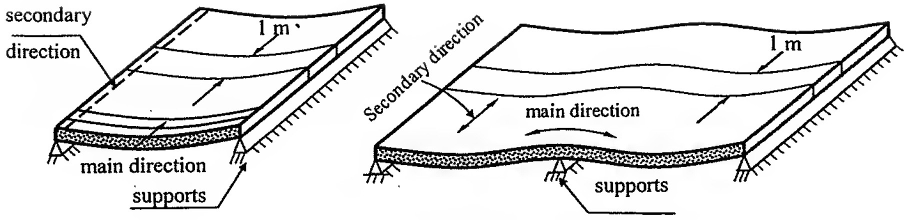


Fig. 1.2 Structural action of one-way slabs

The load transferred to the perpendicular direction is almost equal to zero except near the edge beams (*Beams AB and A′B′*). Thus, a secondary mesh is required in this direction (as shown in Figs. 1.2 and 1.3) to carry this small portion and to keep the main reinforcement in place. Also, the secondary reinforcement is placed to control cracks produced by shrinkage or temperature changes and to help in distributing concentrated loads transversely.



a- Simply supported b- Continuous

Fig. 1.3 One-way slabs supported on two sides

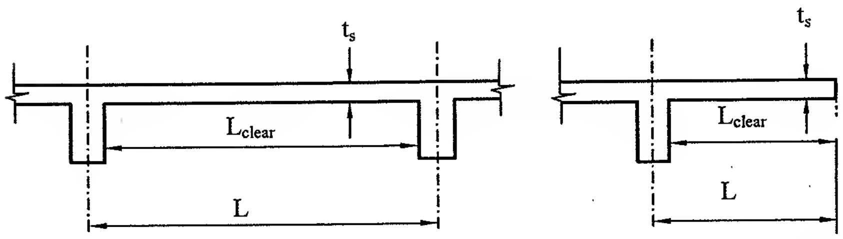
It should be noted that if a slab panel is supported only on two sides, it would act as one-way slab regardless of the ratio of the long side to the short side. Figures 1.3.a and 1.3.b show examples of slabs act as one way because of having supporting beams on two sides only.

1.2.3 Effective Span

The effective span (L_{eff}) for solid slabs is given by the following equation

$$L_{eff} = \min of \begin{cases} \text{max of} \begin{cases} L_{clear} + t_s \\ 1.05 \times L_{clear} \end{cases} & \text{for simple or continuous slabs} \\ \text{CL to CL (L)} \end{cases} \quad \text{.....................(1.1)}$$

$$L_{eff} = \min of \begin{cases} L_{clear} + t_s \\ \text{edge to CL (L)} \end{cases} \quad \text{for cantilever slabs} \quad \text{...(1.2)}$$



Continuous span Cantilever span

Fig. 1.4 Effective spans for solid slabs

1.2.4 Minimum Thickness

The depth of solid slabs is usually controlled by deflection rather than flexural strength requirements. The ECP-203 gives the minimum thickness for one-way slabs reinforced with high grade steel in which the deflection calculations can be ignored as listed in Table 1.1.

Table 1.1 (L_n/t)* ratios for members spanning less than 10 meters or cantilevers spanning less than 2m (f_y=400 N/mm^2). (Deflection calculations can be ignored)

Element	Simply supported	One end continuous	Two ends continuous	Cantilever
Solid slabs	25	30	36	10
Hidden Beams and hollow blocks	20	25	28	8

$^*L_n$ is the clear span.

The values listed in Table (1.1) are valid when using high grade steel 400/600. In the case of using other types of reinforcing steel, the values mentioned in Table 1.1 should be divided by factor ζ, given by:

$$\xi = 0.40 + \frac{f_y}{650} \quad \text{...(1.2)}$$

Where f_y is the yield strength of reinforcing steel in N/mm^2.

The code also provides an absolute minimum thickness for one-way slabs

$$t_{\min} = \begin{cases} L/30 & \textit{simple span} \\ L/35 & \textit{continuous from one end} \\ L/40 & \textit{continuous from two ends} \end{cases} \quad \text{........................(1.3)}$$

where L is the effective span

In addition, the absolute minimum thickness should not be less than 80 mm for slabs subjected to static loads and 120 mm for slabs subjected to dynamic loads. The aforementioned thickness can be reduced in case of prefabricated slabs. To satisfy serviceability requirements for corrosion and fire protection, the concrete cover should not be less than 20mm.

1.2.5 Bending moments

The exact solution for determining the bending moment distribution of solid slabs is complicated. In the case of equal spans with a maximum difference of 20% with equal uniform loads and the live loads are less than the dead loads (p<g), the Egyptian Code gives the following values:

For slabs with two spans

$$w_u = 1.4(\gamma_c \times t_s + flooring) + 1.6 w_{LL}$$

$$M_u = \frac{w_u \times L_{eff}^2}{k}$$

The value of the factor k is given in Fig. 1.5.

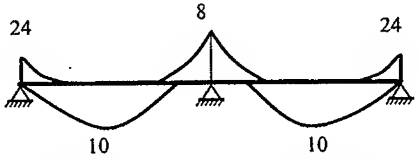


Fig. 1.5 k-factor for continuous slabs with two spans

For slabs with three spans or more

$$M_u = \frac{w_u \times L_{eff}^2}{k}$$

The value of the factor k is given in Fig. 1.6.

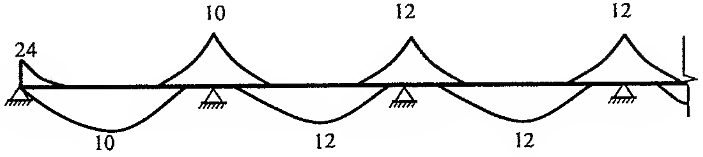


Fig. 1.6 k-factor for continuous slabs with three spans or more

In the case of unequal continuous one-way slabs, the bending moment can be obtained using classical structural analysis or computer programs. In this case, the negative bending moment over supports can be reduced according to a parabolic distribution by $M_l/2$, where M_l is the difference between the bending at the centerline and that at the support face as shown in Fig. 1.7.

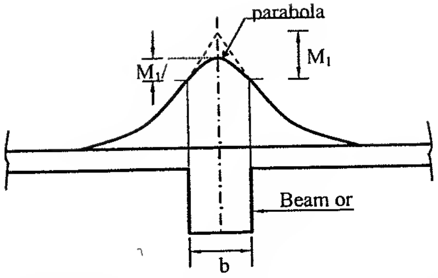


Fig. 1.7 Negative moment reduction in solid slabs

The structural analysis of the slabs shown in Fig. 1.8 may lead to a negative moment at midspan of the interior bay. However, the Egyptian Code requires that a minimum positive bending moment at any span should not be less than $w_u L^2/16$ as shown in Fig. 1.8.

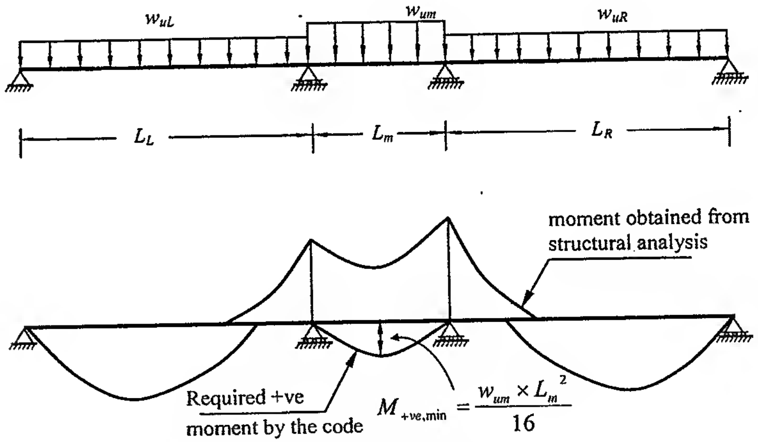


Fig. 1.8 Minimum positive bending moments in one way slabs

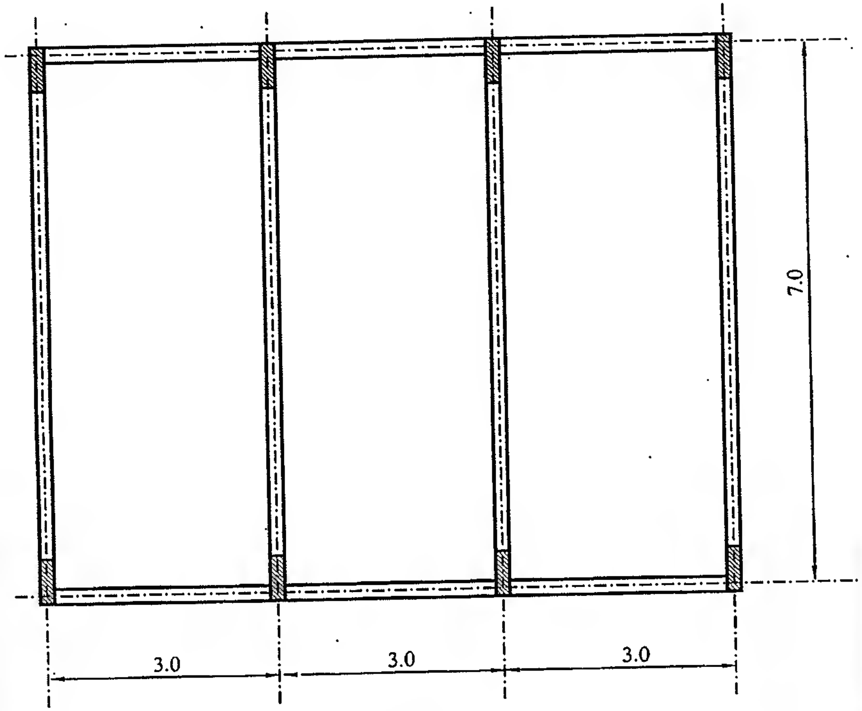
In case of slabs subjected to heavy live loads, the sections at midspan should be designed to withstand a negative bending moments in addition to a positive bending moment. The negative moment at midspan equals

$$M_{min} = \frac{\left(g - \frac{p}{2}\right) L^2}{24} \quad \text{.......................................(1.4)}$$

where g is the dead loads, p is the live loads, and L is the effective span

Example 1.1

Compute and draw the reinforcement details for the reinforced concrete floor shown in the figure below. The floor is to be designed to carry a live load of 3 kN/m^2 and a flooring material of 1.5 kN/m^2 using concrete strength of 30 N/mm^2 and yield strength of steel of 400 N/mm^2. Consider the width of all beams to be 250 mm.



Floor plan

Solution

Step 1: Estimate the thickness of the slab

The plan consists of one way slabs. Assume that the slab thickness t_s is 120 mm for all slabs in the floor and will be checked as follows.

For one way slab, the minimum thickness is given by

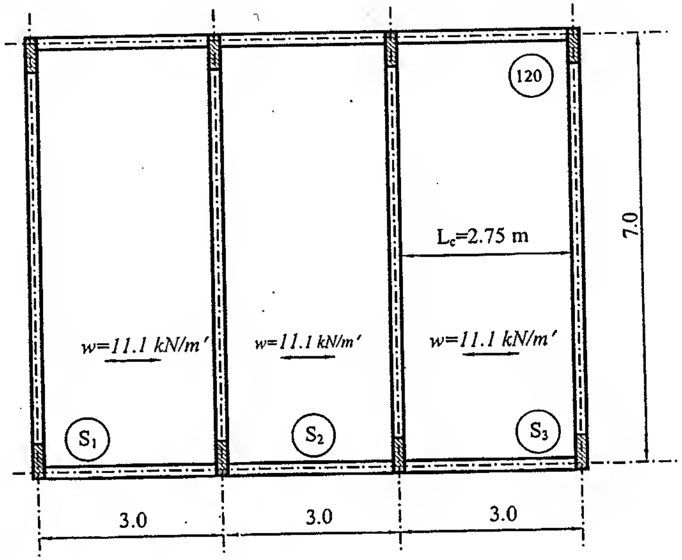
$$t_{\min} = \begin{cases} L/30 & \textit{simple span} \\ L/35 & \textit{continuous from one end} \\ L/40 & \textit{continuous from two ends} \end{cases}$$

Slabs S_1 and S_3 are one way slabs continuous from one end thus

$$t_{\min} = \frac{L}{35} = \frac{3.0\times1000}{35} = 86\ mm\ \ (< t_s\ldots.\text{o.k})$$

Also for deflection calculations may be ignored if the thickness is greater than the values listed in Table 1.1. For the solid slab panel that is continuous from one side with f_y=400 N/mm^2

$$t_{\min} = \frac{L_{clear}}{30} = \frac{(3.0-0.25)\times1000}{30} = 91.67\ mm\ < t_s \text{ (Deflection calculation is not necessary)}$$



Step 2: Calculate the effective span

Since the width of all beams is 250 mm, the clear span is equal to
L_{clear} =L-0.25
where L is the centerline to centerline distance

For the interior panel, the effective span is equal to

$$L_{eff} = \min\begin{cases} \text{max of}\begin{cases}(L_{clear} + t_s \\ 1.05\times L_{clear}\end{cases} \\ \text{CL}\rightarrow\text{CL}\end{cases}$$

$$L_{eff} = \min\begin{cases} \text{max of}\begin{cases}(3-0.25)+0.12 = 2.87\ m \\ 1.05\times(3-0.25) = 2.89\ m\end{cases} \\ 3\text{ m}\end{cases}$$

L_{eff}= 2.89 m

Step 3: Calculate the Loads

$g_s = slab\ weight + flooring = \gamma_c \times t_s + flooring = 25 \times 0.12 + 1.5 = 4.5 kN/m^2$

$p_s = Live\ loads = 3\, kN/m^2$

$w_u = 1.4\, g_s + 1.6\, p_s$

$w_u = 1.4 \times 4.5 + 1.6 \times 3 = 11.10\ kN/m^2$

Taking a strip width of 1m (b=1000 mm), the load acting on this strip is equal to

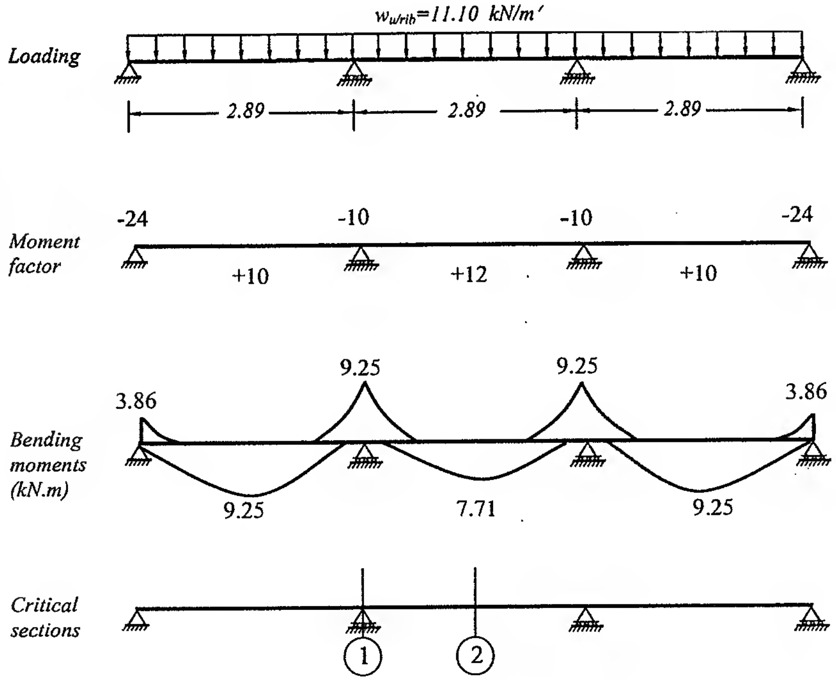
$w_{su} = w_u \times 1 = 11.10\, kN/m'$

Load transferred in x-direction = 11.1 kN/m^2	(main direction)
Load transferred in y-direction = 0	(secondary direction)

Step 4: Bending Moments

Since the slab is continuous with equal spans and equal loading, the bending moments can be obtained using code coefficients as follows

$$M_u = \frac{w_u \times L_{eff}^2}{k}$$



Step 5: Design of reinforcement

Assuming concrete cover of 20 mm

d= 120 –20 =100 mm

$$A_{s,min} = \frac{0.6}{f_y} \times b \times d = \frac{0.6}{400} \times 1000 \times 100 = 150\ mm^2$$

Design of Section 1

M$_{-ve}$= 9.25 kN.m, using the design aids namely R-ω curve

$$R = \frac{M_u}{f_{cu} b d^2} = \frac{9.25 \times 10^6}{30 \times 1000 \times 100^2} = 0.0308$$

From the curve, ω=0.037

$$A_s = \omega \frac{f_{cu}}{f_y} b \times d = 0.037 \frac{30}{400} \times 1000 \times 100 = 276\ mm^2 > A_{s,min}$$

use (2.5 Φ 10 + 2.5 Φ 8/m')   A$_{s,chosen}$=322 mm^2

Design of Section 2

M$_{+ve}$=7.71 kN.m, and using R-ω curve

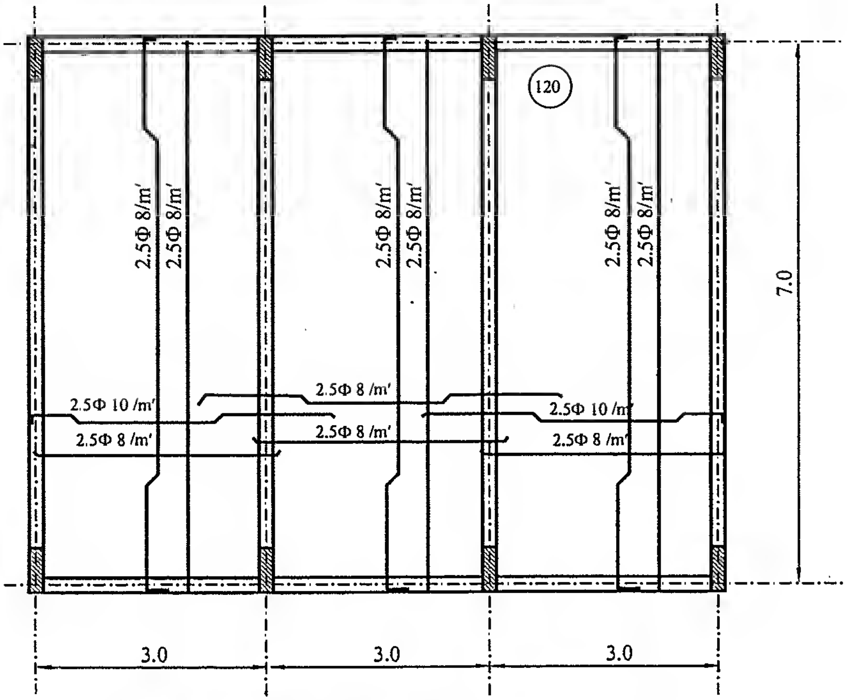
$$R = \frac{M_u}{f_{cu} b d^2} = \frac{7.71 \times 10^6}{30 \times 1000 \times 100^2} = 0.0257$$

From the curve, ω=0.030

$$A_s = \omega \frac{f_{cu}}{f_y} b \times d = 0.030 \frac{30}{400} \times 1000 \times 100 = 229\ mm^2 > A_{s,min}$$

use (5 Φ 8/m')   A$_{s,chosen}$=250 mm^2

Because the slabs are one-way in the x-direction, the moment in the y-direction equals to zero. However, a secondary reinforcement mesh with cross sectional area of at least 25% of the main steel should be provided.
Use (5 Φ 8 /m').



Reinforcement details

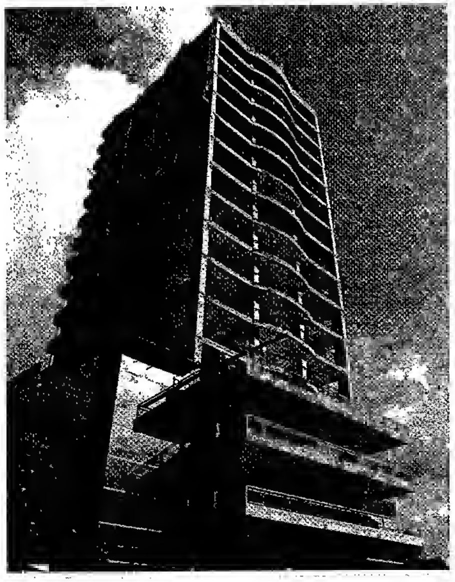


Photo 1.2 Cantilever solid slab in a hotel building.

1.3 Two-Way Slabs

1.3.1 Definition

To classify a slab as a two-way slab, the length of the long side should be less than twice the length of the short side. The short direction is considered the main direction because most of the load is transferred in this direction. The main reinforcement is arranged in the short direction and the secondary reinforcement is arranged in the long direction. Two-way slabs are those that bend in double curvature as shown in Fig. 1.9, and thus require steel reinforcement in two directions to prevent excessive cracking and to limit deflections. The reinforcement is normally positioned parallel to the side of the slab in both directions. The position of the reinforcement is determined by the curvature of the slab. The top steel is placed in the negative curvature areas and the bottom steel in the positive curvature areas.

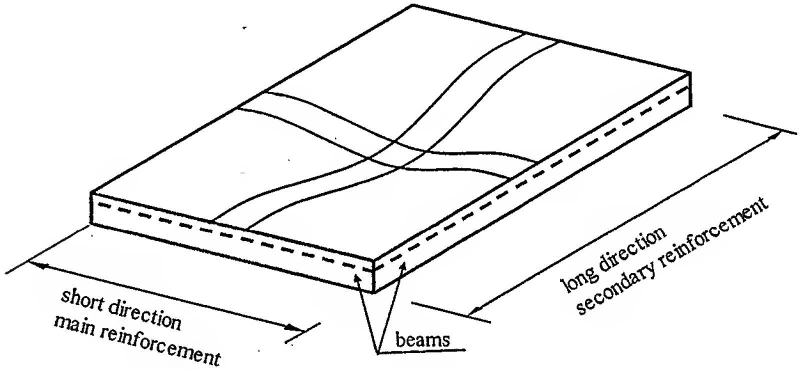


Fig. 1.9 Main and secondary reinforcement in two-way slabs

1.3.2 Elastic Analysis of Plates

Textbooks dealing with the theory of elasticity such as the one by Timoshenko and Krieger contain exact solutions for two-way slabs. The fundamental assumption used in the analysis of plates under pure bending is that the deflection of the plate (*z*) is small in comparison to its thickness *t*. The basic differential equation for the uniformly loaded-simply supported rectangular plate shown in Fig. 1.10 is given by

$$\frac{\partial^4 z}{\partial x^4}+2\frac{\partial^4 z}{\partial x^2\,\partial y^2}+\frac{\partial^4 z}{\partial y^4}=\frac{w}{D} \quad \text{..(1.5)}$$

where z is the deflection of the plate, w is the uniform load and D is the flexural rigidity of the plate (similar to EI in beams) and is given by

$$D=\frac{E\,t^3}{12(1-\nu^2)} \quad \text{..(1.6)}$$

where E is the modulus of elasticity of the plate, t is the plate thickness and ν is Poisson's ratio.

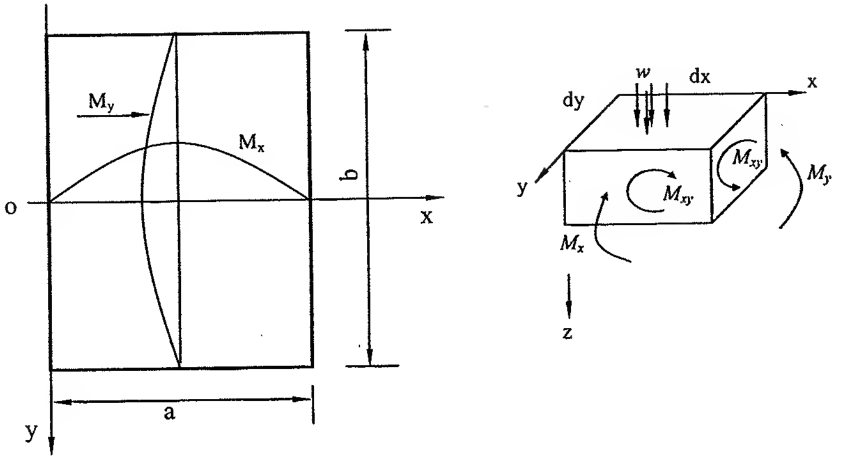


Fig. 1.10 Elastic analysis of plates

Solving the previous differential equation gives the deflection of the plate. The solution must satisfy the conditions at the boundaries of the plate. For example, for a simply supported plate the deflection z along the edges must equal to zero (z=0 and M_x=0 @ x=0 and x=a). Lévy presents one of the famous solutions for this problem in 1899 in the form of a series of sin curves as follows:

$$z = \sum_{m=1}^{\infty} Y_m \sin \frac{m\pi x}{a} \quad \ldots\ldots(1.7)$$

where Y_m is function of y only and determined to satisfy the boundary condition. Having determined the deflection equation, the developed bending moments in the plate can be obtained using the following relations:

$$M_x = -D\left(\frac{\partial^2 z}{\partial x^2} + \nu \frac{\partial^2 z}{\partial y^2}\right) \quad \ldots\ldots(1.8)$$

$$M_y = -D\left(\frac{\partial^2 z}{\partial y^2} + \nu \frac{\partial^2 z}{\partial x^2}\right) \quad \ldots\ldots(1.9)$$

$$M_{xy} = -M_{yx} = D\,(1-\nu)\frac{\partial^2 z}{\partial x\,\partial y} \quad \ldots\ldots(1.10)$$

where $\partial^2 z/\partial x^2$ is the curvature of the slab in x direction, and $\partial^2 z/\partial y^2$ is the curvature in y direction. A positive curvature corresponds to a curve that is concave downwards. The magnitude of the moment is proportional to curvature. Equation 1.5 can be written in the following form:

$$\frac{\partial^2 M_x}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial x\ \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -w \quad \ldots\ldots(1.11)$$

This form indicates that in plates the load *w* results in:

- Bending moments in strips running in the x-direction (M_x)
- Bending moments in strips running in the y-direction (M_y)
- Torsional moments (M_{xy})

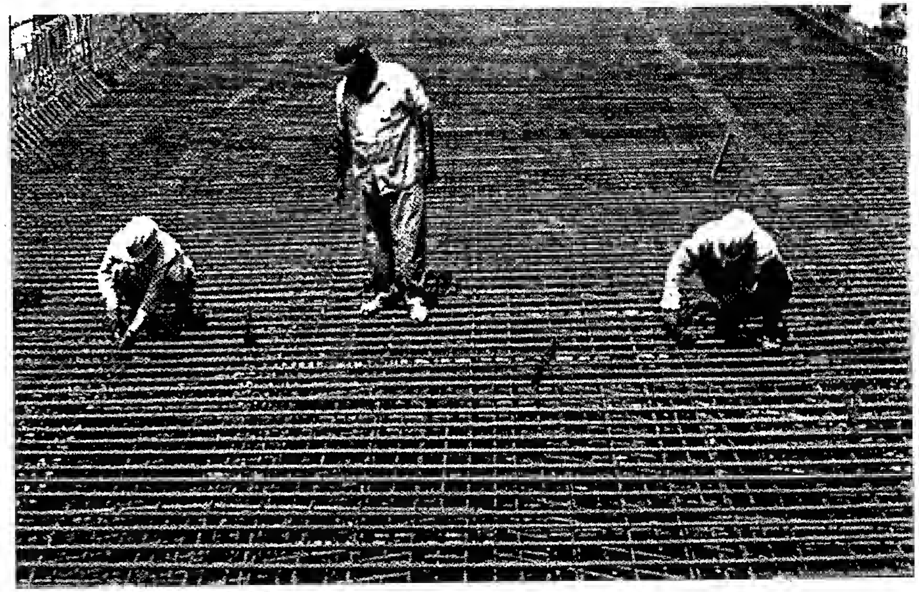


Photo 1.3 Preparation of reinforcement of solid slabs

1.3.3 Load Distribution Factors According to ECP 203

As has been shown in 1.3.2, the exact analysis of two-way slabs is complicated and involves many mathematical computations. However, for simplicity a strip of 1.0 m width can be analyzed in each direction as a wide beam. The bending moments resulting from the analysis of such a strip is quite different from those obtained using the exact plate analysis mentioned in section 1.3.2. This is attributed to the disregard of the torsional moment represented by the term $\partial^2 z/\partial x \partial y$. For example, for a uniformly loaded-simply supported square slab, the load transferred in x direction equals the load transferred in y direction (=*w/2*). Thus, the maximum bending moments developed in a strip of 1.0 m width of the slab is equal

$$M_{\max} = \frac{(w/2)\times a^2}{8} = 0.5\times\frac{w\times a^2}{8} \quad \text{.................................(1.12)}$$

However, according to the theory of plates the maximum bending for square plate (ν=0.2, Poisson's ratio for concrete) is only equal to

$$M_{\max,theo} = 0.044\,wa^2 = 0.35\times\frac{w\times a^2}{8} \quad \text{.............................(1.13)}$$

The difference is quite large (%30) and attributed to the torsional moments developed in the plate. As the rectangularity ratio increases the load transferred in the short direction increases and the load transferred in the long direction decreases. The theoretical analysis of the plates is the basis of the values adopted in the Egyptian code in which the bending moment of a simply supported slab equals

$$M_a = \frac{(w\ \alpha)a^2}{8} \quad (short\ \ direction)$$

$$\text{............................(1.14)}$$

$$M_b = \frac{(w\ \beta)b^2}{8} \quad (long\ \ direction)$$

where α and β represent the percentage of loads causing bending moments in each direction.

Comparing Eq. 1.13 to Eq. 1.14 gives α=0.35 for square plates, which is identical to the value adopted by the Egyptian code. Table 1.2 shows the ECP 203 values for α and β for different plate rectangularity ratios. The slab load w_u transferred in the short direction is denoted as w_α and the load transferred in the longitudinal direction is denoted as w_β as shown in Fig. 1.11, where

$$w_\alpha = \alpha\cdot w_u \quad \text{...(1.15.a)}$$

$$w_\beta = \beta\cdot w_u \quad \text{...(1.15.b)}$$

The distribution factors α and β can be also represented by following set of equations

$$\alpha = \frac{r}{2} - 0.15 \quad \text{...(1.16.a)}$$

$$\beta = \frac{0.35}{r^2} \quad \text{...(1.16.b)}$$

in which r is the rectangularity ratio given by Eq. 1.17

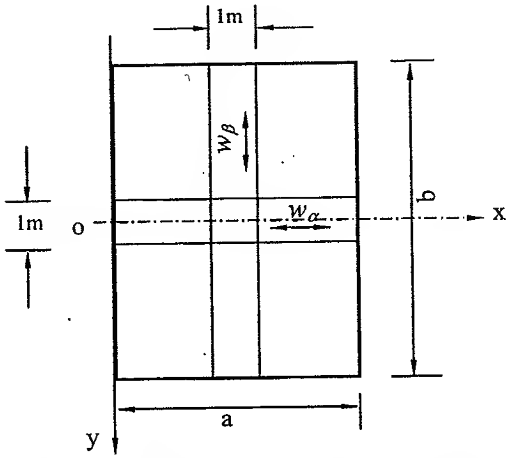


Fig. 1.11 Load distribution for two-way slabs

Table 1.2 Values of α and β for solid slab with live loads less than 5 kN/m²

r	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
α	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
β	0.35	0.29	0.25	0.21	0.18	0.16	0.14	0.12	0.11	0.09	0.08

The load transferred in each direction is affected by the continuity condition of the adjacent slabs. This was taken into consideration when determining the rectangularity ratio of slabs by using the coefficient *m*. The value of *m* depends on the support condition and is defined as the ratio between the two inflection points to the effective span. This coefficient is applied in each direction (a, b) to determine the effective rectangularity ratio *r* as follows

$$r = \frac{m_b\times b}{m_a\times a} \quad \text{...(1.17)}$$

where

$$m_a\ or\ m_b = \begin{cases} 1.0 & simple\ span \\ 0.87 & continuous\ from\ one\ end \\ 0.76 & continuous\ from\ two\ ends \end{cases} \quad \text{........................(1.18)}$$

The use of the values listed in Table 1.2 is limited to slabs with live loads not more than 5 kN/m^2. If the live loads exceed this limit, the value of α and β should be taken from Table 2.4 in which the torsional moment is neglected.

Two-way slabs designed according to the previously mentioned procedure have proven over the years a very satisfactory performance under service loads. This is attributed to the stress redistribution at higher loads. As a result, the design of two-way slabs is generally based on empirical moment coefficients even for unequal spans, which might not give actual prediction of the stresses but give the total amount of reinforcement. That is why they say, *"the total amount of reinforcement in a slab seems more important than the exact distribution"*. The designer can clearly establish his design on numerical analysis or on a finite element method as long as they meet all the ECP 203 requirements for deflection, cracking and reinforcement.

1.3.4 Minimum Thickness

To control cracking, deflection and to ensure good performance for two-way slab, the code states that the minimum thickness t_{min} should be

$$t_{min} = \begin{cases} a/35 & \text{simple span} \\ a/40 & \text{continuous from one end} \\ a/45 & \text{continuous from two ends} \end{cases} \qquad \text{.................................(1.19)}$$

where a is the short effective span

Although thin reinforced concrete two-way slabs have high flexural resistance, their deflections are often large. From the deflection point of view, the thickness is considered acceptable if it is greater than t calculated using the following equation

$$t = \frac{a\left(0.85+\dfrac{f_y}{1600}\right)}{15+20/(b/a)+10\,\beta_p} \geq 100\ mm \qquad \text{...............................(1.20)}$$

Where a is the span in the short direction, b is the span in the long direction, β_p is the ratio between the perimeter of the continuous edges to the total perimeter, and f_y is the steel yield strength N/mm^2. It is clear that the slab thickness required for deflection considerations is higher in case of square slabs with large spans. For example, a slab reinforced with high-grade steel and continuous from all edges with dimensions of (5mx5m) requires 122 mm thickness, while a slab with dimensions of (6m x 6m) requires 146 mm.

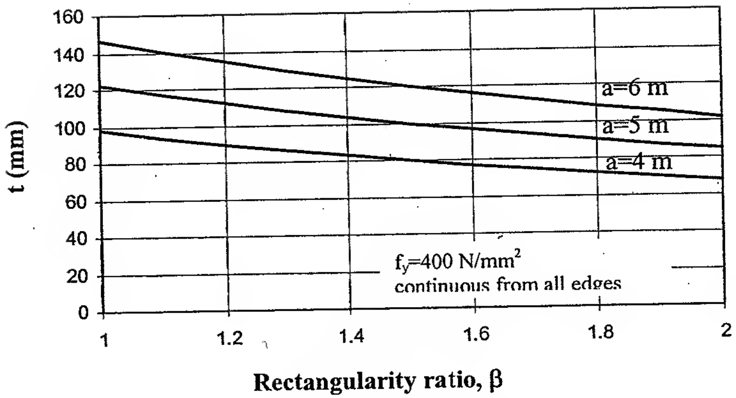


Fig. 1.12 Minimum thickness of 2-way slabs for deflection control

1.3.5 Related Code Provisions

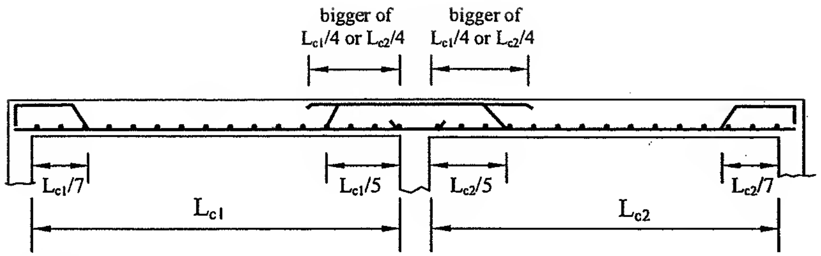
The minimum area of steel for one and two-way slabs is given by

$$A_{s,min} = \frac{0.6}{f_y}\, b\ d \qquad \text{for all types of steel ....(1.21a)}$$

Or

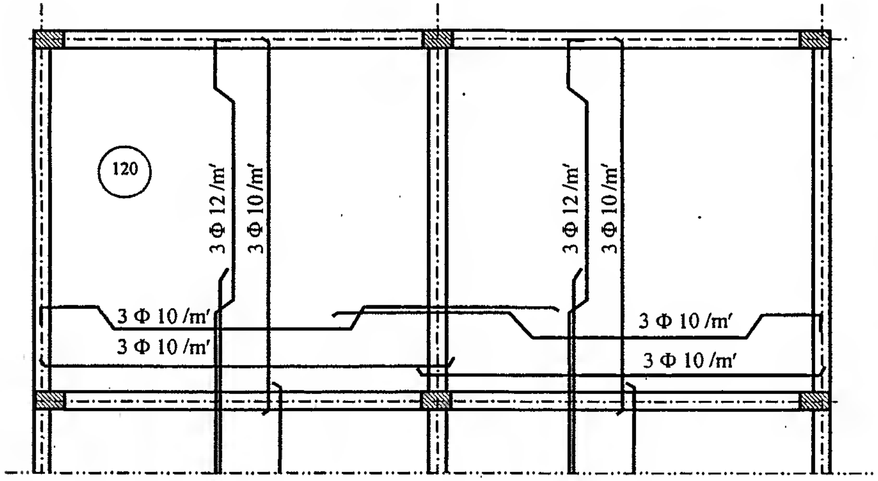
$$A_{s,min} = \begin{cases} \dfrac{0.25}{100} b \times d & \text{for mild steel } (f_y = 240 \text{ and } 280\ \text{N/mm}^2) \\ \\ \dfrac{0.15}{100} b \times d & \text{for high grade steel } (f_y = 360 \text{ and } 400\ \text{N/mm}^2) \end{cases} \qquad \text{.............(1.21b)}$$

- At least one third of the reinforcement must extend from the support to the support.
- The maximum distance between bars is 200 mm
- The area of the secondary steel mesh should be at least 20% of the main area of steel with a minimum of 4 bars per meter.
- The minimum bar diameter is 6 mm for straight bars and 8mm for bent bars.
- Slabs with a thickness of more than 160 mm should be reinforced with top steel mesh not less than 20% of the main steel with a minimum of 5ϕ8/m' for mild steel and 5ϕ6/m' for high grade steel.
- Under normal conditions and for spans that do not differ more than 20%, half of the reinforcement can be bent at the fifth of the clear span and extends to the adjacent span one fourth the bigger of the two spans as shown in Fig. 1.13.



A. Minimum code requirement for bar extension

Fig. 1.13a Reinforcement details for solid slabs

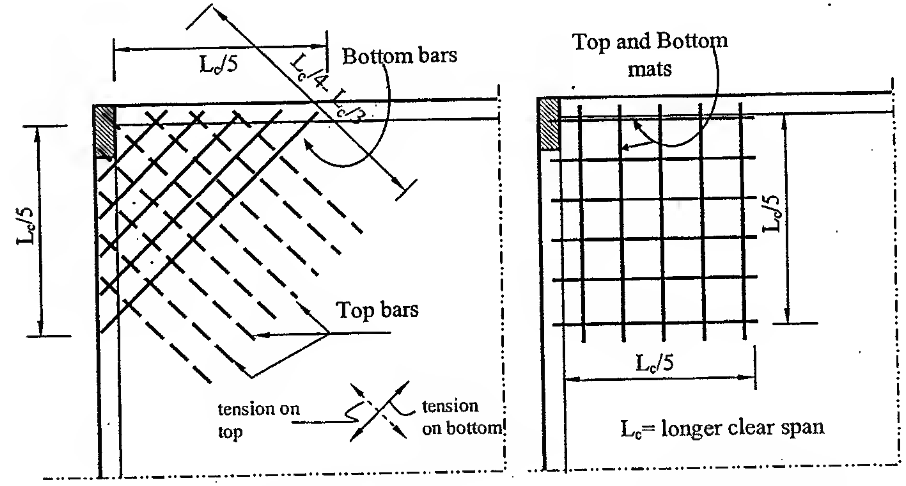


B. Example of reinforcement for two-way slabs

Fig. 1.13b Reinforcement details for solid slabs

1.3.6 Corner Reinforcement

Twisting moments are developed at the corners of exterior two-way slabs. The magnitude of these moments is usually small for slabs spanning less than 4 to 5 meters. However, for bigger spans (>5 ms) these moments tend to crack the slab. Special reinforcement should be added to control cracking and to resist the torsional moments. Bottom bars are placed perpendicular to the slab diagonal while top bars are placed in the direction of the diagonal as shown in Fig. 1.14.a. Alternatively, the diagonal reinforcement can be replaced by top and bottom mats as shown in Fig. 1.14.b. The amount of this reinforcement can be taken as the same area and the spacing as the main positive reinforcement (per meter). The reinforcement should extend about one fifth of the clear span in either direction as shown in Fig. 1.14.



a- using skewed bars

b-using top and bottom

Fig. 1.14 Corner reinforcement at an exterior two-way slab

Example 1.2

A reinforced concrete floor is to be constructed as shown in Fig. EX 1.2. Beams with a cross section of (250 mm x 600 mm) are provided on all column lines. The floor is to be designed to carry a live load of 3 kN/m^2 and a flooring material of 2 kN/m^2 using concrete strength of 25 N/mm^2 and reinforcing steel having a yield strength of 360 N/mm^2. Calculate and draw the reinforcement required for the floor.

Solution

Step 1: Estimate the thickness of the slab

The plan consists of one way, two way and cantilever slabs. Assume that the slab thickness t_s is 120 mm for all slabs of the floor.

For one way slab

$$t_{\min} = \begin{cases} L/30 & simple span \\ L/35 & continuous\ from\ one\ end \\ L/40 & continuous\ from\ two\ ends \end{cases}$$

Since slabs S_6 and S_8 are one-way slabs continuous from two ends, the minimum thickness is given by

$$t_{\min} = \frac{L}{40} = \frac{2.4 \times 1000}{40} = 60\ mm (< t_s \ldots\text{o.k})$$

Also for deflection calculations to be ignored the thickness should be at least:

$$\xi = 0.40 + \frac{f_y}{650} = 0.40 + \frac{360}{650} = 0.95$$

$$t_{\min} = \frac{L_n}{(36/\xi)} = \frac{(2.4 - 0.25) \times 1000}{(36/0.95)} = 56.73\ mm\ (< t_s \ldots\text{o.k})$$

For two way slabs
The biggest slab is S10 (4.8x 5.4 m)

$$t_{\min} = \begin{cases} a^*/35 & simple\ span \\ a^*/40 & continuous\ from\ one\ end \\ a^*/45 & continuous\ from\ two\ ends \end{cases}$$

a* is the short span

This span is considered continuous from one end thus

$$t_{\min} = \frac{a}{40} = \frac{4.8 \times 1000}{40} = 120\ mm\ (= t_s \ldots\text{o.k})$$

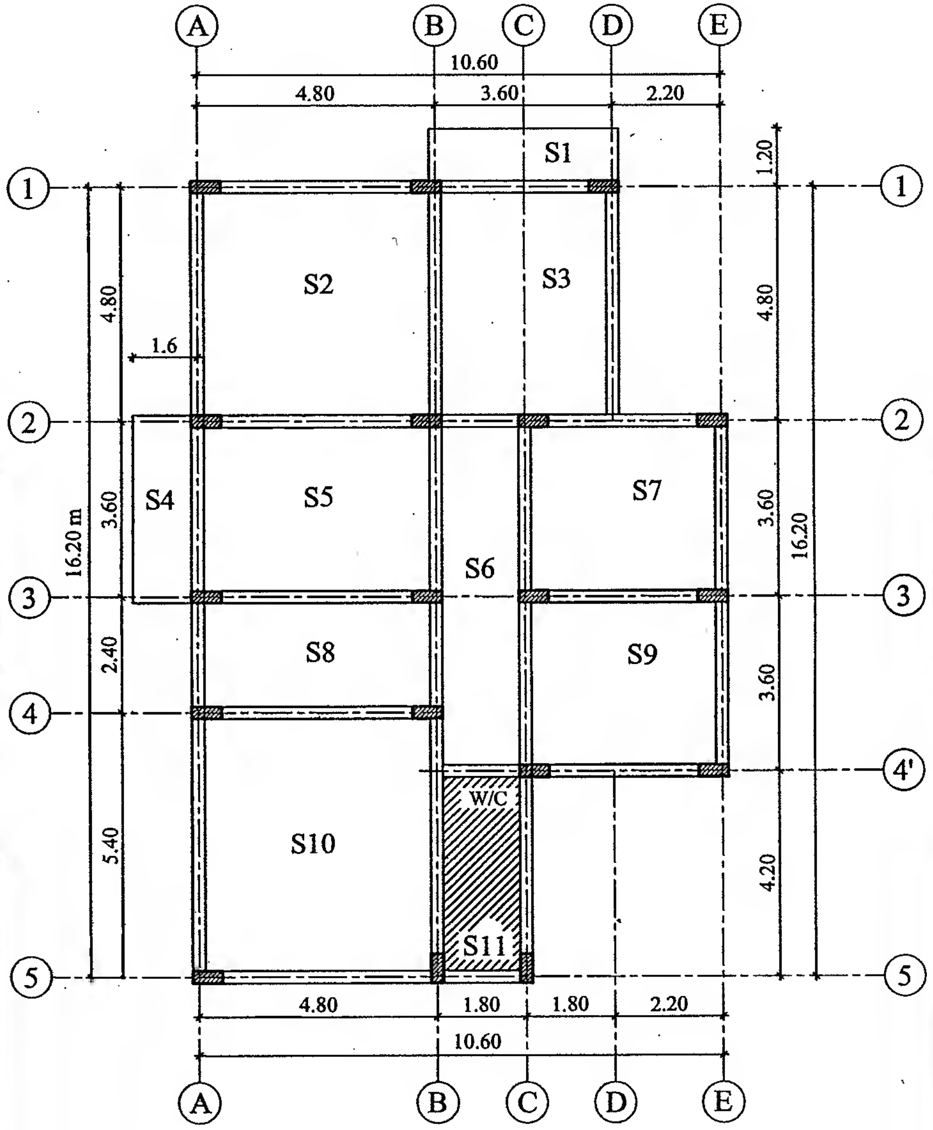


Fig. EX 1.2 Structural Plan

For cantilever slabs

There is no direct recommendation for the minimum thickness for cantilever slabs except for deflection calculations. The minimum thickness for cantilever slabs reinforced with high grade steel (f_y=360 N/mm^2) equals

$$t_{\min} = \frac{L}{10} = \frac{1.6\times1000}{10} = 160\ mm > t_s$$

From economic point of view, it is better to use thickness of 120 mm and to check the deflection.

Step 2: Effective span

Since the width of all beams are 250 mm, the clear span equals

L_{clear} =L-0.25

where L is the centerline to centerline distance

For interior span, the effective length equals to

$$L_{eff} = \min\begin{cases}\text{max of}\begin{cases}L_{clear}+t_s\\1.05\times L_{clear}\end{cases}\\ \text{CL to CL}\end{cases} = \min\begin{cases}\text{max of}\begin{cases}(L-0.25)+0.12\\1.05\times(L-0.25)\end{cases}\\ L\end{cases}$$

For cantilever slabs

$$L_{eff} = \min\begin{cases}L_{clear}+t_s\\ \text{edge to CL}\end{cases} = \min\begin{cases}(L-0.125)+0.12\\ L\end{cases}$$

Slab	L_x	$L_{clear,x}$	$L_{clear,x}$ $+t_s$	1.05 x $L_{clear,x}$	$L_{eff,x}$	Ly	$L_{clear,y}$	$L_{clear,y}$ $+t_s$	1.05 x $L_{clear,y}$	$L_{eff,y}$
S1	3.60	-	-	-	3.60	1.20	1.08	1.20	1.13	1.20
S2	4.80	4.55	4.67	4.78	4.78	4.80	4.55	4.67	4.78	4.78
S3	3.60	3.35	3.47	3.52	3.52	4.80	4.55	4.67	4.78	4.78
S4	1.60	1.48	1.60	1.55	1.60	3.60	-	-	-	3.60
S5	4.80	4.55	4.67	4.78	4.78	3.60	3.35	3.47	3.52	3.52
S6	1.80	1.55	1.67	1.63	1.67	7.20	6.95	7.07	7.30	7.20
S7	4.00	3.75	3.87	3.94	3.94	3.60	3.35	3.47	3.52	3.52
S8	4.80	4.55	4.67	4.78	4.78	2.40	2.15	2.27	2.26	2.27
S9	4.00	3.75	3.87	3.94	3.94	3.60	3.35	3.47	3.52	3.52
S10	4.80	4.55	4.67	4.78	4.78	5.40	5.15	5.27	5.41	5.40
S11	1.80	1.55	1.67	1.63	1.67	4.20	3.95	4.07	4.15	4.15

Step 3: Calculation of loads

w_u =1.4 g_s +1.6 p_s= 1.4 (25 x t_s + flooring) + 1.6 x LL

$$w_u = 1.4\times(25\times0.120+2.0)+1.6\times3 = 11.8\ kN/m^2$$

Taking a strip of 1m, the load acting on this strip equals to

$w_{su} = w_u \times 1 = 11.80\,kN/m'$ and b=1000 mm

For one-way and cantilever slabs

w_α=w_{su} and w_β=0

For two-way slabs

The load distribution factors are determined using the rectangularity ratio r

$$r = \frac{m_b \times b\,(bigger)}{m_a \times a\,(smaller)}$$

where b and a are the effective spans and

$$m_a\ or\ m_b = \begin{cases}1.0 & simple\,span\\0.87 & continuous\ from\ one\ end\\0.76 & continuous\ from\ two\ ends\end{cases}$$

The values of α and β are obtained from Table 1.2

$w_\alpha = \alpha \,.\, w_{su}$

$w_\beta = \beta \,.\, w_{su}$

Please note that w_α is the load transferred in the short direction (not x-direction) and w_β is the load transferred in the long direction. For example for slab S8 w_α runs in y-direction and w_β in x-direction,, while for slab S10 w_α runs in x-direction and w_β in y-direction.

Slab	$L_{eff,x}$	m_x	$L_{eff,y}$	m_y	r	α	β	w_α	w_β
S1	3.60	1	1.20	0.87	3.46	1.00	0.00	11.80	0.00
S2	4.78	0.87	4.78	0.87	1.00	0.35	0.35	4.13	4.13
S3	3.52	0.87	4.78	0.87*	1.36	0.53	0.19	6.24	2.24
S4	1.60	0.87	3.60	1	2.59	1.00	0.00	11.80	0.00
S5	4.78	0.76	3.52	0.76	1.36	0.53	0.19	6.24	2.24
S6	1.67	0.76	7.20	0.87	4.94	1.00	0.00	11.80	0.00
S7	3.94	0.87	3.52	0.87	1.12	0.41	0.28	4.83	3.30
S8	4.78	0.87	2.27	0.76	2.41	1.00	0.00	11.80	0.00
S9	3.94	0.87	3.52	0.87	1.12	0.41	0.28	4.83	3.30
S10	4.78	1	5.40	0.87	1.02	0.36	0.34	4.23	3.99
S11	1.67	1	4.15	1	2.48	1.00	0.00	11.80	0.00

* In Y- direction, slab S3 is not continuous with S6 (one way slab in X- direction).

Bending moment and reinforcement

The floor slab considered in this example has panels that contain variation in loads and spans of more than 20%. Exact analysis of such a floor is lengthy and quite tedious. Due to the fact that R/C slabs have an ability to redistribute the moments, it has become a common practice to extend the use of the moment coefficients mentioned in 1.2.5 to most types of floors.

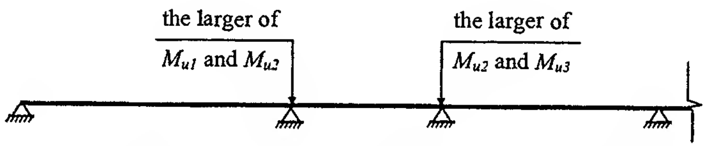
The procedure followed to obtain the design moments in the floor of this example:

1. The positive moment in any span equals to

$$M_u = \frac{w_u \times L_{eff}^2}{k}$$

2. The negative moment developed at the support connecting two adjacent unequal spans is conservatively taken as the larger of M_{u1} and M_{u2} given by

$$M_{u1} = \frac{w_{u1} \times L_1^2}{k_1} \quad , \quad M_{u2} = \frac{w_{u2} \times L_2^2}{k_2}$$



$$M_{u1} = \frac{w_{u1} \times L_1^2}{10} \qquad M_{u2} = \frac{w_{u2} \times L_2^2}{12} \qquad M_{u3} = \frac{w_{u3} \times L_3^2}{12}$$

For cantilever slabs and assuming that the hand rail weight is 1.5 kN/m′, the bending moment equals to

$$M_u = \frac{w_u \times L_{eff}^2}{2} + 1.5 \times L_{eff}$$

The effective depth equals

d = t_s-15 = 105 mm   in the main direction (α)

d= t_s-25 = 95 mm   in the secondary direction (β)

$$R = \frac{M_u \times 10^6}{f_{cu}\ b\ d^2} = \frac{M_u \times 10^6}{25 \times 1000 \times d^2} = \frac{M_u \times 40}{d^2}$$

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = \omega \times \frac{25}{360} \times 1000 \times d \ = 69.44 \times \omega \times d \ > A_{s,\min}$$

$$A_{s,\min} = \frac{0.6}{f_y} b\, d = \frac{0.6}{360} \times 1000 \times 105 = 175\ mm^2$$

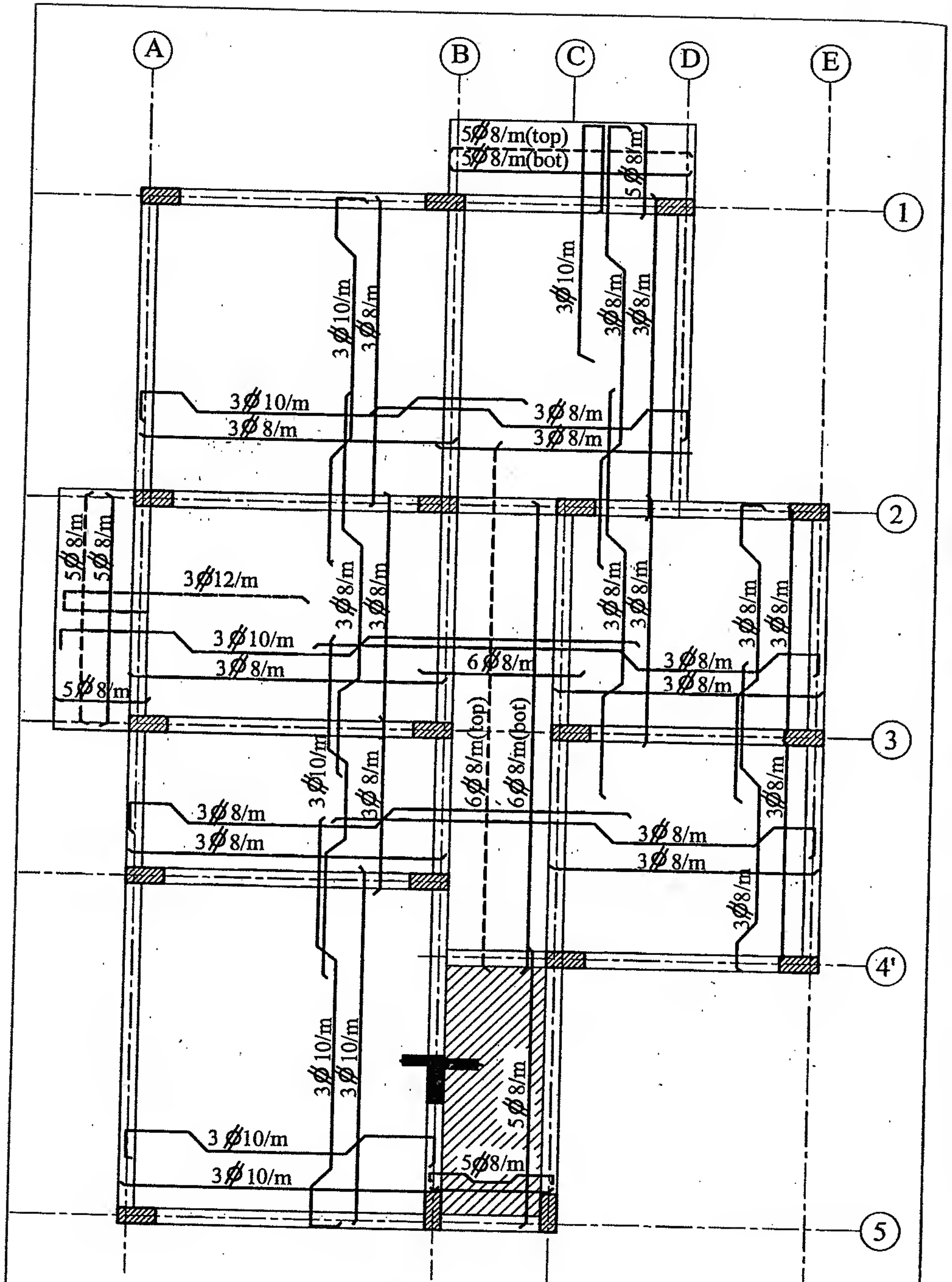
Design of Slabs in X-Direction

Slab	$L_{eff,x}$ (m)	w_{ux} (kN/m′)	k	M_{ux} (kN.m)	d mm	R	ω	A_s required	A_s chosen
S1	3.60	0.00	0	0.00	95	0.0000	0.000	-	5 Φ 8/m′
S2	4.78	4.13	10	9.43	105	0.0342	0.041	299	3 Φ 8/m′+ 3 Φ 10/m′
S3	3.52	6.24	10	7.72	105	0.0280	0.033	243	6 Φ 8/m′
S4	1.60	11.80	2	17.40	105	0.0631	0.079	573	3 Φ 10/m′+ 3 Φ 12/m′
S5	4.78	2.24	10	5.11	95	0.0226	0.027	176	3 Φ 8/m′+ 3 Φ 10/m′
S6	1.67	11.80	12	2.74	105	0.0099	0.012	175	3 Φ 8/m′+ 3 Φ 10/m′
S7	3.94	3.30	10	5.11	95	0.0226	0.027	176	6 Φ 8/m′
S8	4.78	0.00	0	0.00	95	0.0000	0.000	175	6 Φ 8/m′
S9	3.94	3.30	10	5.11	95	0.0226	0.027	176	6 Φ 8/m′
S10	4.78	3.99	8	11.39	95	0.0505	0.062	408	6 Φ 10/m′
S11	1.67	11.80	8	4.11	105	0.0149	0.017	175	5 Φ 8/m′

Design of Slabs in Y-Direction

Slab	$L_{eff,y}$ (m)	w_{uy} (kN/m′)	k	M_{uy} (kN.m)	d mm	R	ω	A_s required	A_s chosen
S1	1.20	11.80	2	10.3*	105	0.0374	0.045	325	3 Φ 8/m′+ 3 Φ 10/m′
S2	4.78	4.13	10	9.43	95	0.0418	0.051	333	3 Φ 8/m′+ 3 Φ 10/m′
S3	4.78	2.24	10	5.11	95	0.0226	0.027	176	6 Φ 8/m′
S4	3.60	0.00	-	0.00	95	0.0000	0.000	175	5 Φ 8/m′
S5	3.52	6.24	12	6.44	105	0.0234	0.028	201	6 Φ 8/m′
S6	7.20	0.00	-	0.00	95	0.0000	0.000	175	5 Φ 8/m′
S7	3.52	4.83	10	5.98	105	0.0217	0.026	187	6 Φ 8/m′
S8	2.27	11.80	12	5.07	105	0.0184	0.022	175	3 Φ 8/m′+ 3 Φ 10/m′
S9	3.52	4.83	10	5.98	105	0.0217	0.026	187	6 Φ 8/m′
S10	5.40	4.23	10	12.33	105	0.0447	0.054	396	6 Φ 10/m′
S11	4.15	0.00	-	0.00	95	0.0000	0.000	175	5 Φ 8/m′

$^*M_u = 11.8 \times 1.2^2 / 2 + 1.5 \times 1.2$



Reinforcement Details

2

Hollow Block Slabs

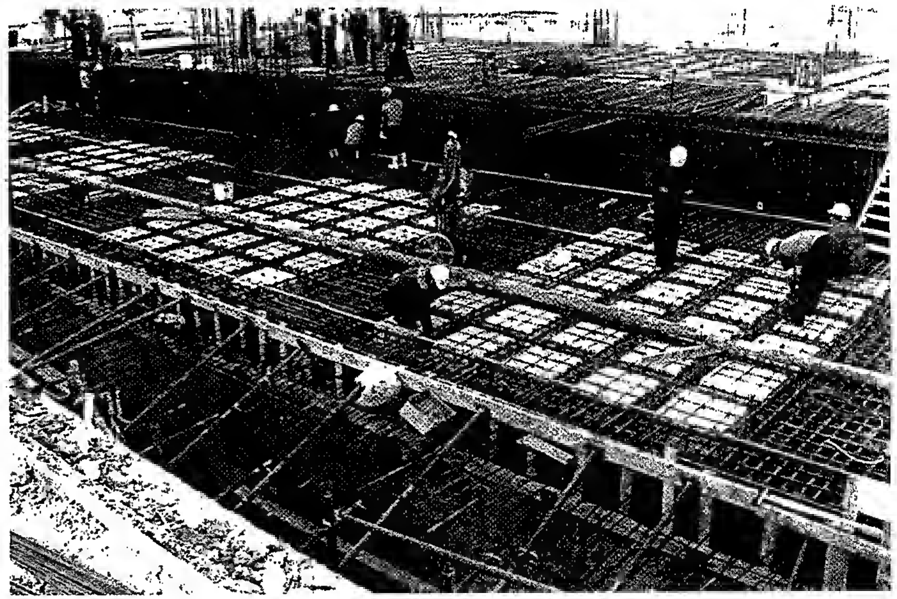


Photo 2.1 Reinforcement placement in ribbed slab during construction

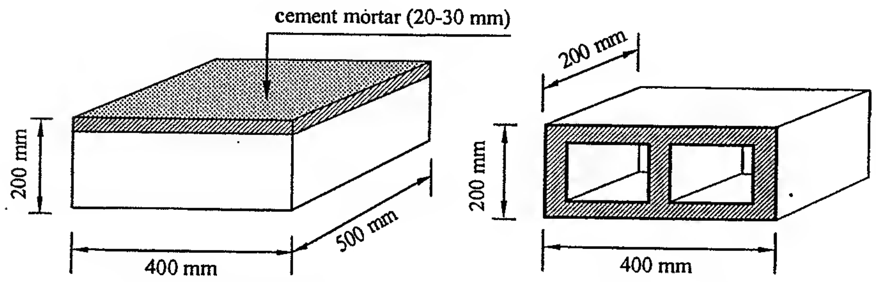
2.1 Introduction

Hollow block floors are formed typically using blocks made of concrete with lightweight aggregate. The void in the blocks reduces the total weight of the slab significantly.

The main advantage of using hollow blocks is the reduction in weight by removing the part of the concrete below the neutral axis. An additional advantage is the ease of construction, especially if the floor is designed with no projected beams (*hidden beams)*. Hollow block floors proved economic for spans of more than 5m with light or moderate live loads, such as hospitals, office or residential buildings. They are not suitable for structures having heavy live loads such as warehouses or parking garages.

The blocks do not contribute to the strength of the slab; as a matter of fact it is an additional weight on the slab. Thus, in recent years these blocks were made of polystyrene which is 1/15 of the weight of concrete blocks as shown in Fig. 2.1. Thus, a reduction in the reinforcement can be achieved. The values listed in Table 2.1 include the weight of the concrete ribs and 50mm top concrete slab. If for any reason the thickness of the top flange was increased more than 50 mm, the additional weight should be included in the calculations. The total ultimate load for per square meter w_{su} is given by

$$w_u = 1.4\times[\textit{weight of blocks} / m^2(\textit{Table } 2.1)+\textit{flooring}]+1.6\,w_{LL} \quad .... \quad (2.1)$$



Foam block, w=0.16 kN/m^2
(polystyrene)

Concrete block, w=2.7 kN/m^2
(without concrete slab weight)

Fig. 2.1 Weight comparison between concrete and foam blocks.

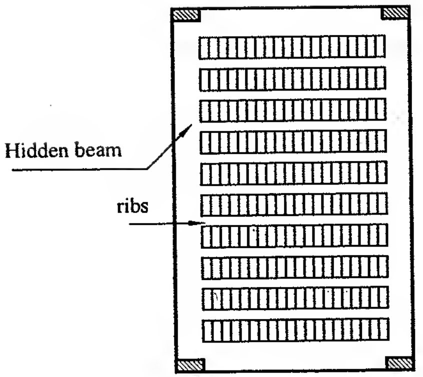
Table 2.1 Weight of material used in hollow block floors* (kN/m^2)

Concrete blocks 400x200x150		Concrete blocks 400x200x200		Concrete blocks 400x200x250		Foam blocks 500x400x200	
One way	Two-way	One-way	Two-way	One-way	Two-way	One-way	Two-way
3.03	3.36	3.30	3.80	4.10	4.78	0.70	1.20

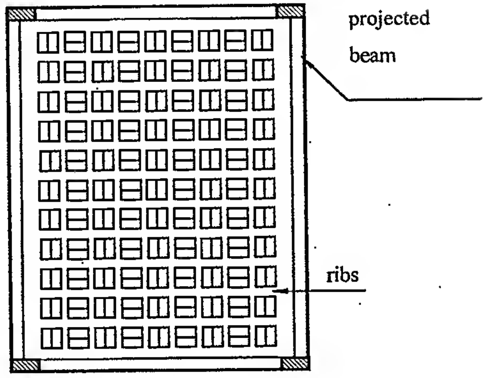
* include the weight of the concrete ribs and 50mm top concrete slab

Hollow block slabs are classified into: a)one-way hollow block slabs or b)two-way hollow block slabs depending on the arrangement of the ribs on plan. Fig. 2.2.A shows a one-way hollow block slab in which the ribs are arranged in one direction. Fig. 2.2.B on the other hand, shows a two-way hollow block slabs in which the ribs are arranged in two directions

To avoid shear failure, the blocks are terminated near the support and replaced by solid parts. Solid parts are also made under partitions, brick walls and concentrated loads.



A. One-way hollow blocks with hidden beams



B. Two-way hollow blocks with projected beams

Fig. 2.2 One and two-way hollow block slabs

2.2 One-Way Hollow Block Slabs

2.2.1 General

One-way hollow blocks are used frequently in construction even for slabs with a rectangularity ratio less than 2. This is attributed to the ease of placing the blocks in one direction. The arrangement of the ribs controls the direction of the slab regardless of its rectangularity ratio. The ribs are positioned in the shorter direction, thus all the loads are transferred in this direction as shown in Fig. 2.2.A

2.2.2 Arrangement of blocks

The number of blocks in each direction must be specified on the construction drawings. Thus, the layout of the blocks must be positioned so that enough solid parts are present near the supporting beams. For floors with hidden beams, the solid part must be wide enough to carry all the applied loads. The normal width of the solid part ranges between 0.8-2.0 for floors with hidden beams and ranges between 0.2-0.5 for floors with projected beams.

Having determined the dimension of the solid parts, the clear length of the blocks can be attained as shown in Fig. 2.3. For example, in the case of using 400×200×200 blocks in a one-way slab, the clear distance in the rib direction equals to

$$L_{c1} = 200 \times n_1 + 100 \times n_{cr}$$

where n_1 is the number of blocks in rib direction and n_{cr} is the number of cross ribs.

The clear distance in the perpendicular direction equals

$$L_{c2} = 400 \times n_2 + 100\ (n_2 - 1)$$

$$L_{c2} = 500 \times n_2 - 100$$

Where n_2 is the number of blocks perpendicular to the ribs. The number of ribs=n_2-1

Photo 2.2 One-way hollow blocks during construction

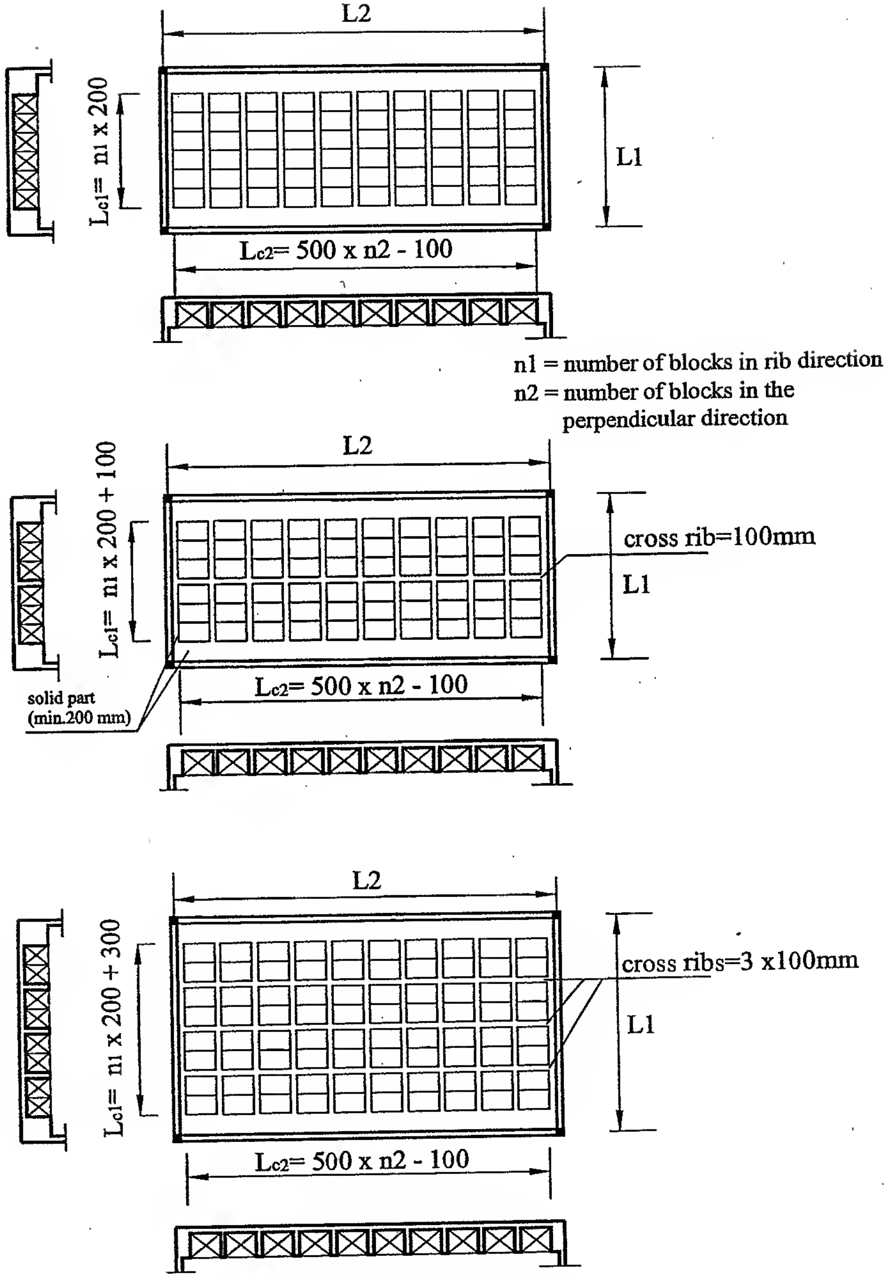
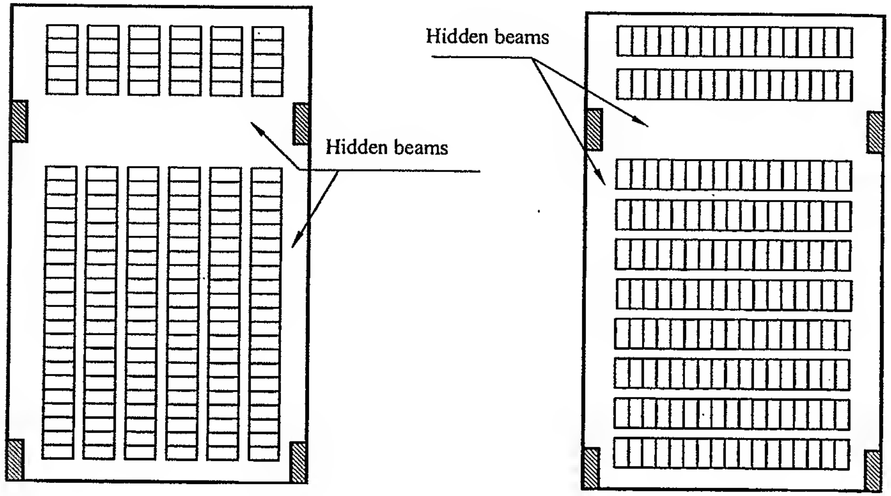


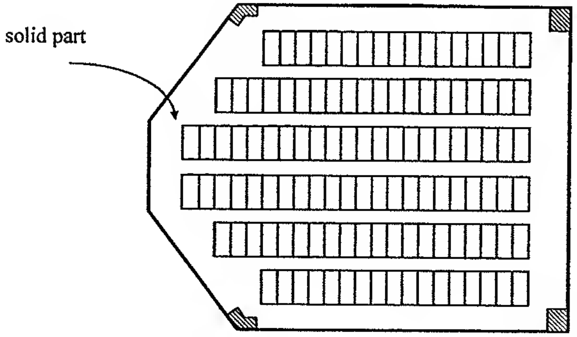
Fig. 2.3 Arrangement of blocks and cross ribs in one-way slabs

Hollow block slabs can be used to form a cantilever slab as shown in Fig. 2.4. The ribs can be either made continuous as in Fig. 2.4.A, or simply supported on two hidden beams as shown in Fig. 2.4.B. For slabs with irregular shapes, the blocks can be arranged along the perimeter as shown in Fig 2.4.C.



A. Continuous ribs spanning in the long direction

B. Ribs spanning as simply supported on hidden beams.



C. Ribs arrangement in irregular plans

Fig. 2.4 Hollow blocks arrangement in cantilever and irregular slabs.

2.2.3 Code Provisions

- The thickness of the slab (t_s) should not be less than 50 mm or the 1/10 of the clear spacing between blocks (e/10) as shown in Fig. 2.5.
- The maximum clear distance (e) is 700 mm
- The minimum rib width (b) is 100mm or 1/3 the total slab thickness (t/3)

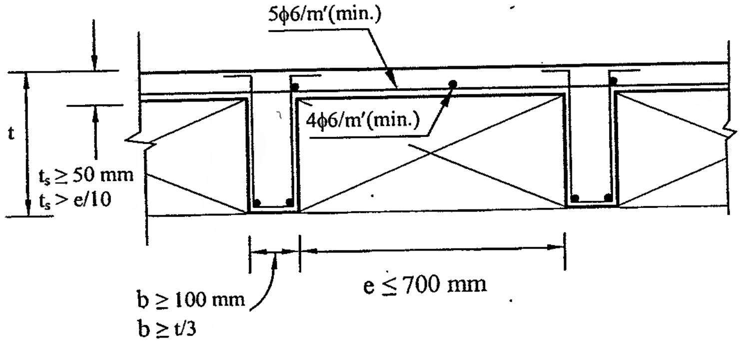


Fig. 2.5 Minimum dimensions required by the code

- To avoid cracking due to shrinkage in the top concrete flange, the cement blocks should be watered prior to concrete placing. A light reinforcing steel mesh should be placed in the top slab for added durability and strength. This mesh also helps in case the slab is subjected to concentrated or moving loads and reduces cracking and shrinkage in the concrete. The code requires that reinforcement in the direction of the ribs should be at least 5ϕ6/m' perpendicular to the ribs and 4ϕ6/m' parallel to the ribs.

- Transversal ribs or *cross ribs* are added to one-way hollow block floors for better distribution of the applied loads. They also help in distributing the concentrated loads due to walls in the transverse direction. The bottom reinforcement is taken as the reinforcement in the main ribs, and the top reinforcement should be at least ½ of the bottom reinforcement. The code requires that in case of large spans or heavy live loads that the floor should be equipped with cross ribs with the conditions shown in Table 2.2. Fig. 2.3 illustrate the use of cross ribs in one way slabs and block arrangement

Table 2.2 Cross-rib requirements by the ECP 203

Live loads	Span	Condition
≤ 3 kN/m^2	≤ 5m	No cross rib required
≤ 3 kN/m^2	> 5m	One cross rib
>3 kN/m^2	4m to 7m	One cross rib
>3 kN/m^2	> 7m	Three cross rib

2.2.4 Design of One-way Hollow Block Slabs

2.2.4.1 Design of Ribs

The ribs can be considered as a beam having a flange width B which is equal to the distance between ribs. The ribs carries a uniformly distributed load w_{ur} equals to

$$w_{ur} = w_u \times B \quad (kN/m') \quad \text{..} \quad (2.2)$$

where w_u is the slab load in kN/m^2 and B=e+b

Depending on the sign of the bending moment, the sections may be designed either as T-sections or rectangular sections. Since Section (A) is subjected to positive bending it is designed as T-section, whereas section (B) is subjected to negative bending and it is designed as rectangular section as shown in Fig. 2.6.

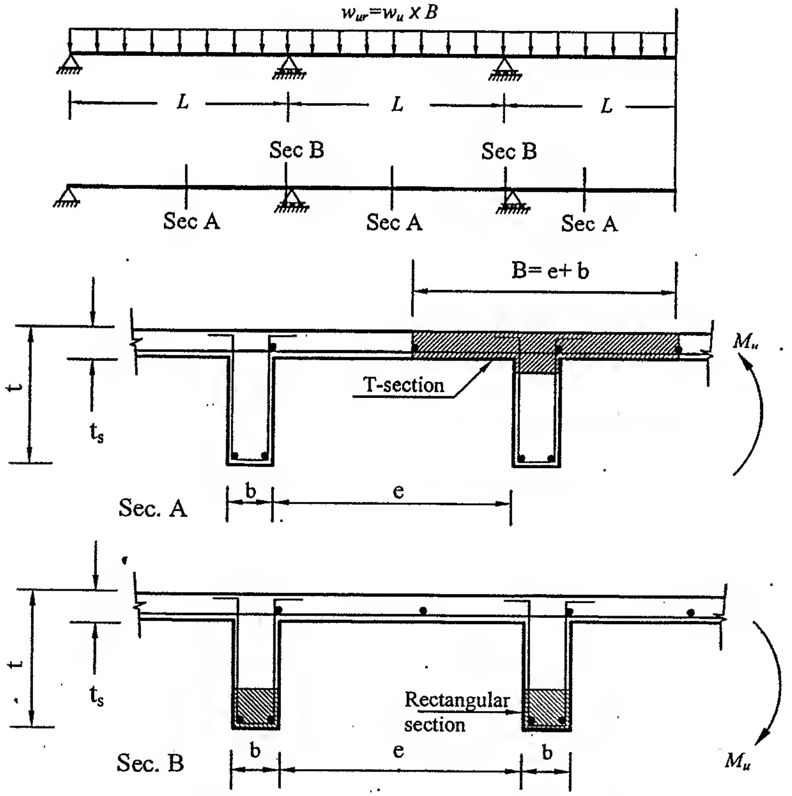


Fig. 2.6 Design of ribs

An example of the reinforcement of a one way hollow block floor is shown in Fig. 2.7

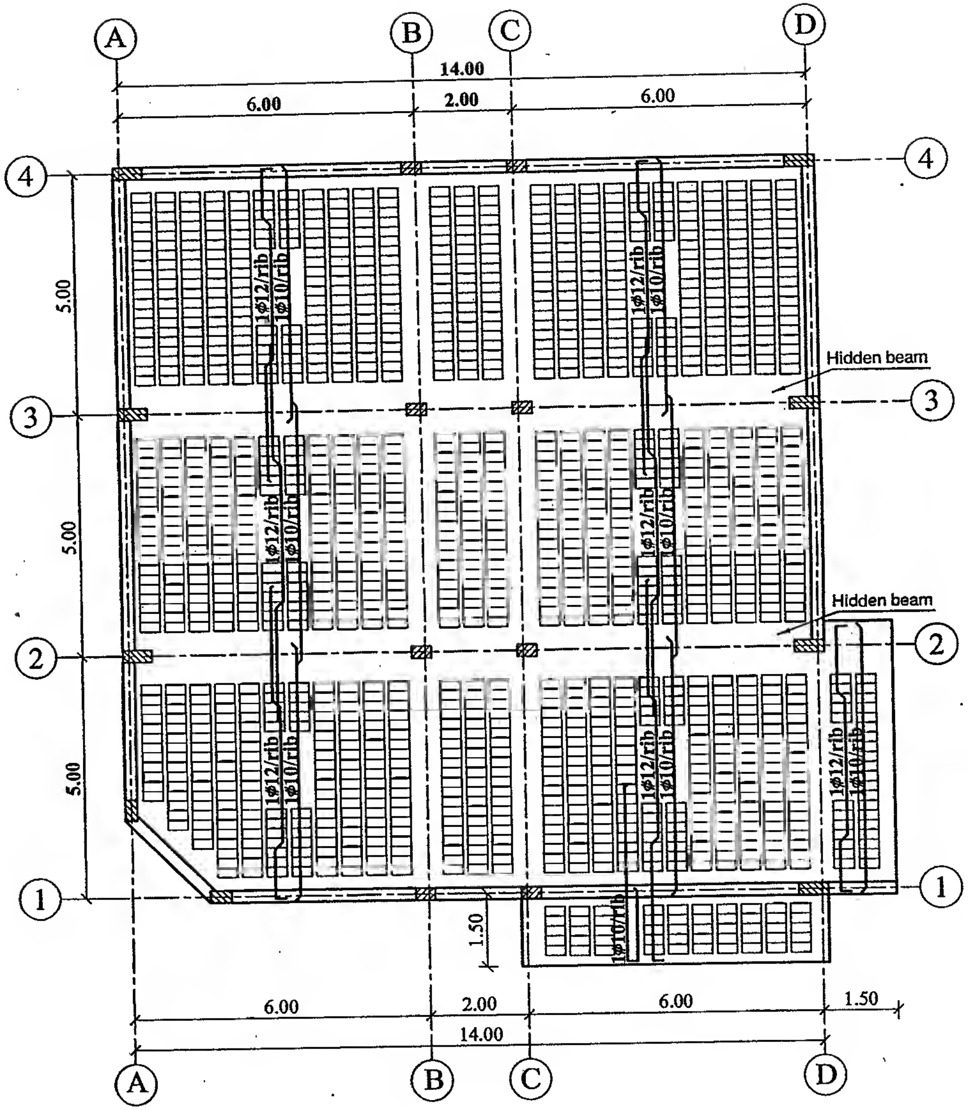
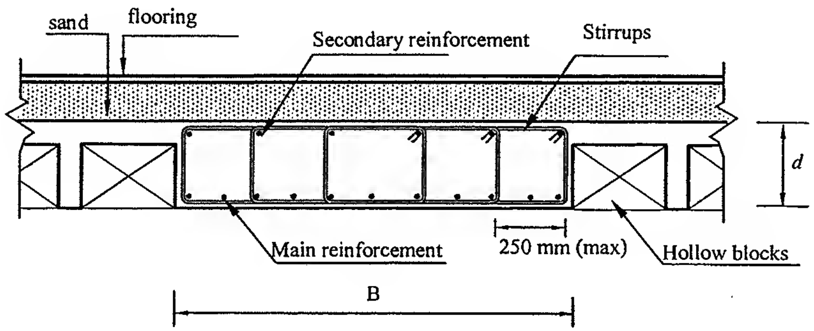


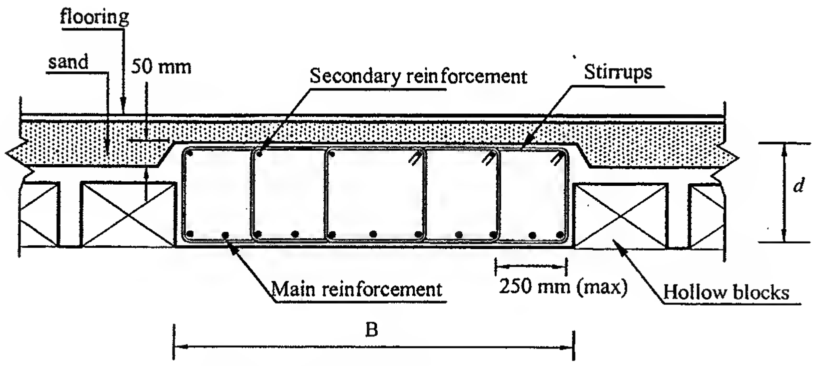
Fig. 2.7 Reinforcement of a one way hollow blocks slab with hidden beams

2.2.4.2 Design of Hidden Beams

Hidden beams are usually used in hollow block floors to give flexibility and a sense of spaciousness. Due to their limited depth, hidden beams are heavily reinforced and their width exceeds their depth as shown in Fig. 2.8.A. To reduce the amount of reinforcement, the depth of the hidden beam is increased by 50 mm as shown in Fig. 2.8.B.



A: Hidden beam



B: Hidden beam with increased thickness

Fig. 2.8 Cross section in a hidden beam

Since the width of the hidden beam is greater than its depth, the ECP 203 requires that the applied shear stress, $q_u = Q_u/(B \cdot d)$, be less than the concrete shear strength q_{cu} without any shear reinforcement contribution. Thus, it is customary not to use bent bars in hidden beams because the shear reinforcement contribution is not allowed by the code. For hidden beams, the concrete shear strength q_{cu} is given by

$$q_{cu} = 0.16\sqrt{\frac{f_{cu}}{\gamma_c}} \ge q_u \quad (N/mm^2) \quad \dots\dots\dots\dots \quad (2.3)$$

The minimum shear reinforcement for hidden beams can be reduced to

$$\mu_{\min} = \frac{0.4}{f_y} \times \left(\frac{q_u}{q_{cu}}\right) \quad \dots\dots\dots\dots \quad (2.4.a)$$

$$A_{s,\min} = \frac{0.4}{f_y} \times b \times s \left(\frac{q_u}{q_{cu}}\right) \quad \dots\dots\dots\dots \quad (2.4.b)$$

where q_u is the ultimate shear strength at the critical section and q_{cu} is the concrete shear strength. The stirrups should be arranged so that the distance between stirrups should not exceed 250 mm as shown in Fig, 2.8. It has to be mentioned that the role of the stirrups in such case is to keep the longitudinal bars in place and to confine concrete in the cross section.

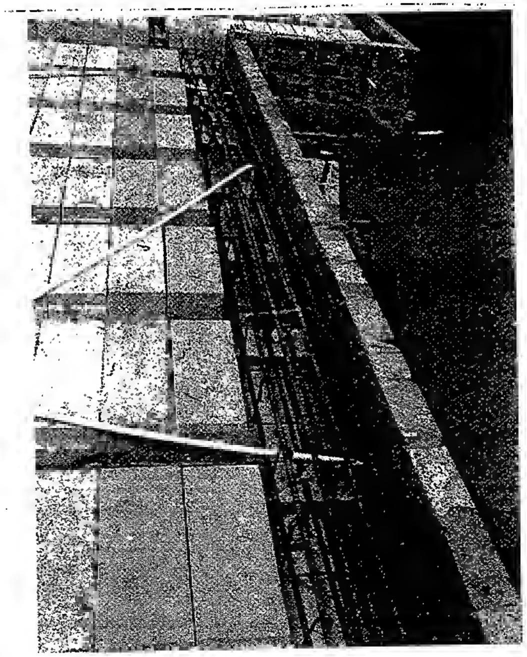


Photo 2.3 Reinforcement placement in a hidden beam

Example 2.1

A hollow block floor with hidden beams is constructed with concrete blocks over several spans as shown in figure. The characteristic material strength is

f_{cu}= 40 N/mm^2 and

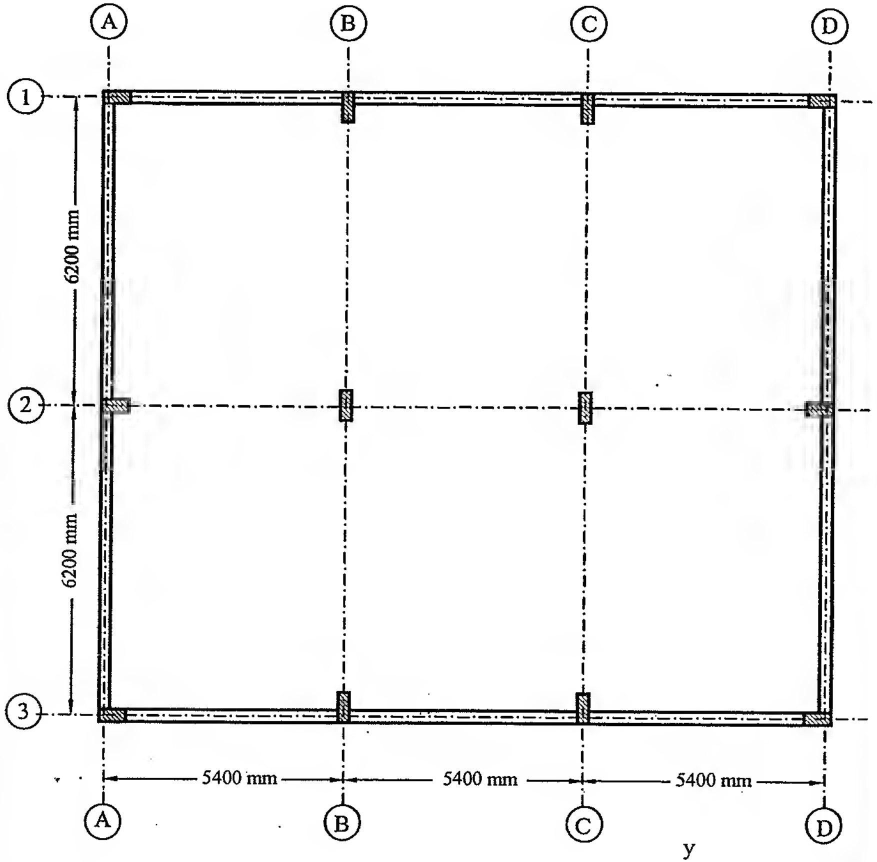
f_y=400 N/mm^2

The applied loads are

Live loads=4 kN/m^2

Flooring =2 kN/m^2

Design the floor using one-way hollow block slabs with hidden beams.



Floor layout

Solution

Step 1: Dimensioning

Choose 400x200x200 concrete blocks

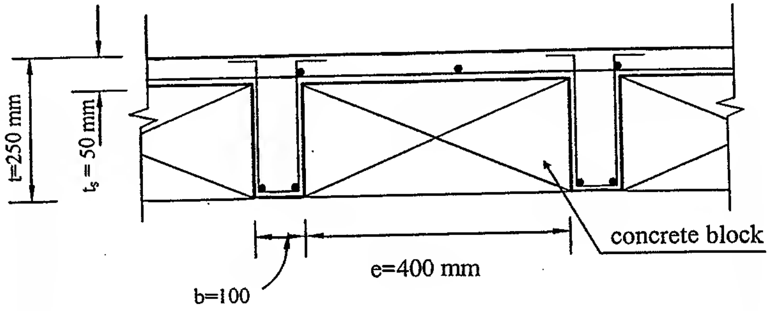
Thus, e= block width=400 mm

$$t_s = bigger\ of \begin{cases} 50\ mm \\ e/10 = 40\ mm \end{cases}$$

t_s=50 mm

$t = t_{block}+t_s$ = 200+50 =250 mm

$$b = bigger\ of \begin{cases} 100\ mm \\ 250/3 = 83.3\ mm \end{cases} \quad \rightarrow\rightarrow b= 100\ mm$$



Step 2: Calculation of ultimate loads.

For concrete blocks of dimensions 400x200x200, the self weight of the blocks and the concrete ribs is 3.3 kN/m^2 (refer to Table 2.1)

g_s= self weight + flooring=3.3+2 =5.3 kN/m^2

p_s=4 kN/m^2

$$w_u = 1.4 g_s + 1.6 p_s = 1.4\times5.3+1.6\times4 = 13.82\ kN/m^2$$

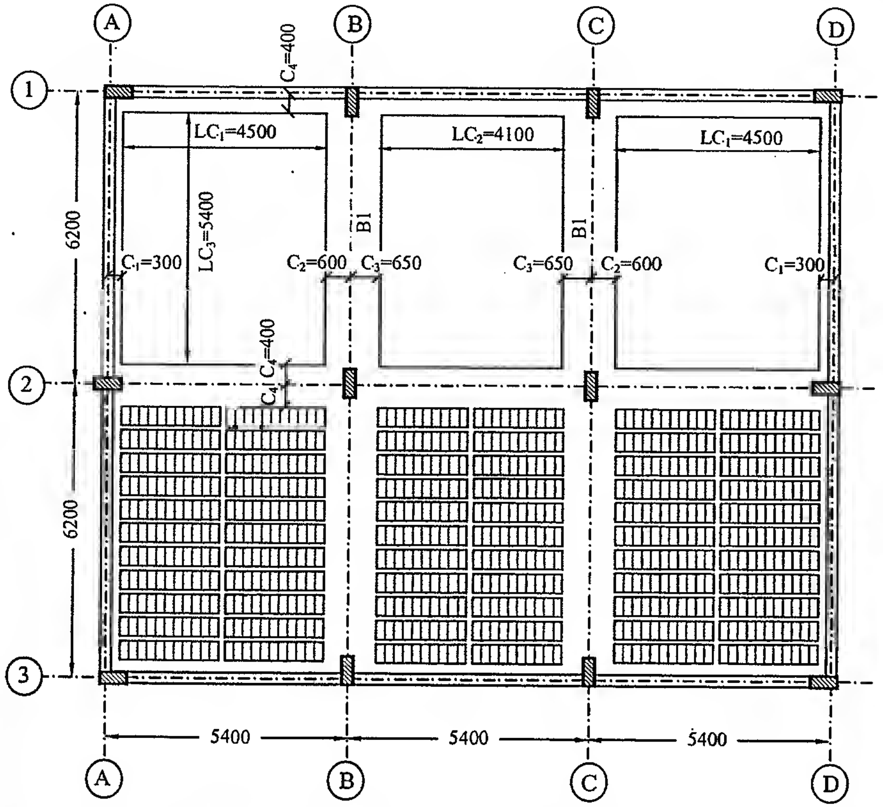
Since this is a one way hollow blocks, all the load is carried in the direction of the ribs.

$w_{u/rib} = w_u\,(e+b)$ = 13.82 (400+100) /1000=6.91 kN/m′

Step 3: Arrangement of blocks

The code requires the use of one cross rib for live loads >3 kN/m^2 and spans from 4-7 m (n_{cr}=1). Since the roof contains hidden beams the distance c measured from the centerline is taken from 300-700 mm.

Let us assume that clear span for the blocks are 0.6-1.0 m less than centerline distance.



Step 3.1: Arrangement of the blocks in x-direction

Step 3.1.1: Exterior bay

(Let $C_1+C_2=900$)

$L_{C1} = 5400 - (C_1 + C_2) = 5400 - 900 = 4500$ mm

$200\times n_1 + 100\times n_{cr} = 4500\ mm$

where, n_1=No of blocks and n_{cr}= No. of cross ribs

$200\times n_1 + 100\times 1 = 4500$ $n_1=22$

$C_1=300$ mm (near projected beam) $C_2=600$ mm (near hidden beam)

Step 3.1.2: Interior bay

Assume $C_3=650$ mm (value close to C_2)

$L_{C2} = 5400 - 2C_3 = 5400 - 1300 = 4100$ mm

$200\times n_2 + 100\times n_{cr} = 4100\ mm$ where, n_2=no of blocks

$200\times n_2 + 100\times 1 = 4100$ $\longrightarrow$ $n_2=20$ $\longrightarrow$ $C_3=650$ mm

Step 3.2: Arrangement of the blocks in y-direction

Assume $C_4 = 300$ mm

$L_{C3} = 6200 - 2\ C_4 = 6200 - 2\times 300 = 5600$ mm

If we assume that n_3= No. of blocks, then the number of ribs $=(n_3-1)$

$400\times n_3 + 100\ (n_3 - 1) = 5600$

$5600 = 500\times n_3 - 100$

$n_3 = 11.4$ block

Round down to the nearest number ($n_3=11$), and recalculate C_4

$6200 - 2\ C_4 = 500\times 11 - 100$ $C_4=400$ mm

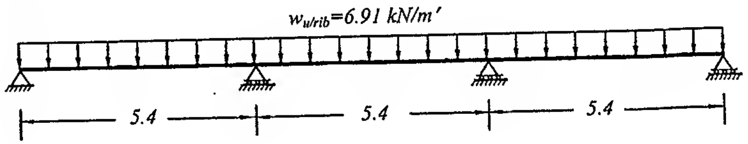
Thus use $C_4=400$ mm

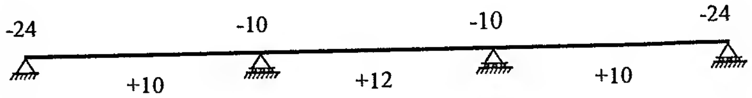
Step 4: Design of Ribs

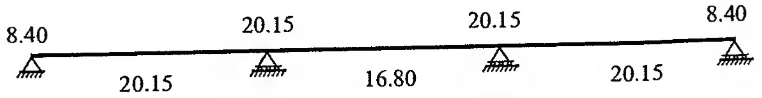
Step 4.1: Calculation of the Bending Moments in Ribs

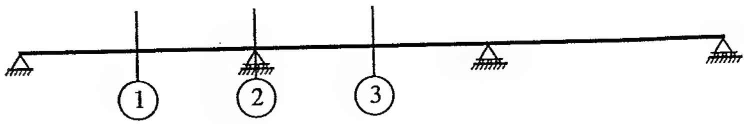
Since the ribs are continuous over the supports with equal loads and equal spans, the code coefficients for ***slabs*** are applied

$$M_u = \frac{w_{u/rib}\times L^2}{k}$$









Step 4.2: Design of rib critical sections (Continuous rib)

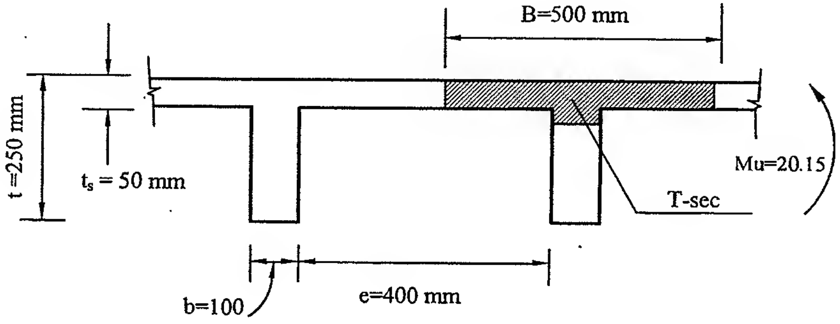
Design of Section 1

This section is subjected to a positive bending (20.15 kN.m), the compression flange form a T-section as shown in the Figure.

From the figure B=500 mm.

Assuming concrete cover of 30 mm

d= 250 –30 =220 mm



Using the C1-J curve

$$d = C1\sqrt{\frac{M_u}{f_{cu}B}}$$

$$220 = C1\sqrt{\frac{20.15\times10^6}{40\times500}} \qquad C1=6.93$$

The point is outside the curve use $(c/d)_{min}=0.125$

c=0.125 x 220=27.5 mm

a=0.8 x c = 22 mm <t_s .....o.k.

use j=0.825

$$A_s = \frac{M_u}{j\times d\times f_y} = \frac{20.15\times10^6}{0.825\times220\times400} = 277\ mm^2$$

$$A_{s\,min}{}^{(1)} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{40}}{400}\times100\times220 = 78.2\ mm^2 \ \lrcorner < A_s \quad o.k \\ 1.3A_s = 1.3\times277 = 360\ mm^2 \end{cases}$$

use 2 Φ 14 (307 mm^2)/rib(1)

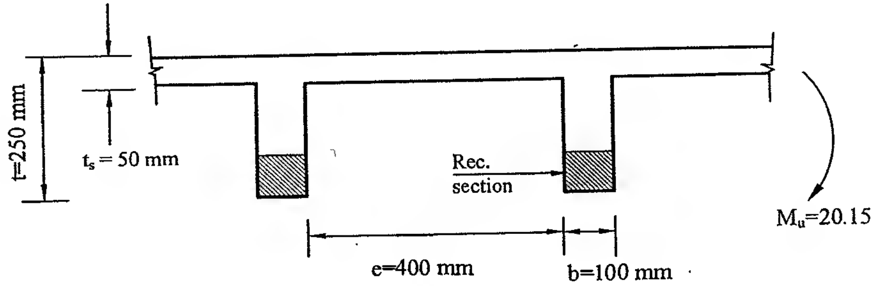
---

(1) The minimum area of steel for beams was used

Design of Section 2

This section is subjected to a negative bending moment (20.15 kN.m), thus the rib will be designed as rectangular section

b=100 mm



Using R-ω curve

$$R = \frac{M_u}{f_{cu}bd^2} = \frac{20.15\times10^6}{40\times100\times220^2} = 0.1041$$

From the curve it can be determined that ω=0.14

$$A_s = \omega\frac{f_{cu}}{f_y}b\times d = 0.14\frac{40}{400}\times100\times220 = 307\ mm^2\ > A_{s,min}$$

use (2 Φ 14, 307.9 mm^2)/rib

Design of Section 3

This section is subjected to a positive bending (16.8 kN.m), the compression flange form a T-section. Using the C1-J curve

$$d = C1\sqrt{\frac{M_u}{f_{cu}B}}$$

$$220 = C1\sqrt{\frac{16.8\times10^6}{40\times500}}\ ,\ C1=7.6$$

The point is outside the curve,

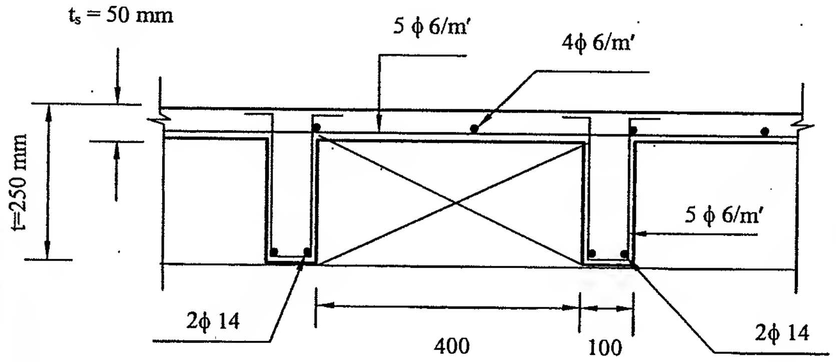
use $c/d)_{min}=0.125$ →→→ use j=0.826

c=0.125 x 220=27.5 mm

a=0.8 x c = 22 mm <t_s .....o.k.

$$A_s = \frac{M_u}{j \times d \times f_y} = \frac{16.8\times10^6}{0.826\times220\times400} = 231.3\ mm^2$$

use (1Φ12+1Φ14, 267 mm^2)/rib



Step 5: Design of hidden Beam (B1)

The slab over the hidden beam will be increased by 50 mm.

t=300 mm

B=600+650=1250 mm

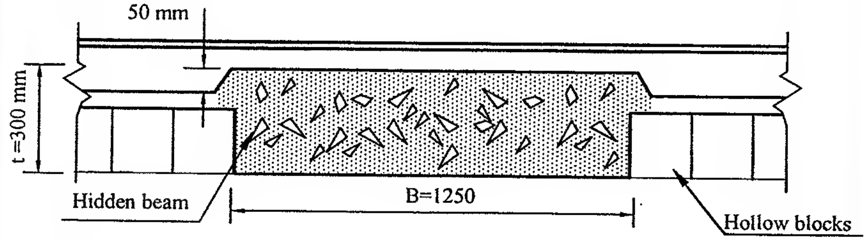
The self weight of the beam $=\gamma_c$ B t = 25 × 1.25× 0.3 = 9.375 kN/m′

But this weight should subtracted from the self weight of the blocks

Self weight of the blocks = o.w x B=3.3 × 1.25 = 4.125 kN/m′

Net weight =9.375-4.125=5.25 kN/m′, factored net weight =1.4 × 5.25 =7.35 kN/m′

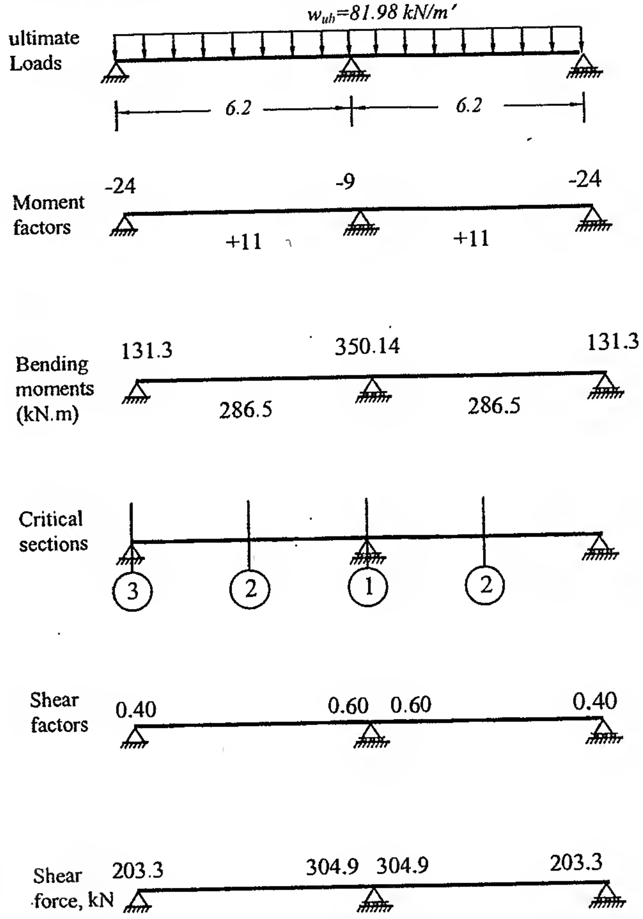
w_{ub} = w_u × spacing + net weight=13.82 × 5.4 + 7.35= 81.98 kN/m′



Note: w_{ub} calculations can be simplified by assuming that the increase in the hidden beam weight is about 10→12% of the slab weight as follows:

$$w_{ub} = 1.1 \times w_u \times spacing = 1.1\times13.82\times5.4 = 82.1\ kN/m'$$

Step 5.1: Design for flexure



Design of section 1

All sections in the hidden beam are rectangular sections. Assuming cover of 30 mm, d=270 mm. Using R-ω curve

$$R = \frac{M_u}{f_{cu} b d^2} = \frac{350.14\times10^6}{40\times1250\times270^2} = 0.096$$

From the curve it can be determined that ω=0.126

$$A_s = \omega \frac{f_{cu}}{f_y} B \times d = 0.126\,\frac{40}{400}\times 1250\times 270 = 4248\ mm^2$$

$$A_{s\,\min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{40}}{400}\times 1250\times 270 = 1201\ mm^2 \ \lrcorner < A_s \quad o.k \\ 1.3A_s = 1.3\times 4248 = 5523\ mm^2 \end{cases}$$

use (14Φ 20, 4398 mm²)

The design of other critical sections is summarized in the following table

Sec	M_u	R	ω	A_s	Chosen	
2	286.5	0.079	0.100	3379.7	3456	11 Φ 20
3	131.3	0.036	0.043	1459.5	1885	6 Φ 20

Step 5.2: Design for Shear

The critical section for shear is at d/2 from the face of the middle support. Assume that the width of the column is 600 mm. The critical section is at section 1 as shown in figure with code coefficient of 0.6

$$Q_u = 0.6\ w_{uh}\ L - w_{uh}\left(\frac{c}{2}+\frac{d}{2}\right) = 0.6\times 81.98\times 6.2 - \frac{81.98}{1000}\times\left(\frac{600}{2}+\frac{270}{2}\right) = 269.3\ kN$$

$$q_u = \frac{Q_u}{B\times d} = \frac{269.3\times 1000}{1250\times 270} = 0.80\ N/mm^2$$

The code requires that the beam shear stress be less than concrete shear strength given by

$$q_{cu} = 0.16\sqrt{\frac{f_{cu}}{1.5}} = 0.16\sqrt{\frac{40}{1.5}} = 0.826\ N/mm^2$$

Since $q_u < q_{cu}$ the shear strength of the beam is sufficient, Provide minimum area of stirrups. Assuming spacing of 200 mm and using mild steel f_y=240 N/mm²

$$A_{st,\min} = \frac{0.4}{f_y} B\times s = \frac{0.4}{240}\times 1250\times 200 = 416\ mm^2$$

The previous amount can be reduced according to the code item 4-2-2-1-6-c

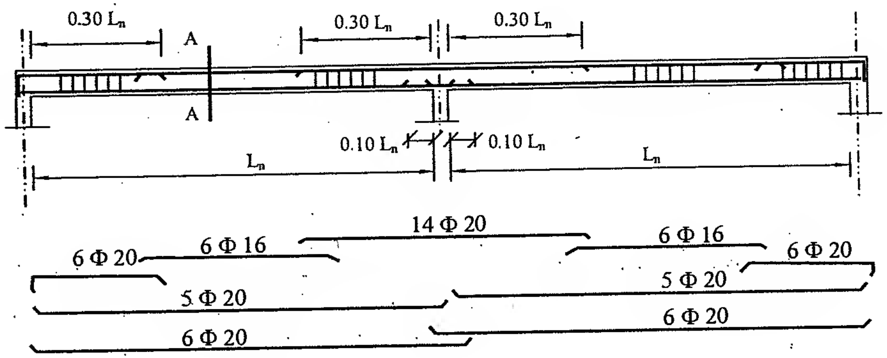
$$A_{st,\min} = 416\times\left(\frac{q_u}{q_{cu}}\right) = 416\times\left(\frac{0.80}{0.826}\right) = 405\ mm^2$$

Assuming 6 branches, the area of one branch equals

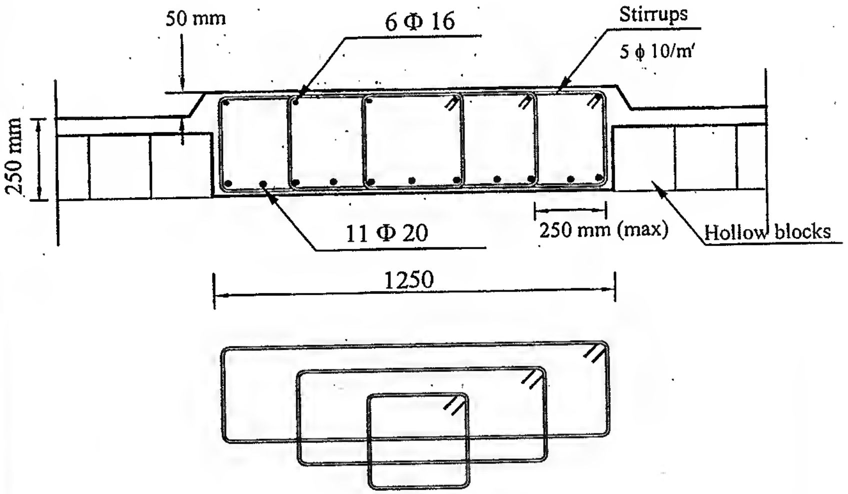
$$A_{sb} = \frac{416}{6} = 69.44\ mm^2 \qquad \text{Choose } \phi 10\ (78.5\ mm^2)$$

Use ϕ 10/200 mm (5φ 10/m′).

Since we have six branches, use 6Φ 16 as a secondary reinforcement

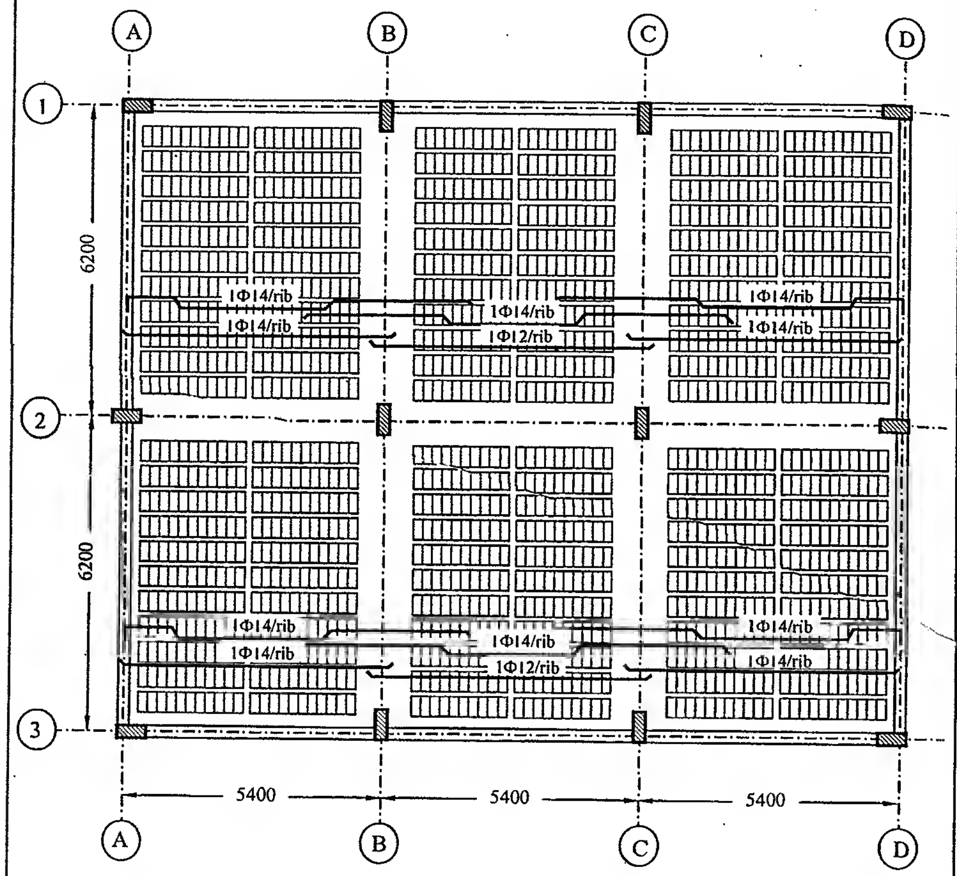


b. Longitudinal section in the hidden beam



a. Section A-A

Reinforcement details for the hidden beam



Reinforcement layout for the hollow blocks floor

2.3 Two-Way Hollow Block Slabs

In these types of floors the ribs run in two directions and the load is distributed in both directions as shown in Fig. 2.9. It is more economical to use two-way slabs if the shorter span exceeds 6 meters. However, the placing of the blocks in two directions during the construction is more difficult.

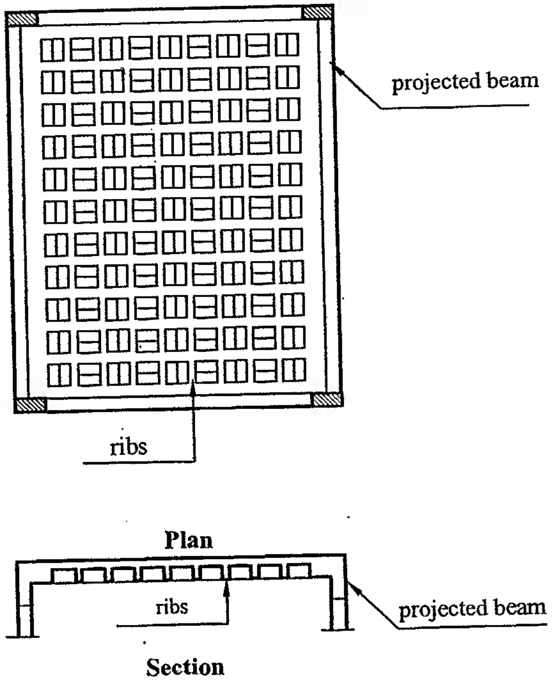
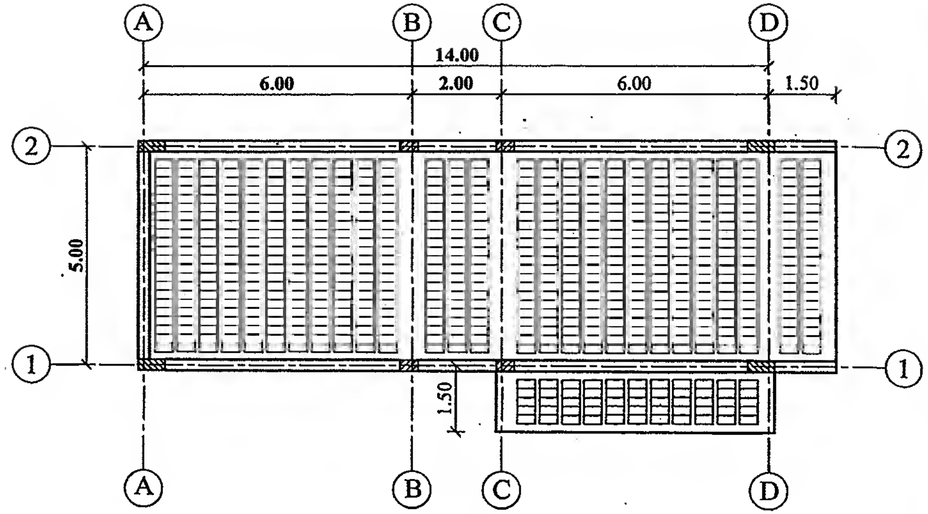


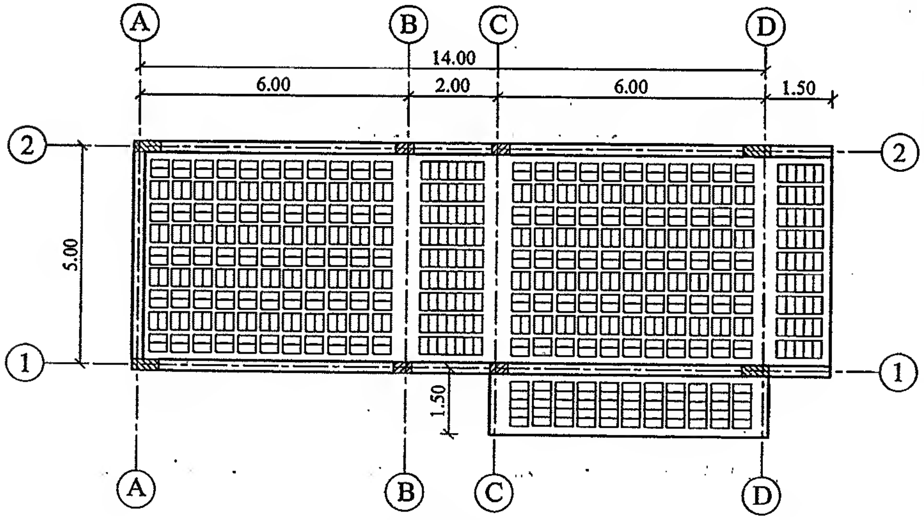
Fig. 2.9 Arrangement of the blocks in two-way hollow block slabs

In typical hollow block floors, the designer has to choose between the simplicity of construction one-way hollow block and the economy that might be offered by utilizing two way hollow blocks.

Figure 2.10 shows two options for arranging hollow-blocks in the same roof. Note that cantilever hollow block slabs are designed as one-way hollow block slabs.



a) One way slab hollow block slabs



b) One way and two way slab hollow block slabs

Fig. 2.10 Alternative solutions for arranging blocks in hollow block slabs

2.3.1 Method of Analysis

The analysis of two-way hollow blocks is carried out according to the type of supporting beams. The code distinguishes between the two cases:

Case A: two-way hollow block floors with hidden beams are designed in a similar fashion to the flat slabs.

Case B: two-way hollow block floors with projected beams are divided in two sub-cases depending on the live loads and the compression flange as shown Table 2.3.

Table 2.3 load distribution values that should be used in designing two-way hollow block floors with projected beams

Compression flange case	Live load value	
	$LL \leq 5\ kN/m^2$	$LL > 5\ kN/m^2$
Complete compression flange	Use Table (2.4) (Marcus)	Use Table (2.5) (Grashoff)
Incomplete compression flange	Use Table (2.5) (Grashoff)	Use Table (2.5) (Grashoff)

2.3.2 Design of two-way hollow slabs with projected beams

2.3.2.1 Design of ribs

Two-way hollow blocks have some capability to develop torsion but not as much as the solid slab. About 15-20% of the total load is consumed in torsional action. Marcus developed the distribution for such types of slabs in which the distribution load factors are reduced to account for some torsion action. The values are listed in Table 2.4.

Table 2.4 α and β values for two-way hollow block slabs (Marcus values)

r	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	0.396	0.473	0.543	0.606	0.660	0.706	0.746	0.778	0.806	0.830	0.849
β	0.396	0.333	0.262	0.212	0.172	0.140	0.113	0.093	0.077	0.063	0.053

In some cases where the live loads are considerably high, it is more advisable to neglect the torsion action and distribute the load according to Grashoff's factors listed in Table 2.5.

Table 2.5 α and β values for two-way hollow block slabs with high live loads (Grashoff's values)

r	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	0.500	0.595	0.672	0.742	0.797	0.834	0.867	0.893	0.914	0.928	0.941
β	0.500	0.405	0.328	0.258	0.203	0.166	0.133	0.107	0.086	0.072	0.059

The total factored load in the slab is given by:

$$w_u = 1.4\times[\text{weight of blocks/m}^2(\text{Table 2.1})+\text{flooring}]+1.6\,w_{LL} \quad\text{.............}(2.4a)$$

For ribs running in the short direction, the design loads are given by:

$$w_{\alpha/rib} = \alpha\, w_u\ (b+e) \quad\text{...}\ (2.4b)$$

For ribs running in the long direction, the design loads are given by:

$$w_{\alpha/rib} = \beta\, w_u\ (b+e) \quad\text{...}(2.4c)$$

The analysis and the design of the ribs in the two directions are carried out in similar manners to those followed in one way hollow block slabs. Fig. 2.11 shows an example of the reinforcement for a floor comprising one-way and two-way hollow block slabs.

2.3.2.2 Design of Projected beams

Projected beams are designed to carry the following loads

1- Own-weight
2- Wall load (if any)
3- Load transmitted from the slabs

The distribution of the slab load to the projected beams is carried out as shown in Fig. 2.12. The triangular or trapezoidal loads can be replaced by equivalent uniform loads for calculating bending moment and shear forces using Table 2.6.

Fig. 2.6 Coefficients of equivalent uniform loads on beams

$L/2x$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	0.667	0.725	0.769	0.803	0.830	0.853	0.870	0.885	0.897	0.908	0.917
β	0.500	0.554	0.582	0.615	0.642	0.667	0.688	0.706	0.722	0.737	0.750

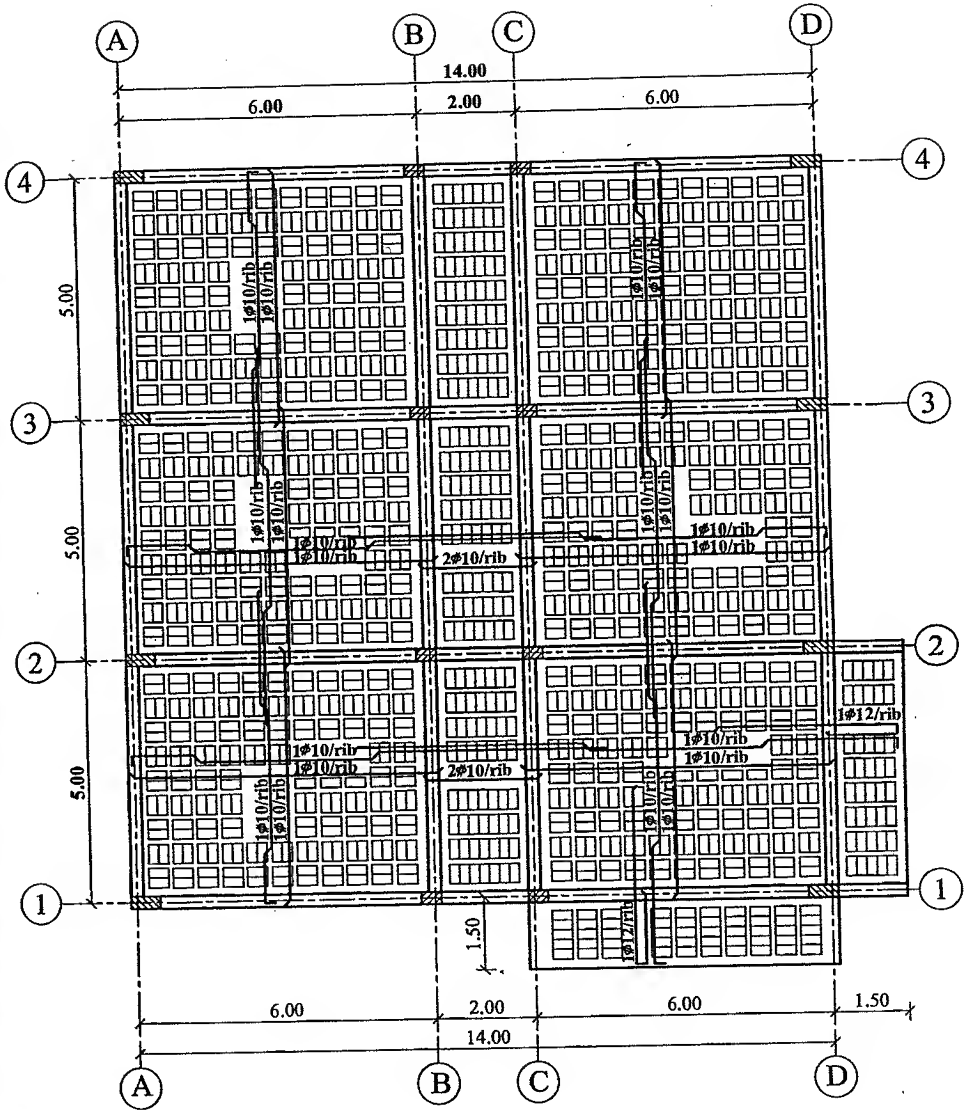


Fig. 2.11 Reinforcement of a typical hollow block floor

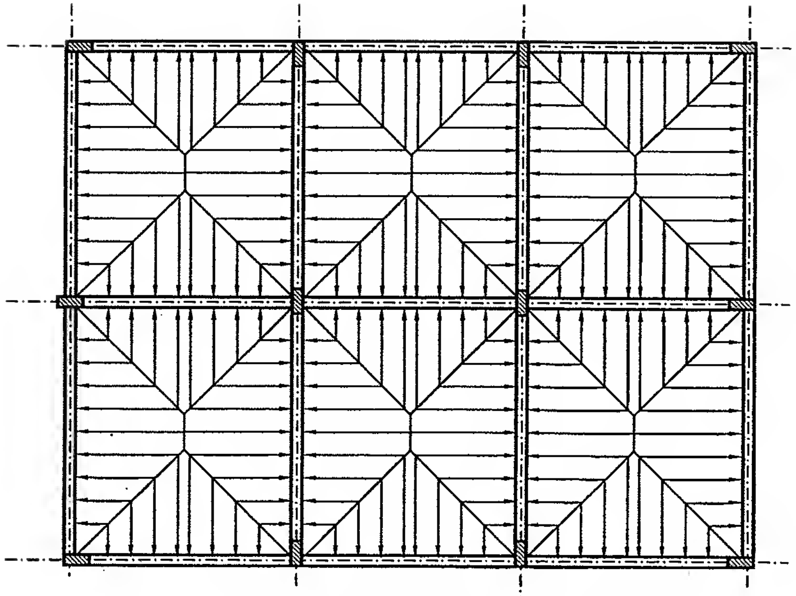


Fig. 2.12 Load distribution on the projected beams.

The bending moments and shear forces in the projected beams are obtained using methods of structural analysis (or code coefficients if applicable). Sections of positive moments are designed as T-sections while those of negative moments are designed as rectangular sections as shown in Fig. 2.13.

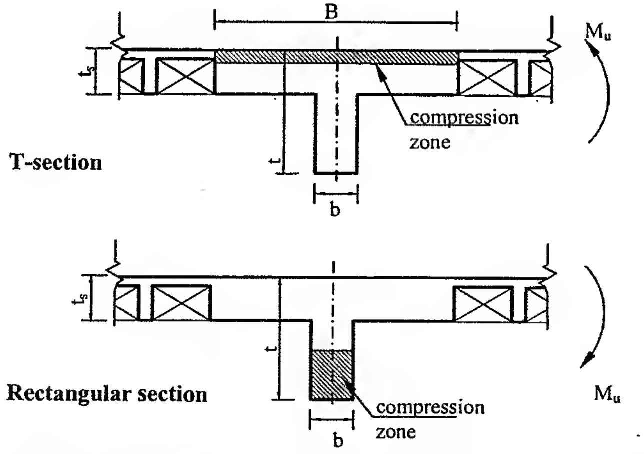
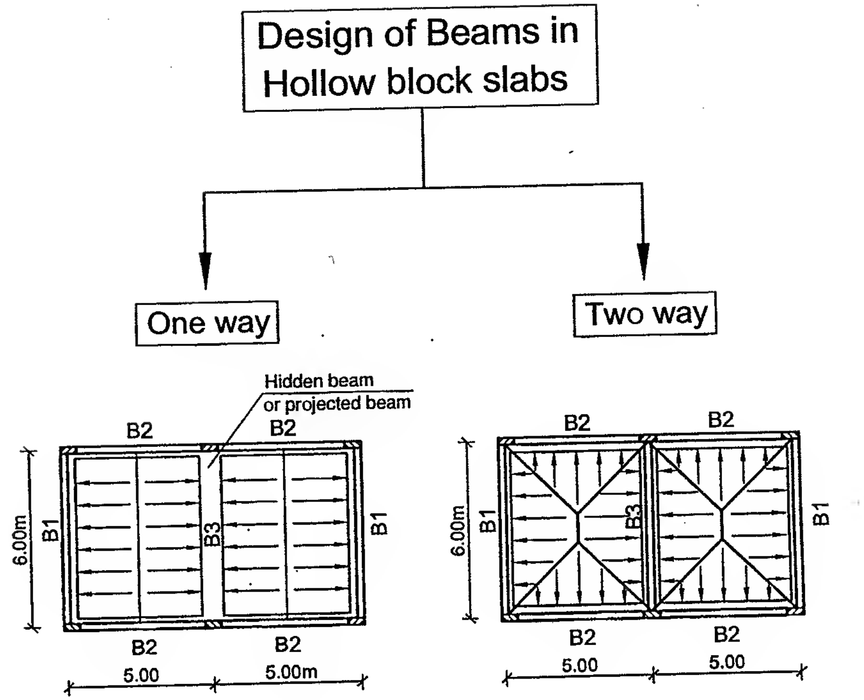


Fig. 2.13 Type of section in projected beams according to the applied moment.

Fig 2.14 shows a summary of the load calculations for beams in hollow block roofs.



$W_{B1} = O.W. + W_u \times 2.50$

$W_{B2} = O.W.$

$W_{B3} = O.W. + W_u \times 5.00$

r = 6/5 =1.20

From Table 2.6 — $\alpha = 0.769$

$W_{B1} = O.W. + W_u \times 0.769 \times 2.50$

$W_{B2} = O.W. + W_u \times 0.67 \times 2.50$

$W_{B3} = O.W. + W_u \times 0.769 \times 5.00$

Note: Wall loads may be added (if any)

Fig.2.14 Calculation of loads of beams in hollow block roofs

Example 2.2

Redesign example 2.1 using two-way hollow blocks with projected beams

Solution

Step 1: Dimensioning

Choose 400x200x200 blocks

Take e=400 mm

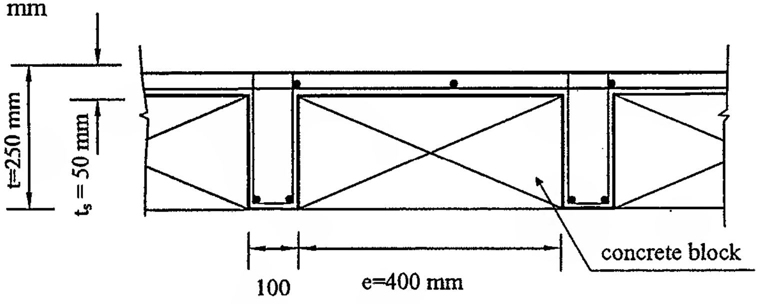
$$t_s = bigger\ of \begin{cases} 50\ mm \\ e/10 = 40\ mm \end{cases}$$

t_s=50 mm

$t = t_{block} + t_s$ = 200+50 =250 mm

$$b = bigger\ of \begin{cases} 100\ mm \\ 250/3 = 8.33\ mm \end{cases}$$

b= 100 mm



Step 2: Calculating Ultimate Loads.

For concrete blocks 400x200x200 and from Table (2.1) the self weight of the blocks and the concrete ribs is 3.8 kN/m^2

g_s= self weight + flooring=3.8+2 =5.8 kN/m^2

p_s=4 kN/m^2

$$w_u = 1.4 g_s + 1.6 p_s = 1.4 \times 5.8 + 1.6 \times 4 = 14.52\ kN/m^2$$

$$r = \frac{L_1}{L_2} = \frac{6.2}{5.4} = 1.148$$

Since this is two way hollow blocks with projected beams and L.L.< 5kN/m^2, use Table 2.3 (Marcus). The load distribution factors are

α=0.507 and β=0.299

$w_{\alpha/rib}$ = α w_u (e+b) =0.507 x 14.52 (400+100) /1000=3.68 kN/m'

$w_{\beta/rib}$ = β w_u (e+b) = 0.299 x 14.52 (400+100) /1000=2.17 kN/m'

Step 3: Arrangement of Blocks

Since the roof contains projected beams the distance c measured from the centerline is taken from 300-500 mm. Let us assume that the clear span for the blocks is 0.6-1.0 m less than centerline distance

Transversal direction

Assume C_1=500 mm

L_{c1} = 5400 – 2C_1= 5400 - 1000 = 4400 mm

$500 \times n_1 + 400 = 4400\ mm$ where, n_1=no of ribs →→ n_1=8

Longitudinal direction

Assume C_2 = 400 mm

L_{c2}= 6200 – 2 C_2 =6200-2x 400=5400 where, n_2=no of ribs

$500 \times n_2 + 400 = 5400$ n_2 =10

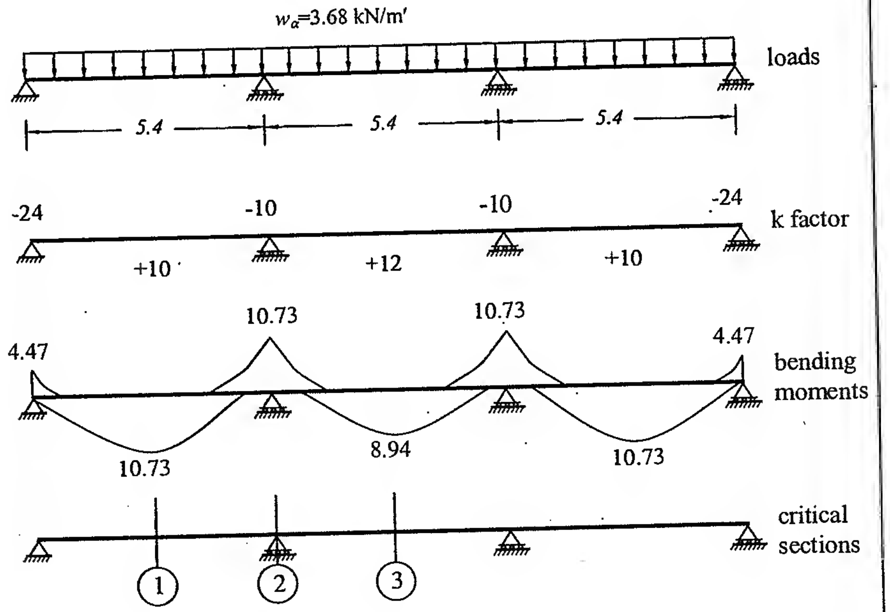
Step 4: Design of ribs

Step 4.1: Transversal direction

Step 4.1.1: Calculation of the bending moments

Since the ribs are continuous over the supports with equal loads and equal spans, the code coefficients (*k*) for slabs are applied. The transversal direction is the shorter direction thus w_α is transferring in this direction

$$M_u = \frac{w_\alpha \times L^2}{k} = \frac{3.68 \times 5.4^2}{k}$$

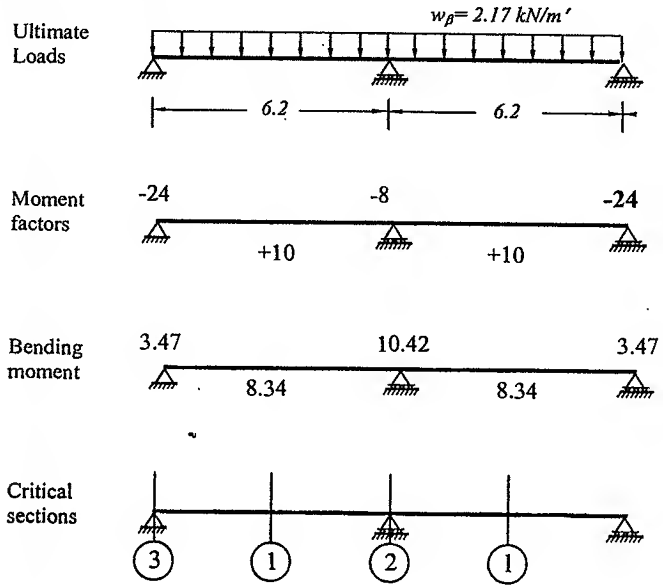


Step 4.2: Design of the Longitudinal Direction

Step 4.2.1 Calculation of the bending moments

Since the ribs are continuous over the supports with equal loads equal span, the code coefficients (*k*) for slabs are applied. The longitudinal direction is the shorter direction thus w_β is transferring in this direction

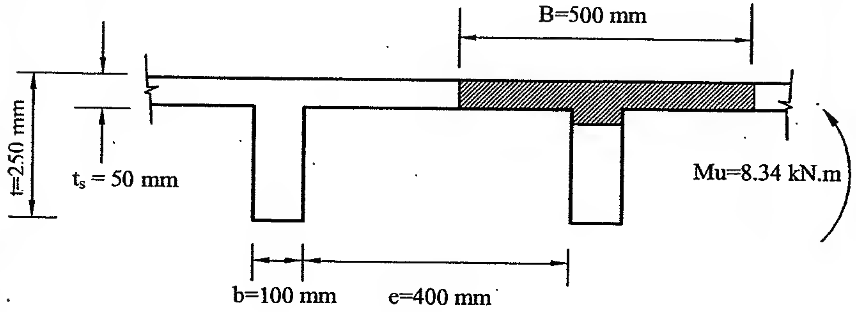
$$M_u = \frac{w_\beta \times L^2}{k} = \frac{2.17\times 6.2^2}{k}$$



Step 4.2.2: Design of Rib Critical Sections

Design of Section 1

This section has positive bending (8.34 kN.m), the compression flange form a T-section as shown in Fig.
Since this is the secondary direction, d= 250 –30-15 =205 mm



Using the C1-J curve

$$d = C1\sqrt{\frac{M_u}{f_{cu}B}}$$

$$205 = C1\sqrt{\frac{8.34\times10^6}{40\times500}} \qquad C1=10.04$$

The point is outside the curve use $c/d)_{min}$=0.125

c=0.125 x 205=25.5 mm

a=0.8 x c = 20.6 mm <t_s .....o.k.

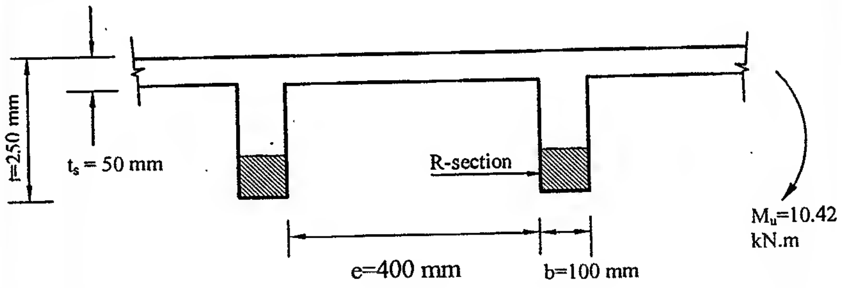
use j=0.825

$$A_s = \frac{M_u}{j\times d\times f_y} = \frac{8.34\times10^6}{0.825\times205\times400} = 123.3\ mm^2 > A_{s,min}$$

use 2 Φ 10 (157 mm^2)/rib

Design of Section 2

This section is subjected to negative bending moment (10.42 kN.m), thus the rib will be designed as rectangular section
b=1000 mm



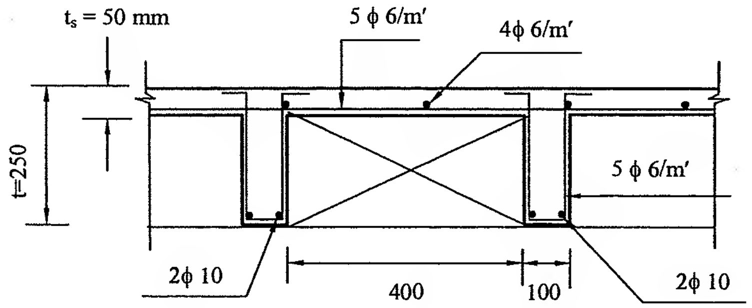
Using R-ω curve

$$R = \frac{M_u}{f_{cu} b d^2} = \frac{10.42\times10^6}{40\times100\times205^2} = 0.061$$

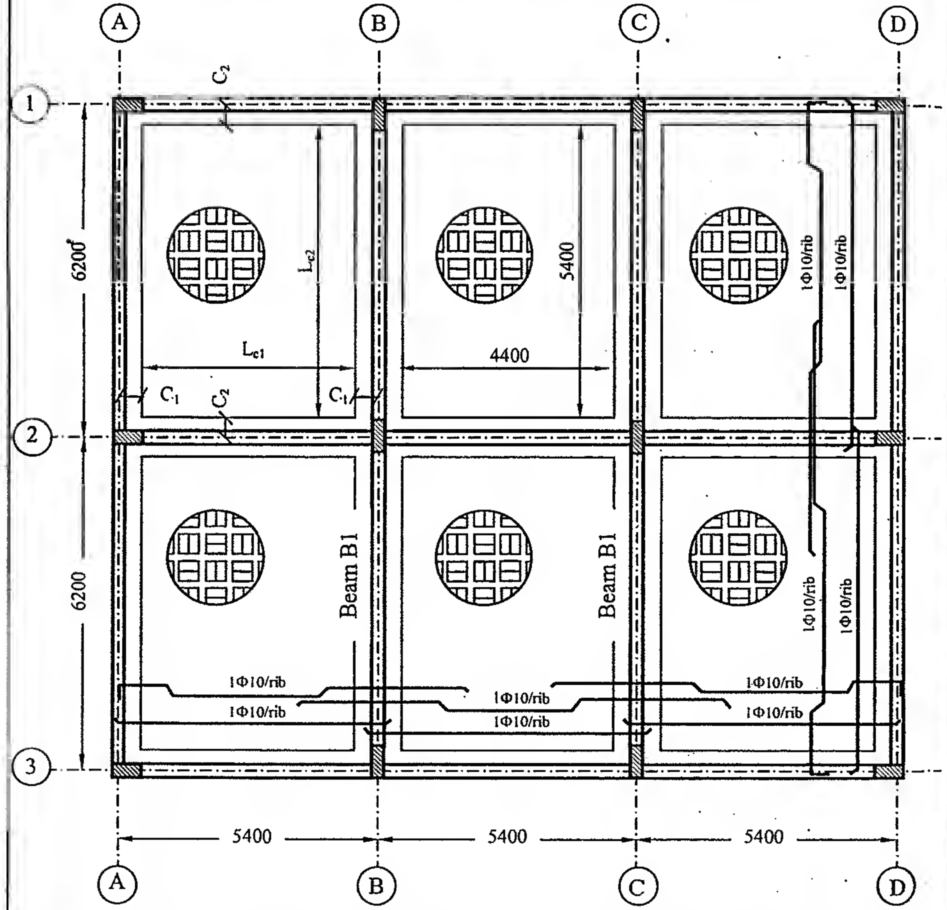
From the curve it can be determined that ω=0.075

$$A_s = \omega\frac{f_{cu}}{f_y} b\times d = 0.075\frac{40}{400}\times100\times205 = 153\ mm^2 > A_{s,min}$$

use (2 Φ 10, 157 mm^2)/rib



Rib reinforcement details in transverse direction



Reinforcement details and blocks arrangement for the two-way hollow blocks

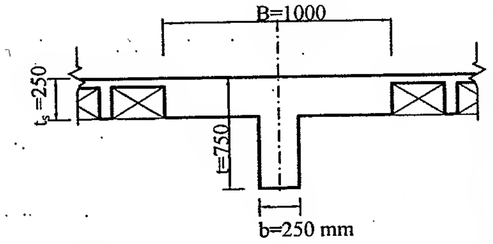
Step 5: Design Of Projected beam (B1)

Assume that the projected beam has the cross section shown in figure.

The self weight of the web of the beam equals $=\gamma_c\, b\, t = 25 \times 0.25 \times 0.75 = 4.68$ kN/m′

The weight of the flanged part of the beam is approximately taken into consideration when calculating the slab load transmitted to the beam

The load distribution over the beams will be the same as regular solid slab and the load distribution factors of the beams will be used



$$r = \frac{L_1}{L_2} = \frac{6.2}{5.4} = 1.148$$

α=0.75 and β=0.56

Load for bending

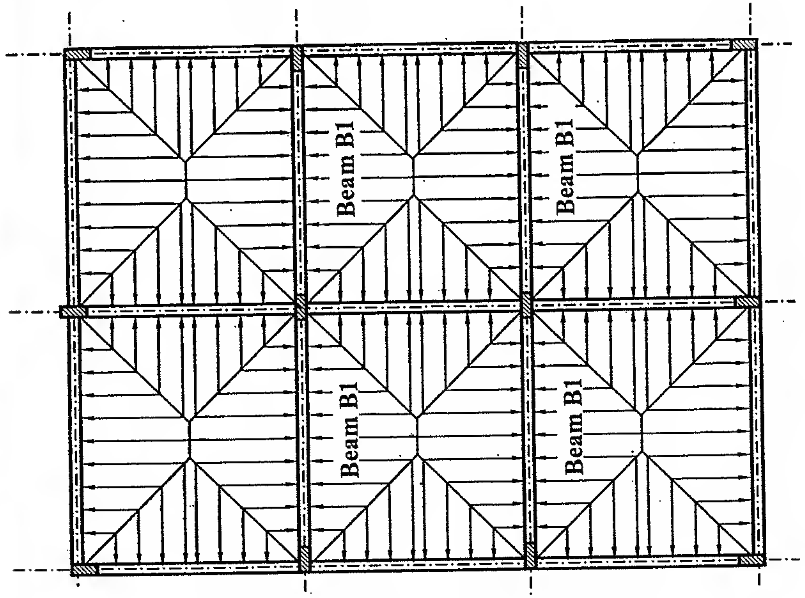
$w_{ub} = w_u \times \alpha \times \text{spacing} + 1.4 \times \text{o.w}$

$w_{ub} = 14.52 \times 0.75 \times 5.4 + 1.4 \times 4.68 = 65.37$ kN/m′

Load for shear

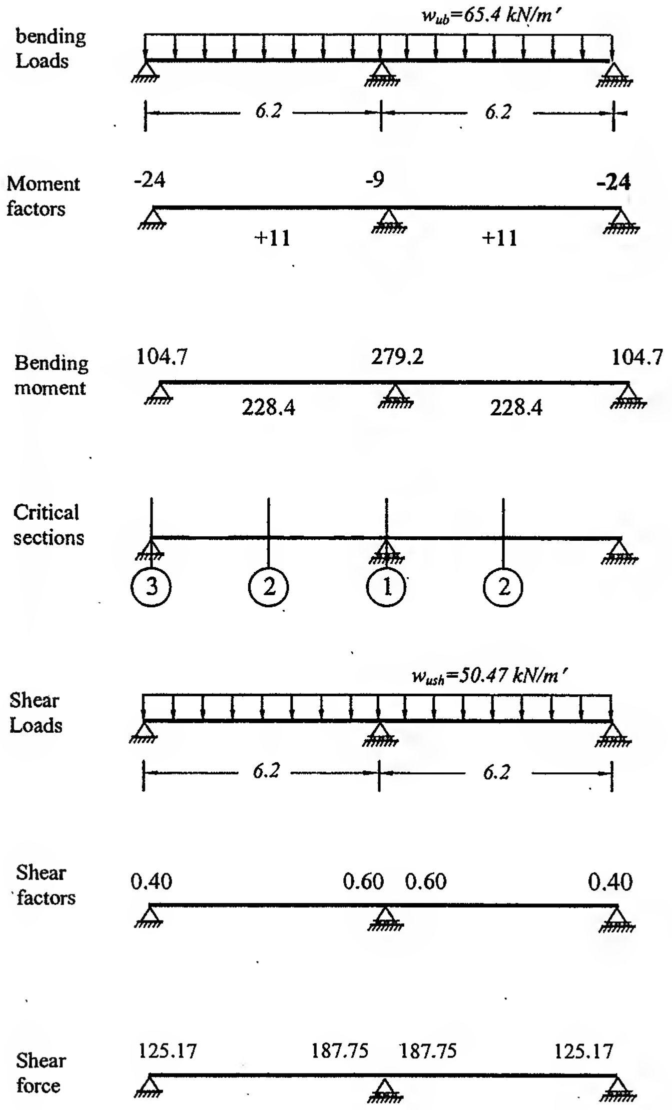
$w_{ush} = w_u \times \beta \times \text{spacing} + 1.4 \times \text{o.w}$

$w_{ush} = 14.52 \times 0.56 \times 5.4 + 1.4 \times 4.68 = 50.47$ kN/m′



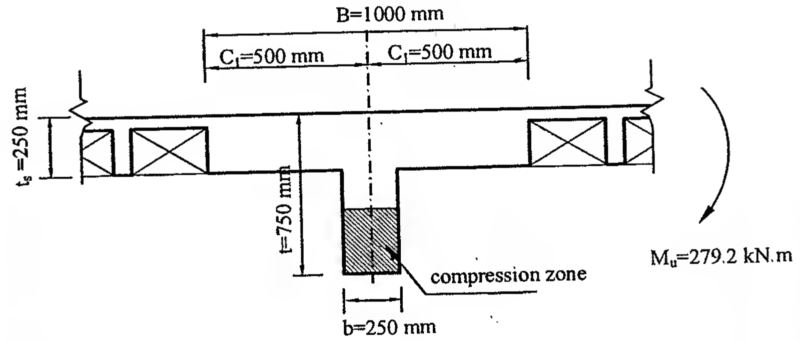
Load distribution on the projected beams.

Step 5.1: Design for Flexure



Design of Section 1

This section is subjected to –ve moment, thus it is a rectangular section
Assume concrete cover of 50 mm, d=700 mm



Using R-ω curve

$$R = \frac{M_u}{f_{cu} b d^2} = \frac{279.2\times10^6}{40\times250\times700^2} = 0.057$$

From the curve it can be determined that ω=0.07

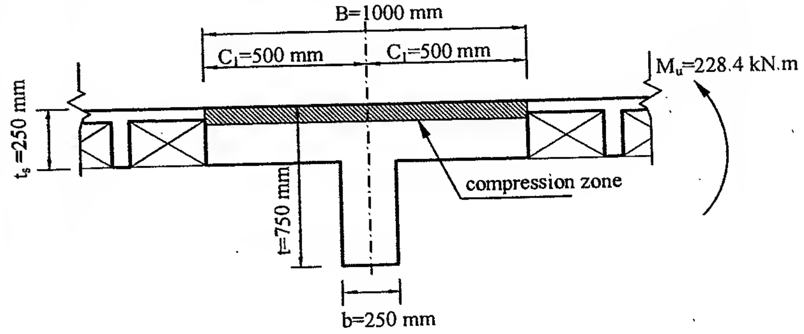
$$A_s = \omega \frac{f_{cu}}{f_y} b\times d = 0.07\,\frac{40}{400}\times250\times700 = 1230\,mm^2 > A_{s,min}$$

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{40}}{400}\times250\times700 = 622.6\ mm^2 \ \lrcorner < A_s \quad o.k \\ 1.3A_s = 1.3\times1230 = 1599\,mm^2 \end{cases}$$

use (5 Φ 18, 1272 mm^2)

Design of Section 2

This section is subjected to a positive bending moment (228.43 kN.m), the compression flange form a T-section as shown in Fig., B=500+500=1000 mm



Using the C1-J curve

$$d = C1\sqrt{\frac{M_u}{f_{cu} B}}$$

$$700 = C1\sqrt{\frac{228.43\times10^6}{40\times1000}} \qquad C1=9.26$$

The point is outside the curve use $c/d)_{min}=0.125$

$c=0.125 \times 700=87.5$ mm

$a=0.8 \times c = 70$ mm $<t_s$ …..o.k.

use $j=0.825$

$$A_s = \frac{M_u}{j\times d\times f_y} = \frac{228.43\times10^6}{0.825\times700\times400} = 988\,mm^2$$

use (4Φ18, 1017 mm^2)

Step 6.2: Design for Shear

The critical section for shear is at d/2 from the face of the support. Assume the width of the column is 600 mm. The critical section is at section 1 as shown in figure with code coefficient of 0.6

$$Q_u = 0.6\,w_{us}\,L - w_{ub}\left(\frac{c}{2}+\frac{d}{2}\right) = 0.6\times50.47\times6.2-\frac{50.47}{1000}\times\left(\frac{600}{2}+\frac{700}{2}\right) = 154.946\,kN$$

$$q_u = \frac{Q_u}{b\times d} = \frac{154.94\times1000}{250\times700} = 0.88\,N/mm^2$$

Concrete shear strength is given by

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{1.5}} = 0.24\sqrt{\frac{40}{1.5}} = 1.24\,N/mm^2$$

Since $q_u<q_{cu}$, no need for web reinforcement, provide minimum area of stirrups

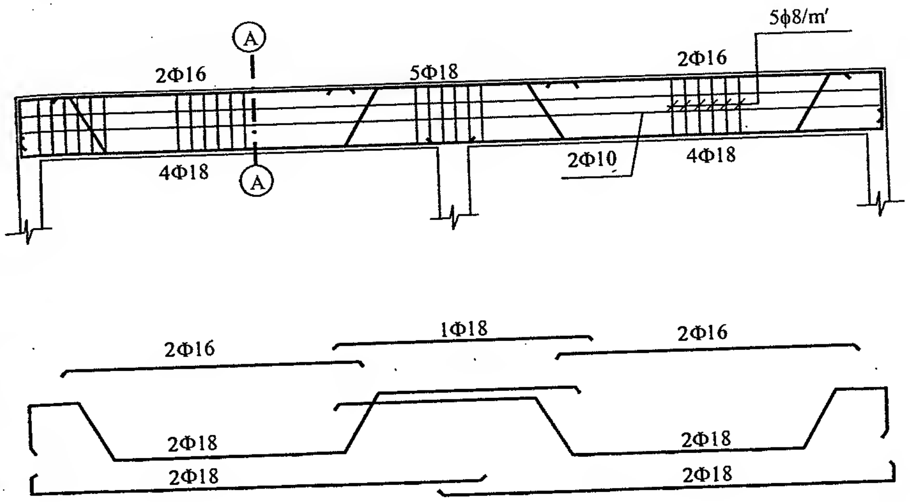
Assuming spacing of 200 mm and using mild steel f_y=240 N/mm^2

$$A_{st,min} = \frac{0.4}{f_y} b\times s = \frac{0.4}{240}\times250\times200 = 83.33\,mm^2$$

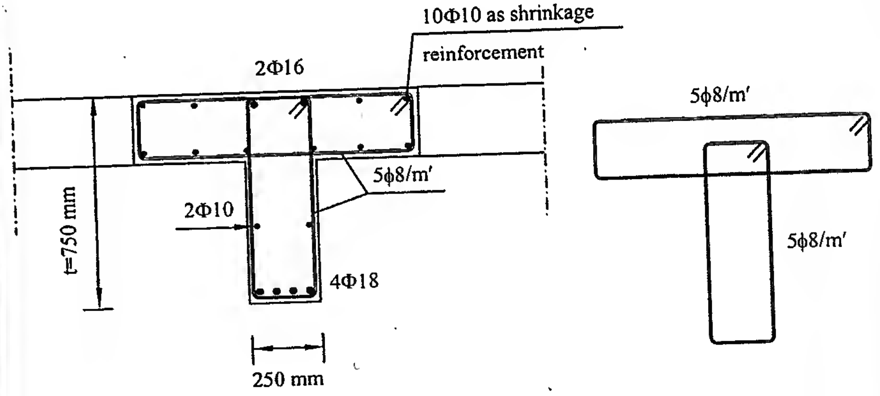
Assuming 2 branches, the area of one branch equals

$$A_{sb} = \frac{83.33}{2} = 41.67\,mm^2 \qquad \text{choose } \phi8\ (50\ mm^2)$$

Use ϕ 8/200 mm



Elevation



Cross section A-A in the projected beam

Reinforcement details of the projected beam

3

PANELED BEAMS

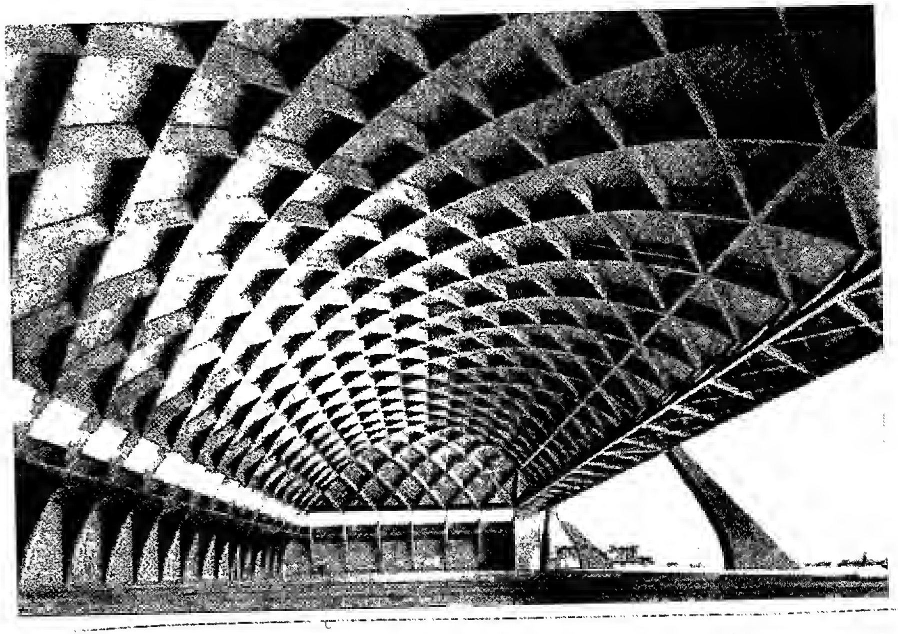


Photo 3.1 Reinforced concrete airplane hanger, Italy.

3.1 Introduction

A Paneled beams system is normally utilized when the dimensions of the floor are relatively large so that it becomes uneconomical to either use solid slabs, or hollow block slabs. In a paneled beams system, the floor is strengthened with a series of beams with equal depth spanning usually in two perpendicular directions. These beams divide the large floor into a number of small panels that can be easily designed as solid slabs as shown in Fig. 3.1. The spacing between the beams ranges normally from 2 to 4 meters. In this system, all beams are of the same depth and are supported directly either on columns or on edge beams.

Because the deflection is equal at the point of intersection for any two beams, the load transferred in the short direction is much larger than the long direction. This is because it takes more loads to deflect a short beam than it does for a long one. If the ratio of the long span to the short span (L_L/L_s) exceeds 1.5, there is no structural advantage for using paneled beams system. Almost the entire load in this case is transferred in the short span, similar to the case of one-way slabs.

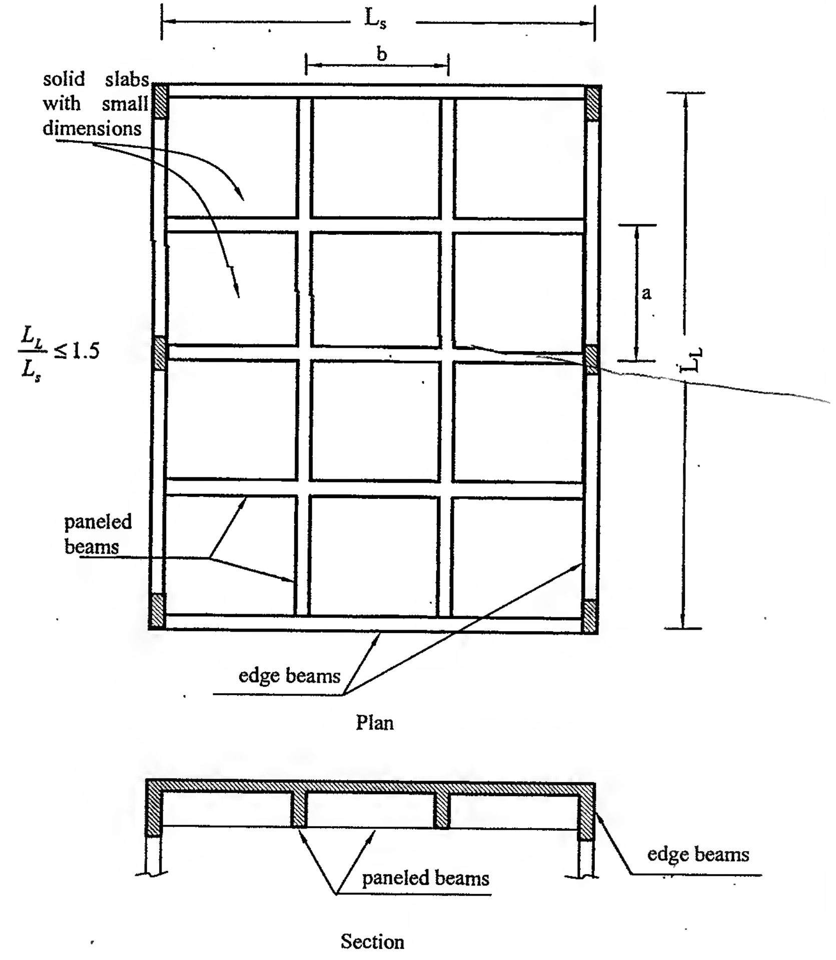


Fig. 3.1 Layout of paneled beams floor

3.2 Load Distribution

To illustrate the behavior of the paneled beams system, let us assume a very simple system which consists of two beams with the same depth intersecting at point A as shown in Fig. 3.2. The dead and the live loads are transferred through these two beams. Denoting the load transferred in the short direction as w_α and the load transferred in the long direction as w_β.

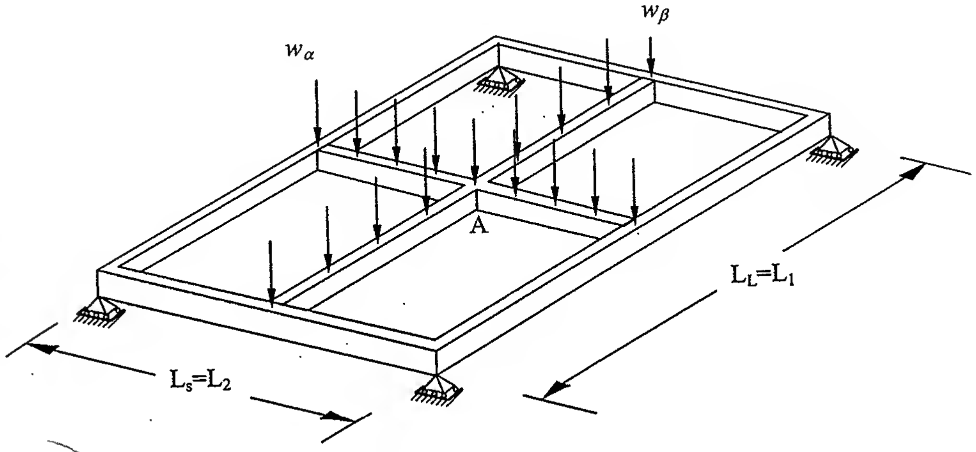


Fig. 3.2 Load distribution in paneled beams

The deflection of the short span beam Δ_1 equals

$$\Delta_1 = \frac{5 \times w_\alpha \times L_2^4}{384\,EI} \qquad (3.1)$$

The deflection of the long span beam Δ_2 equals

$$\Delta_2 = \frac{5 \times w_\beta \times L_1^4}{384\,EI} \qquad (3.2)$$

But since the deflection at point A is the same, $\Delta_1 = \Delta_2$

$$\frac{5 \times w_\alpha \times L_2^4}{384\,EI} = \frac{5 \times w_\beta \times L_1^4}{384\,EI} \qquad (3.3)$$

$$\frac{w_\alpha}{w_\beta} = \frac{L_1^4}{L_2^4} \qquad (3.4)$$

but the total load $w = w_\alpha + w_\beta$

$$\frac{w_\alpha}{w_\beta + w_\alpha} = \frac{L_1^4}{L_2^4 + L_1^4} \qquad \text{...(3.5)}$$

Defining $w_\alpha = w\,\alpha$ and $w_\beta = w\,\beta$

$$\frac{w \times \alpha}{w} = \frac{L_1^4}{L_2^4 + L_1^4} = \frac{L_1^4 / L_2^4}{1 + L_1^4 / L_2^4} \qquad \text{...(3.6)}$$

Denoting r as rectangularity ratio

$r = \frac{Long\ span}{Short\ span} = \frac{L_1}{L_2}$, Thus the load distribution factors α and β equals

$\alpha = \frac{r^4}{1+r^4}$ **and similarly** $\beta = \frac{1}{1+r^4}$

The load distribution factor in the short direction α is always larger than the load distribution factor β in the long direction by the magnitude of r^4. Table 3.1 lists the values of α and β for different rectangularity ratio. This table is used to calculate the load transferred in each direction.

Table 3.1 α and β values for paneled beam slabs (Grashoff's values)

r	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	0.500	0.595	0.672	0.742	0.797	0.834	0.867	0.893	0.914	0.928	0.941
β	0.500	0.405	0.328	0.258	0.203	0.166	0.133	0.107	0.086	0.072	0.059

The difference in magnitude between the part of the load transferred in the short beam and that in the long beam can be explained by considering the structure in Fig 3.3. For the same amount of deflection, it takes more force to displace a shorter beam than a longer one. If the two beams are of the same length, the load transferred in both directions will be the same ($P_1=P_2=P/2$). The developed bending moment in each beam will be PL/8 instead of PL/4 in case of a simple beam(50% reduction), because each beam supports the other. However, if the long span is twice that of the short span, the long beam will provide little support to the short beam because the flexural stiffness is considerably low. A computer analysis reveals that the developed bending moment in the short beam is about (90%) of the simple beam moment with length L(P_1=0.9 P) and the developed bending moment in the long direction is about 10% of the simple beam moment with length 2L. This explains the limitation imposed by the code concerning the ratio of the long span to the short span ($L_1/L_2<1.5$). At this ratio, about 75% percent of the load is transferred in the short direction (P_1=0.75P).

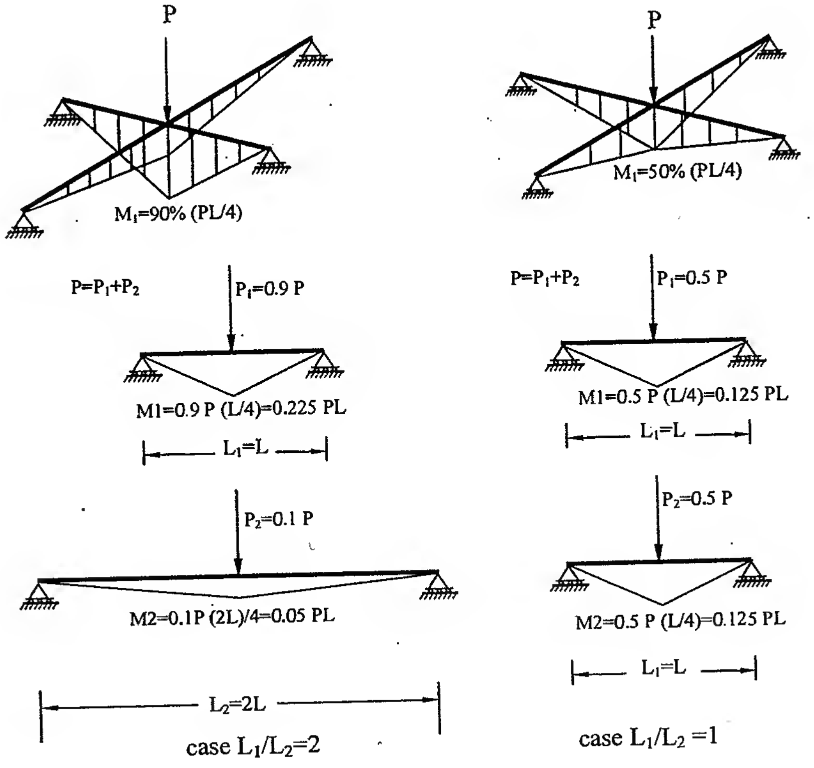
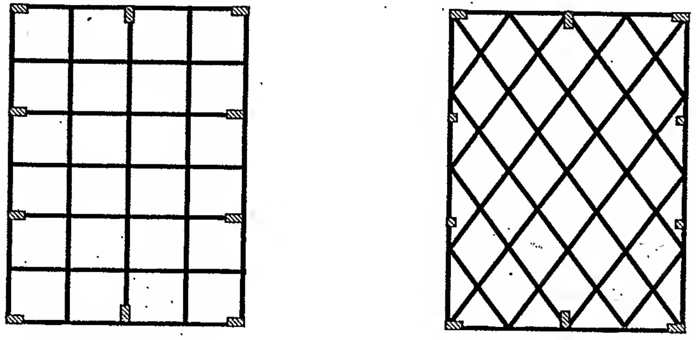


Fig. 3.3 Bending moment and load distribution for different paneled beams

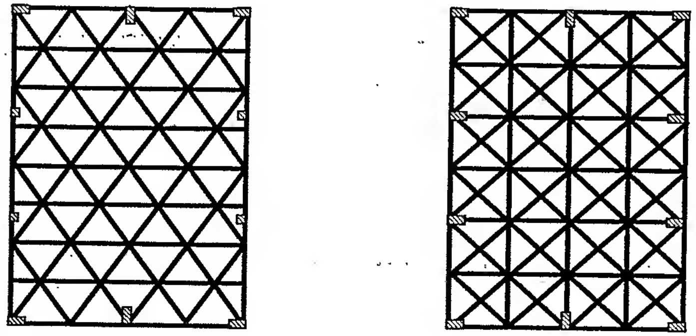
3.3 Code Provisions

- The paneled beams could either be arranged in two perpendicular directions, or in skew, triangular, or quadruple grids as shown in Fig. 3.4.
- All beams should have the same depth with a maximum rectangularity ratio of 1.5
- The internal force in the paneled beams should be determined based on structural analysis, equilibrium and compatibility. The use of any simplified method to calculate the forces is valid as long as the solution is compatible with the actual behavior.



Rectangular grid

Skew grid



Triangular grids

Quadruple grid

Fig. 3.4 Types of paneled beams systems

3.4 Simplified Design Method

In the simplified method, the paneled beams floor of dimension $L_L \times L_s$ behaves as two-way slab that has rectangularity ratio $r=L_L/L_s$. This two-way slab consists of a system of intersecting beams surrounding a number of two-way slabs of smaller dimensions. The design process consists of two steps namely

1. Design of the small solid slabs to resist the loads directly transferred to them
2. Design of the floor beams having dimensions $L_L \times L_s$ as follows
    - Assume the dimensions of the beams
    - Calculate the design loads
    - Distribute the design loads
    - Design of paneled beams in both the short and long directions
    - Design of edge beams

a-Beam dimensions

The thickness of the paneled beams is usually assumed as a ratio form the short span as follows

$$t = \frac{short\ span\ (L_s)}{12-16} \quad \text{...(3.7)}$$

Since the thickness (t) of the beams in both directions is the same, the reinforcement in the short direction is the main direction and the effective depth equals the beam thickness minus the cover as shown in Fig. 3.5. Reinforcement running in the long direction (secondary direction) is placed on the top of the main reinforcement and the effective depth equals the thickness minus the cover and the bar diameter of the main direction as shown in Fig. 3.5.

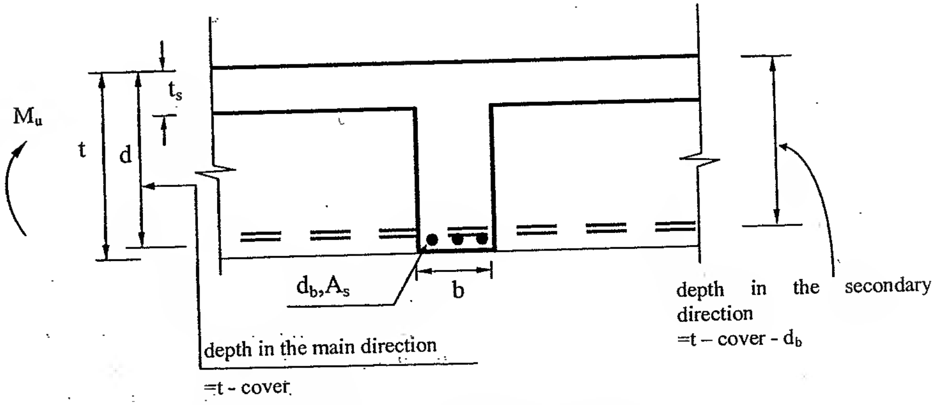


Fig. 3.5 Cross-section in paneled beams floor

b-Design loads

The self-weight of the beams is assumed to be uniformly distributed over the floor. The average weight of the beams equals

$$o w_b = \gamma_c \times b \times (t - t_s) \times \frac{n_1 \times L_L + n_2 \times L_s}{L_L \times L_s} \quad (\text{kN/m}^2) \qquad (3.8.A)$$

To obtain the total loads of the system, beams self-weight is added to the slab self-weight, flooring, partitions, and live loads.

$$w_u = 1.4 \times (o w_b + t_s \times \gamma_c + flooring + partitons) + 1.6\, w_{LL} \quad (\text{kN/m}^2) \qquad (3.8.B)$$

where $\gamma_c = 25\ kN/m^3$

c-Distribution of design loads

In the simplified method, a portion of the load transferred to the beams is determined according to the overall rectangularity ratio of the slab. The transferred load in the short direction is much higher than that transferred in the long direction and is determined using Table 3.1.

$$r = \frac{L_L}{L_s}$$

$$w_{u\alpha} = w_u \times \alpha \qquad\qquad w_{u\beta} = w_u \times \beta$$

d-Design of paneled beams

The design of paneled beams is carried out similar to that of regular beams. However, the distribution of the moments is obtained by assuming that the deflection of the slab is in the form of a *sin* curve. The bending moments developed in each beam is the ratio between the deflection at this location to the deflection at mid span.

For example if we have the paneled beams slab shown in Fig. 3.6 with four beams running in the short direction and three beams running in the long direction, the developed bending moment equals

$$M_{B1} = w_u \times \alpha \frac{L_s^2}{8} \sin(\theta_1) \qquad\qquad M_{B2} = w_u \times \alpha \frac{L_s^2}{8} \sin(\theta_2)$$

$$M_{B3} = w_u \times \beta \frac{L_L^2}{8} \sin(\theta_3) \qquad\qquad M_{B4} = w_u \times \beta \frac{L_L^2}{8} \times 1.0$$

e-Design of edge beams

The thickness of the edge beams is usually equal to or greater than that of the paneled beams. The load distribution on edge beams is affected greatly by the support conditions. If the floor is supported on the corners only, the overall slab is considered supported directly on the edge beam as shown in Fig. 3.7.A . On the other hand, if the paneled beams rest directly on supports, the reaction of the beams is carried directly by the columns. In this case, the load on the edge beam can be estimated as one half the load of the parallel beams as shown in Fig. 3.7.

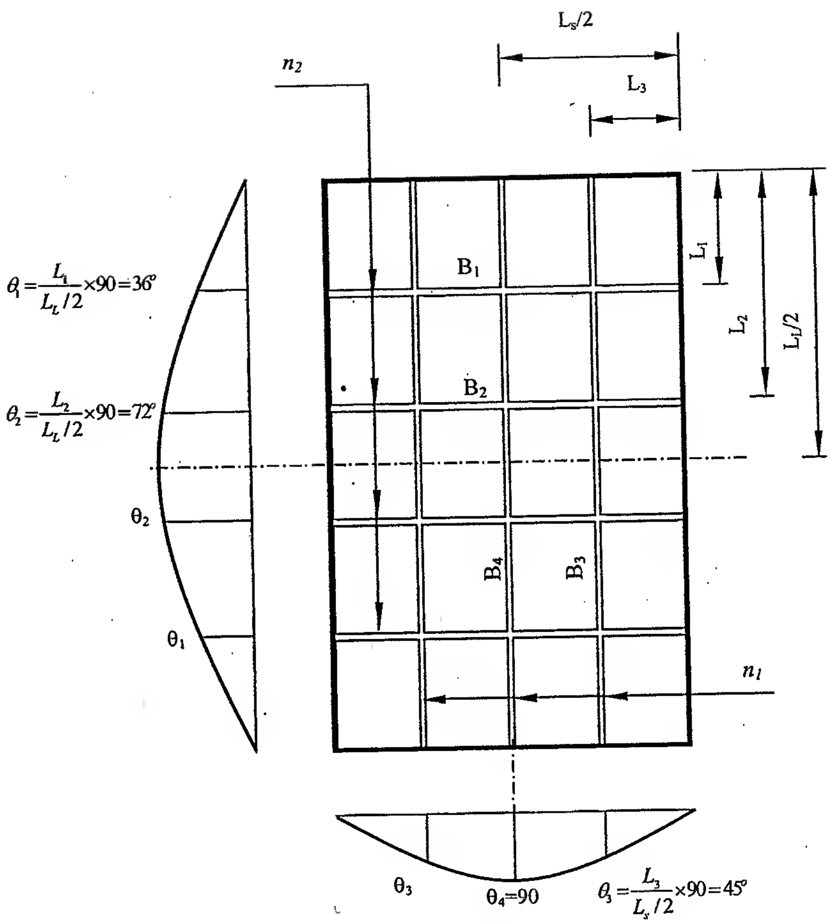
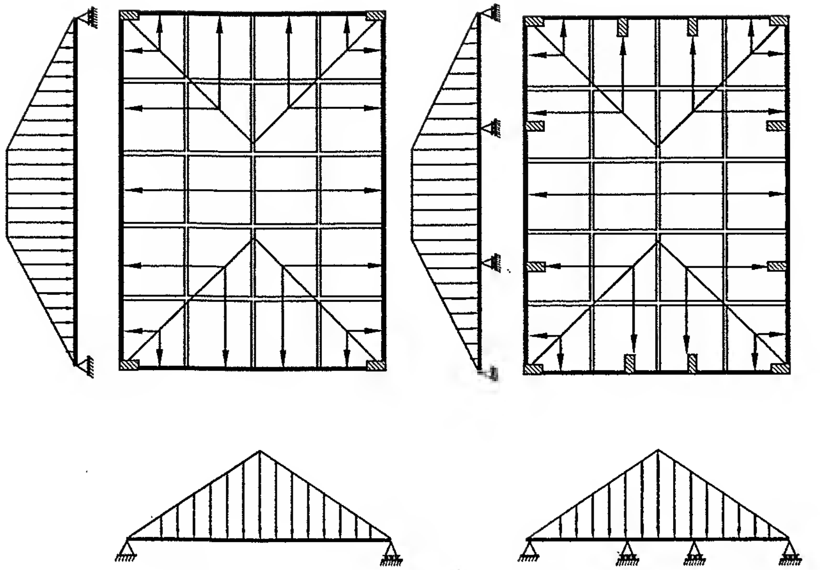
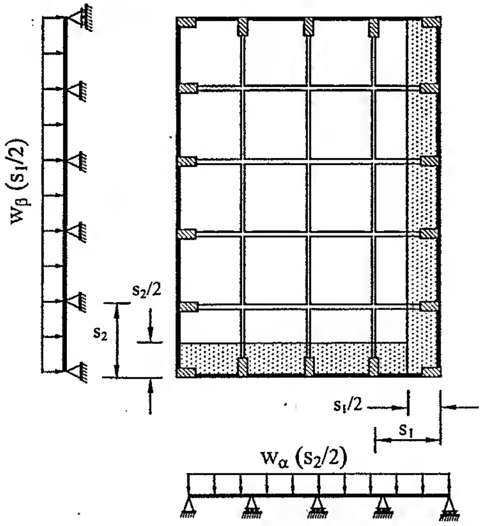


Fig. 3.6 Load distribution and deflection of paneled beams system



A-no intermediate columns

B-paneled beams that are not rested directly on columns



C-paneled beams are rested directly on intermediate columns

Fig. 3.7 Effect of column location on load distribution of edge beams

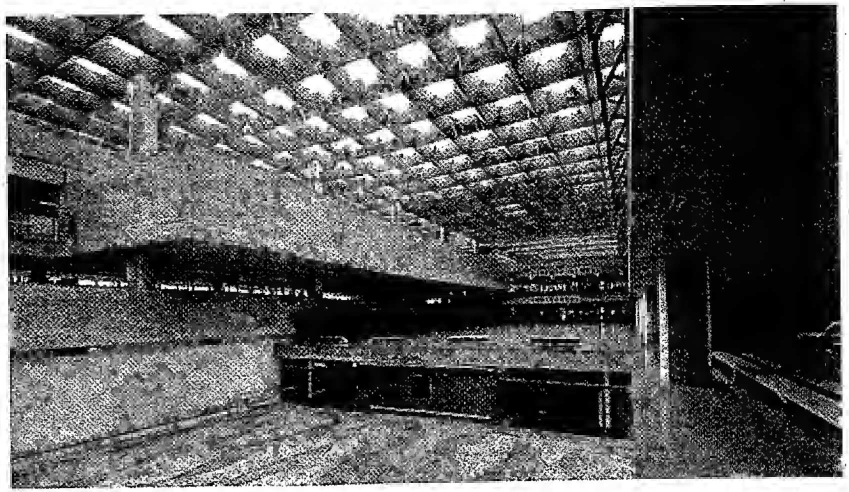


Photo 3.2 Main hall at the Faculty of Architecture Sao Paolo University 1969

Photo 3.3 Ethical Society -entry hall- Missouri-USA

3.5 Design of Skew Paneled Beams

The Egyptian code allows the use of paneled beams that are not perpendicular to each other or that are not parallel to the hall *(skew grid)*. The simplified method discussed in the previous section can not be used in designing skew girds. Structural analysis programs can be used to compute the bending moment and shear developed in the skew system in this case. Alternatively, for a relatively small number of beams, hand calculations can be carried out using equilibrium of forces and compatibility of deflections. For example, for the square hall shown in Fig. 3.8, the beams are perpendicular to each other but are not parallel to the hall directions. Due to symmetry, the problem has only 4 unknowns, which is the percent of loading distributed in each direction (P11, P12, P31, P32). At points 2 and 4 due to symmetry, the load transferred in the two directions is the same (i.e. 50%, P/2). The load acting at each intersection (joint) can be approximated by

$$P = w_u \times a \times b$$

This load is divided between each two perpendicular beams. For example, at point 1 and 3, the equilibrium equation is

$$P = P_{11} + P_{12} \qquad \text{(point1)}$$

$$P = P_{31} + P_{32} \qquad \text{(point3)}$$

The deflection at any point can be easily obtained using the principle of superposition. The compatibility of deflection at point 1 and 3 gives

Δ_1 (beam B3)=Δ_1 (beam B1) (point1)

Δ_3 (beam B3)=Δ_3 (beam B2) (point3)

Solving this set of equations gives the loads and bending moment at each beam.

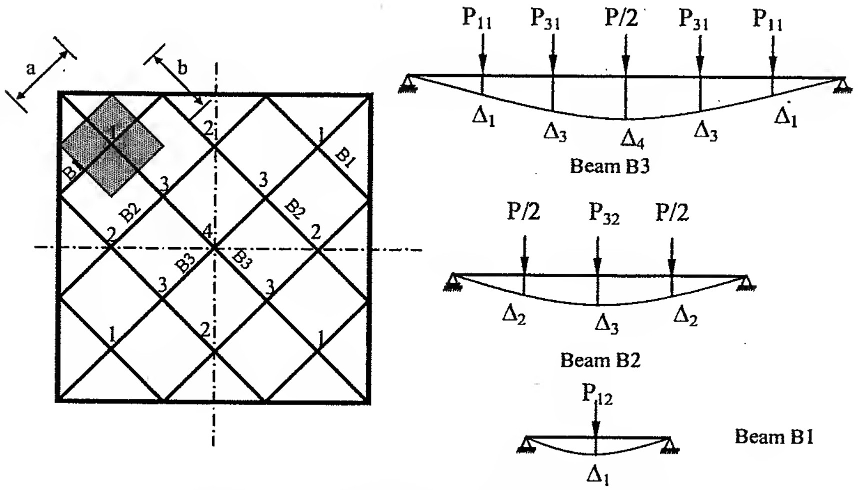


Fig. 3.8 Structural analysis of skew paneled beams

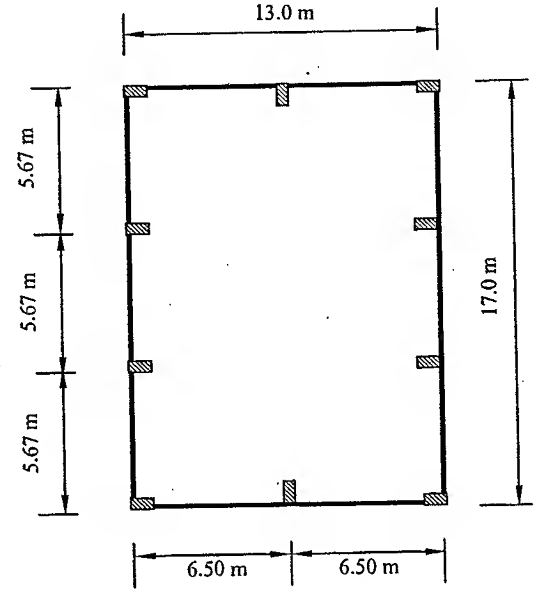
Example 3.1
For the hall shown in figure, it is required to design rectangular paneled beams system to cover the roof, knowing that

f_{cu}=30 N/mm^2

f_y=400 N/mm^2

Loads due to flooring=2.0 kN/m^2

Live loads=3.0 kN/m^2

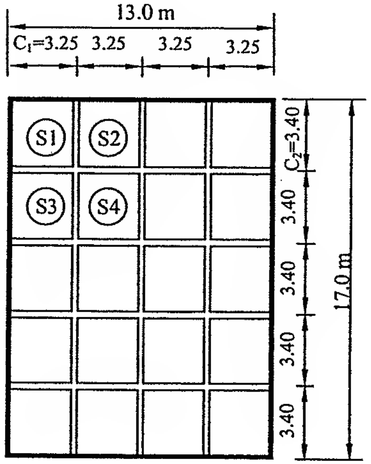


Solution

Step 1: Roof layout

The Hall is divided into small slabs spanning between (3-4 m in each direction). Assuming three beams spanning in the longitudinal direction (four spacing) and four beams spanning in the shorter direction (five spacing) gives

$$C_1 = \frac{13}{4} = 3.25\ m \qquad \text{and} \qquad C_2 = \frac{17}{5} = 3.40\ m$$



Step 2: Design of solid slabs

Since the slab is relatively small (3.4 x 3.25), assume t_s=100 mm

$w_{us}=1.4\ g_s+1.6\ p_s=1.4\ (\gamma_s \times t_s + \text{flooring})+1.6 \times \text{live loads}$

$$w_{us}=1.4\times(25\times0.10+2)+1.6\times3=11.1kN/m^2$$

The slab load distribution factors are determined using table (6-1) in the code.

$w_\alpha=w_{us} \times \alpha = (11.1)\ \alpha$

$w_\beta=w_{us} \times \beta = (11.1)\ \beta$

The rectangularity ratio r is given by

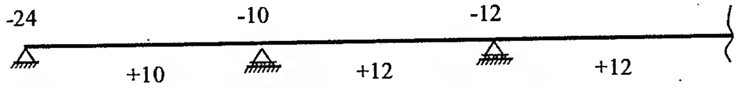
$$r=\frac{m_y\times L_y}{m_x\times L_x}$$

where m equals 1, 0.87, and 0.76 for simple, continuous from one end and continuous from both ends respectively

The following table summarizes the results

Slab	L_x	m_x	L_y	m_y	r	α	β	w_α	w_β
S1	3.25	0.87	3.4	0.87	1.046	0.37	0.32	4.141	3.550
S2	3.25	0.76	3.4	0.87	1.198	0.45	0.24	4.982	2.709
S3	3.25	0.87	3.4	0.76	1.094	0.40	0.29	4.408	3.245
S4	3.25	0.76	3.4	0.76	1.046	0.37	0.32	4.141	3.550

The bending moment for each slab is taken according to code coefficients given in figure below



$d_{\text{(main direction)}} = t_s\text{-}15\text{ mm}=85\text{ mm}$

$d_{\text{(secondary direction)}} = t_s\text{-}25\text{mm}=75\text{ mm}$

For high grade steel the minimum area of steel equals

$$A_{s,\min}=\frac{0.6}{f_y}b\times d=\frac{0.6}{400}\times1000\times85=127.5\ mm^2$$

The following table summarizes the results of the bending moment in x direction and y-directions

X-Direction

Slab	L_x (m)	w_x kN/m′	M_x kN.m	d (mm)	R	ω	As mm²/m′	As chosen	As chosen
S1	3.25	4.141	4.374	85	0.0202	0.024	151.4	5Φ8/m′	251.3
S2	3.25	4.982	4.385	85	0.0202	0.024	151.8	5Φ8/m′	251.3
S3	3.25	3.245	3.427	75	0.0203	0.024	134.5	5Φ8/m′	251.3
S4	3.25	4.141	3.645	85	0.0168	0.020	127.5*	5Φ8/m′	251.3

Y-Direction

Slab	L_y (m)	w_y kN/m′	M_y kN.m	d (mm)	R	ω	As mm²/m′	As chosen	As chosen
S1	3.4	3.550	4.104	75	0.0243	0.029	161.8	5Φ8/m′	251.3
S2	3.4	2.709	3.131	75	0.0186	0.022	127.5*	5Φ8/m′	251.3
S3	3.4	4.408	4.246	85	0.0196	0.023	146.9	5Φ8/m′	251.3
S4	3.4	3.550	3.420	75	0.0203	0.024	134.2	5Φ8/m′	251.3

* the minimum reinforcement (A_{smin}) controls the design

Step 3: Design of the paneled beams roof

Step 3.1: Assume concrete dimensions

Assume that the depth of the beams is shorter span/14

$t=13.0/14=0.93m$ →→Take t=950 mm and b=300 mm

Step 3.2: Calculate design loads

The self-weight of the beams is averaged over the slab using

$$ow_b = \gamma_c \times b \times (t - t_s) \times \frac{n_1 \times L_1 + n_2 \times L_2}{L_1 \times L_2}$$

$$ow_b = 25 \times \frac{300}{1000} \times \frac{(950-100)}{1000} \times \frac{3\times17+4\times13}{17\times13} = 2.97\ kN/m^2$$

$$w_u = 1.4 \times o.w\ of\ beams\,(kN/m^2) + w_{us}\ (kN/m^2)$$

$$w_u = 1.4\times2.97 + 11.1 = 15.26\ kN/m^2$$

Step 3.3: Distribution of the design loads

To determine the distribution of the load in both directions, use the total size of the hall not the individual slab dimension

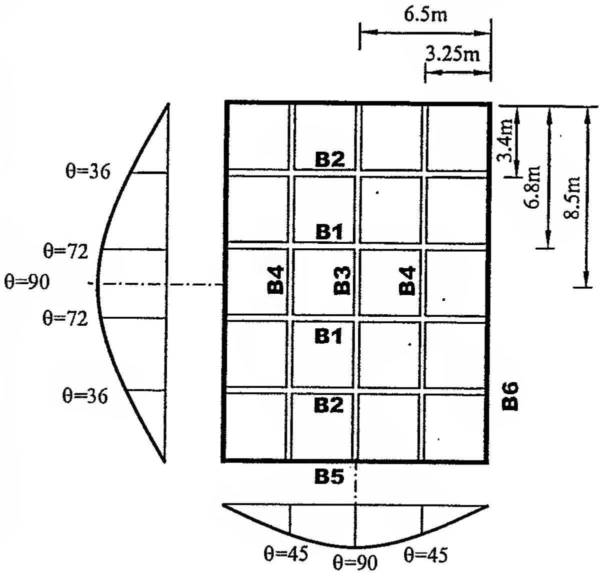
$$r = \frac{\text{Long span}}{\text{Short span}} = \frac{17}{13} = 1.307 < 1.5 \text{ ....o.k}$$

From Table 3.1, it can be determined that

α=0.746 and β=0.253, thus

w_α=0.746 x 15.26 =11.39 kN/m^2, this load is for beams in the short direction (13m)

w_β=0.253 x 15.26 =3.87 kN/m^2, this load is for beams in the long direction (17m)



Step 3.4: Design of paneled beams

Step 3.4.1: Design of beam B1 (short direction)

Design beam B1 for flexure

Since beam B1 is spanning in the short direction, the load considered for design is w_α

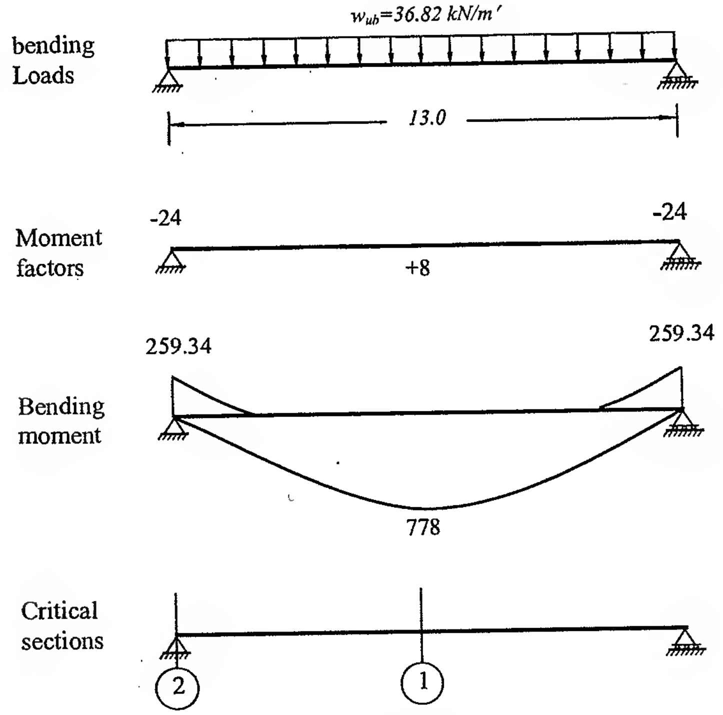
The compatibility angle θ equals the ratio of its distance relative to the centerline of the hall, thus

$$\theta = \frac{6.8}{8.5} \times 90 = 72^\circ$$

w_{ub}= w_α × spacing × sin θ

$$w_{ub} = 11.39\times3.4\times\sin(72) = 36.83\,kN/m'$$

$$M_u = \frac{w_{ub}\times L^2}{8} = \frac{36.83\times13^2}{8} = 778\ kN.m$$



Beam B1

Section 1

This section has positive bending (778 kN.m), the compression flange form a T-section. The effective width equals

$$B = smaller\ of \begin{cases} 16t_s + b & = 16\times100+300 = 1900\ mm \quad \lrcorner \\ \dfrac{L}{5}+b & = \dfrac{13\times1000}{5}+300 = 2900\ mm \\ CL\ to\ CL & = 3400\ mm \end{cases}$$

B=1900 mm

Using the C1-J curve

$$d = C1\sqrt{\frac{M_u}{f_{cu}B}}$$

$$900 = C1\sqrt{\frac{778\times10^6}{30\times1900}} \qquad C1=7.7$$

The point is outside the curve use $c/d)_{min}=0.125$

c=0.125 x 900=112.5 mm

a=0.8 x 112.5 = 90 mm <t_s .....o.k.

use j=0.825

$$A_s = \frac{M_u}{j\times d\times f_y} = \frac{778\times10^6}{0.825\times900\times400} = 2616\ mm^2$$

$$A_{s\,min} = smaller\ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y}b\ d = \dfrac{0.225\sqrt{30}}{400}\times300\times900 = 832\ mm^2 \ \lrcorner < A_s \quad o.k \\ 1.3A_s = 1.3\times2616 = 3401 mm^2 \end{cases}$$

use (4Φ22+4Φ20, A_s=2777 mm^2)

A_s'=(0.1-0.2) A_s choose (3Φ16, 603 mm^2)

Section 2

$$M_u = \frac{w_{ub}\times L^2}{24} = \frac{36.83\times13^2}{24} = 259.34\ kN.m$$

$$R = \frac{M_u}{f_{cu}bd^2} = \frac{259.34\times10^6}{30\times300\times900^2} = 0.0355$$

From the curve it can be determined that ω=0.043

$$A_s = \omega\frac{f_{cu}}{f_y}b\times d = 0.043\frac{30}{400}\times300\times900 = 870mm^2 > A_{s,min}$$

use (4 Φ 20, 1256 mm^2)

Design Beam B1 for Shear

The critical section for shear is at d/2 from the face of the support. Assuming that column width (c) is 500 mm

$$Q_u = \frac{w_{ub}\times L}{2} - w_{ub}(\frac{d}{2}+\frac{c}{2}) = \frac{36.83\times13}{2} - 36.83\left(\frac{0.90}{2}+\frac{0.50}{2}\right) = 213.6\ kN$$

$$q_u = \frac{Q_u}{b\times d} = \frac{213.6\times1000}{300\times900} = 0.791\ N/mm^2$$

The concrete shear strength is given by

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{1.5}} = 0.24\sqrt{\frac{30}{1.5}} = 1.07\ N/mm^2$$

Since $q_u<q_{cu}$ the beam is considered safe, provide minimum area of stirrups

Assuming spacing of 200 mm and using mild steel f_y=240 N/mm^2

$$A_{st,min} = \frac{0.4}{f_y}b\times s = \frac{0.4}{240}\times300\times200 = 100\ mm^2$$

Assuming 2 branches, the area of one branch equals

$$A_{sb} = \frac{100}{2} = 50\ mm^2 \qquad \text{choose } \phi8\ (50\ mm^2)$$

Use 5ϕ8/m'

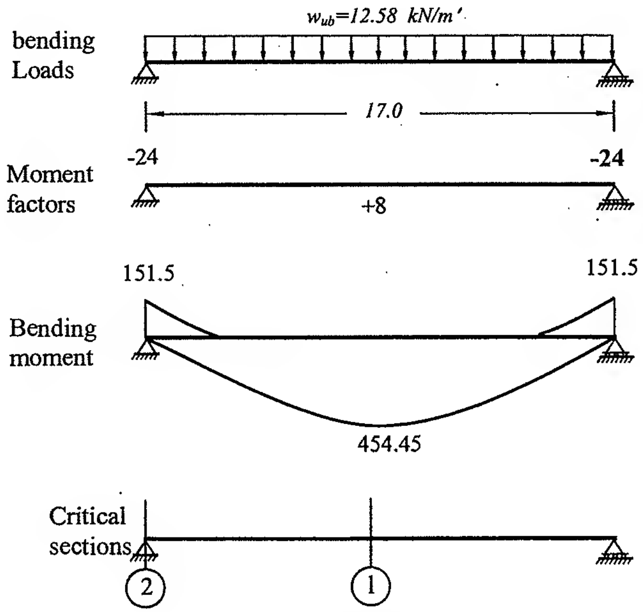
Step 3.4.2: Design of Beam B3 (longitudinal)

Design Beam B3 for flexure

Since this beam spanning in the long direction, the load considered is w_β(3.87 kN/m^2). The compatibility angle θ equals the ratio of the its distance relative to the distance of centerline of the hall, thus

$$\theta = \frac{6.5}{6.5}\times90 = 90^o$$

$$w_{ub} = w_\beta\times spacing\times\sin\theta = 3.87\times3.25\times\sin(90) = 12.58kN/m'$$



Beam B3

Section 1

$$M_u = \frac{w_{ub} \times L^2}{8} = \frac{12.58 \times 17^2}{8} = 454.45\ kN.m$$

This section has positive bending (454.45 kN.m), the compression flange form a T-section.

$$B = smaller\ of \begin{cases} 16t_s + b & = 16 \times 100 + 300 = 1900\ mm \quad \downarrow \\ \frac{L}{5} + b & = \frac{17 \times 1000}{5} + 300 = 3700\ mm \\ CL\ to\ CL & = 3250\ mm \end{cases}$$

B=1900 mm

Because the steel in the longitudinal direction is placed on top of the steel in the short direction, the depth is less than the transverse direction by approximately 50 mm.

d= 850 mm

Using the C1-J curve

$$850 = C1\sqrt{\frac{454.45 \times 10^6}{30 \times 1900}} \qquad \rightarrow C1=9.52$$

The point is outside the curve use $c/d)_{min}$=0.125 use j=0.825

c=0.125 x 850=106.25 mm

a=0.8 x 106.25 = 85 mm <t_s …..o.k.

$$A_s = \frac{M_u}{j \times d \times f_y} = \frac{454.45 \times 10^6}{0.825 \times 850 \times 400} = 1620\ mm^2$$

$$A_{s\,min} = smaller\ of \begin{cases} \frac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \frac{0.225\sqrt{30}}{400} \times 300 \times 850 = 786\ mm^2 \ \downarrow < A_s \quad o.k \\ 1.3A_s = 1.3 \times 1620 = 2106\ mm^2 \end{cases}$$

use (5Φ22, 1900 mm^2)

A_s'=(0.1-0.2) A_s choose (2Φ16, 400 mm^2)

Section 2

$$M_u = \frac{w_{ub} \times L^2}{24} = \frac{12.58 \times 17^2}{24} = 151.5\ kN.m$$

$$R = \frac{M_u}{f_{cu} b d^2} = \frac{151.5 \times 10^6}{30 \times 300 \times 850^2} = 0.0233$$

From the curve it can be determined that ω=0.028

$$A_s = \omega \frac{f_{cu}}{f_y} b \times d = 0.028 \frac{30}{400} \times 300 \times 850 = 535\ mm^2$$

$$A_{s\,min} = smaller\ of \begin{cases} \frac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \frac{0.225\sqrt{30}}{400} \times 300 \times 850 = 786\ mm^2 \\ 1.3A_s = 1.3 \times 535 = 696 mm^2 \ \downarrow > A_s\ (check\ 0.15/100\ b\ d) \end{cases}$$

But not less than $\frac{0.15}{100} \times 300 \times 850 = 382.5\ mm^2$

Use $A_{s,min}$ (696 mm^2)→→use (2 Φ 22, 760 mm^2)

Design of Beam B3 for Shear

Since beam B1 (which have more load than beam B3) requires minimum stirrups for shear, provide minimum stirrups as well for beam B3(5ϕ8/m')

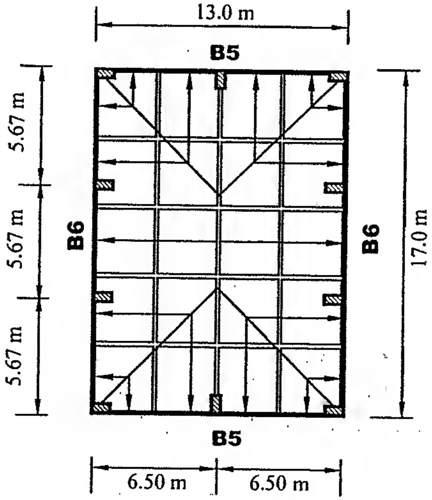
Step 3.4.3:.Design of Beam B2 and B4

Item	Beam B2 short direction	Beam B4 long direction
w_u	11.39	3.87
spacing (m)	3.40	3.25
θ	36	45
$w_{ub}=w_u$ x spacing x sin (θ) kN/m′	22.76	8.89
span (m)	13	17
$M_u=w_{ub}\ L_2/8$ (kN.m)	480.86	321.48
B (mm)	1900	1900
C1	9.8	11.3
c/d (=c/d,min)	0.125	0.125
a (mm)	90 <t_s	85 < t_s
J	0.825	0.825
d (mm)	900	850
A_s (mm^2)	1616	1144
No of bars	5Φ22	6Φ16
Shear design	5ϕ8/m′	5ϕ8/m′

Step 4: Design of edge beams

Step 4.1: Design of edge beam B5

The load distribution on the supporting beams is shown in figure below.



The load on each beam will be calculated using the area method because the load is not symmetrical between each two successive supports.

The thickness of this beam should be at least the thickness of the paneled beams, thus try a beam cross section (250 x 950 mm)

Ultimate beam self weight =1.4 γ_c b t

$$w_{ow} = 1.4 \times 25 \times 0.25 \times 0.95 = 8.31\ kN/m'$$

$$w_{slab} = w_u \times \frac{area\ of\ the\ triangular}{span} = 15.26 \times \frac{0.5 \times 6.5 \times 6.5}{6.5} = 49.6\ kN/m'$$

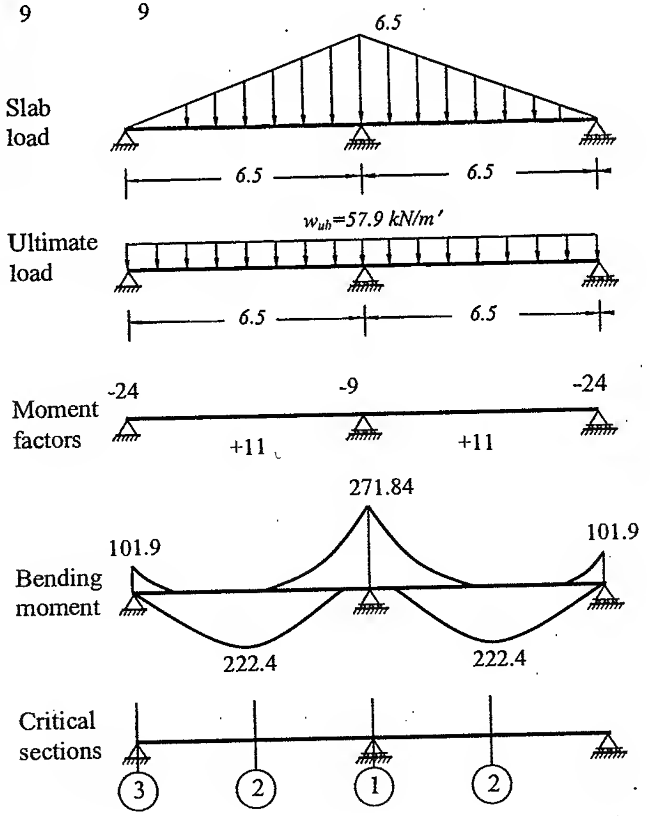
$w_u = w_{ow} + w_{slab}$ =8.31+49.6=57.9 kN/m′

Since the beam has an equal loads and spans, the code coefficients for continuous beam with two spans can be used

Design of beam B5 for flexure

Section 1

$$M_u = \frac{w_{ub} \times L^2}{9} = \frac{57.9 \times 6.5^2}{9} = 271.84\ kN.m$$



Since this section is subjected to negative bending, it will be designed as rectangular section

$$R=\frac{M_u}{f_{cu}bd^2}=\frac{271.84\times10^6}{30\times250\times900^2}=0.0447$$

From the curve it can be determined that ω=0.054

$$A_s=\omega\,\frac{f_{cu}}{f_y}b\times d=0.054\,\frac{30}{400}\times250\times900=916\ mm^2>A_{s,min}$$

use (5 Φ 16, 1005 mm^2)

Section 2

$$M_u=\frac{w_{ub}\times L^2}{11}=\frac{57.9\times6.5^2}{11}=222.4\ kN.m$$

This section has positive bending (222.4 kN.m), the compression flange form a L-section.

$$B=smaller\ of\begin{cases}6t_s+b & =6\times100+250=850\ mm\\ \frac{L_2}{10}+b & =\frac{0.8\times6.5\times1000}{10}+250=770\ mm\ \hookleftarrow\\ CL\ to CL & =1700\ mm\end{cases}$$

B=770 mm

Using the C1-J curve

$$d=C1\sqrt{\frac{M_u}{f_{cu}B}}$$

$$900=C1\sqrt{\frac{222.4\times10^6}{30\times770}}\qquad C1=9.17$$

The point is outside the curve use $c/d)_{min}$=0.125

c=0.125 x 900=112.5 mm

a=0.8 x 112.5 = 90 mm <t_s .....o.k.

use j=0.825

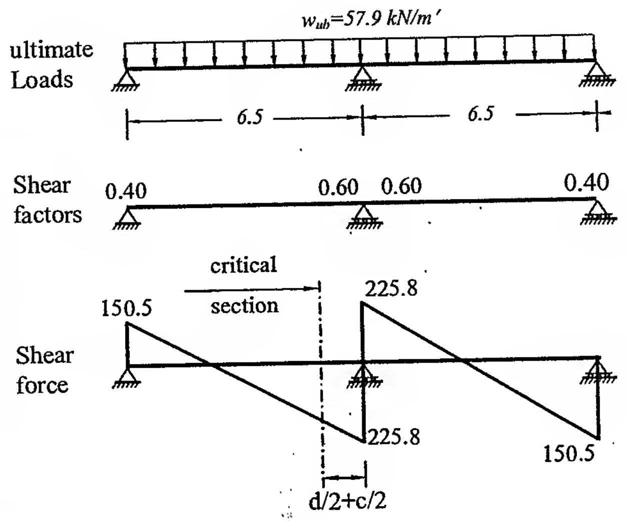
$$A_s=\frac{M_u}{j\times d\times f_y}=\frac{222.4\times10^6}{0.825\times900\times400}=748\ mm^2$$

$$A_{s\,min}=smaller\ of\begin{cases}\frac{0.225\sqrt{30}}{400}\times250\times900=693\,mm^2\ \hookleftarrow<A_s\quad o.k\\ 1.3A_s=1.3\times748=972.4\,mm^2\end{cases}$$

use (4Φ16, 804 mm^2)

Design of Beam B5 for shear

The critical section for shear is at d/2 from the face of the support. Assuming that column width (c) is 250 mm



$$Q_u=0.6\times w_{ub}\times L-w_{ub}(\frac{d}{2}+\frac{c}{2})=225.8-57.9\left(\frac{0.90}{2}+\frac{0.25}{2}\right)=192.5\ kN$$

$$q_u=\frac{Q_u}{b\times d}=\frac{192.5\times1000}{250\times900}=0.856\ N/mm^2$$

The concrete shear strength is given by

$$q_{cu}=0.24\sqrt{\frac{f_{cu}}{1.5}}=0.24\sqrt{\frac{30}{1.5}}=1.07\,N/mm^2$$

Since $q_u<q_{cu}$ the beam is considered safe, provide minimum area of stirrups

Assuming spacing of 200 mm and using mild steel f_y=240 N/mm^2

$$A_{st,min}=\frac{0.4}{f_y}b\times s=\frac{0.4}{240}\times250\times200=83.33\ mm^2$$

Assuming 2 branches, the area of one branch equals

$$A_{sb}=\frac{83.33}{2}=41.67\ mm^2$$ choose ϕ8 (50 mm^2), Choose 5 ϕ 8/m'

Step 4.2 Design of Beam B6

$w_{ow} = 1.4 \times 25 \times 0.25 \times 0.95 = 8.31\ kN/m'$

The weight of the slab will be divided in two parts

$$w_1 = w_u \times \frac{area\ of\ the\ triangular(A_1)}{span} = 15.26 \times \frac{0.5 \times 5.67 \times 5.67}{5.67} = 43.26\ kN/m'$$

$w_{u1}=w_1+w_{ow}$ =43.26 + 8.31=51.5 kN/m'

$$w_2 = w_u \times \frac{area\ of\ A_2}{span} = 15.26 \times \frac{4.01 \times 6.5 + 2 \times 0.5(5.67+6.5) \times 0.83}{5.67} = 97.33\ kN/m'$$

$w_{u2}=w_2+w_{ow}$ =97.33 + 8.31=105.64 kN/m'

Since the beam is equal in spans but the load magnitude differs by more than 20%, code coefficient ***can not*** be used. For simplicity, pattern loading was not considered. Using three moment equations to solve the indeterminate beam, applying the equation at b

$M_a L_1 + 2 M_b (L_1+L_2) + M_c L_2 = -6(R_{ba}+R_{bc})$

The elastic reaction R_{ab} and R_{bc} equals

$$R_{ab} = \frac{w_{u1} \times L^3}{24} = \frac{51.5 \times 5.67^3}{24} = 391.15\, kN.m^2$$

$$R_{bc} = \frac{w_{u2} \times L^3}{24} = \frac{105.6 \times 5.67^3}{24} = 802\, kN.m^2$$

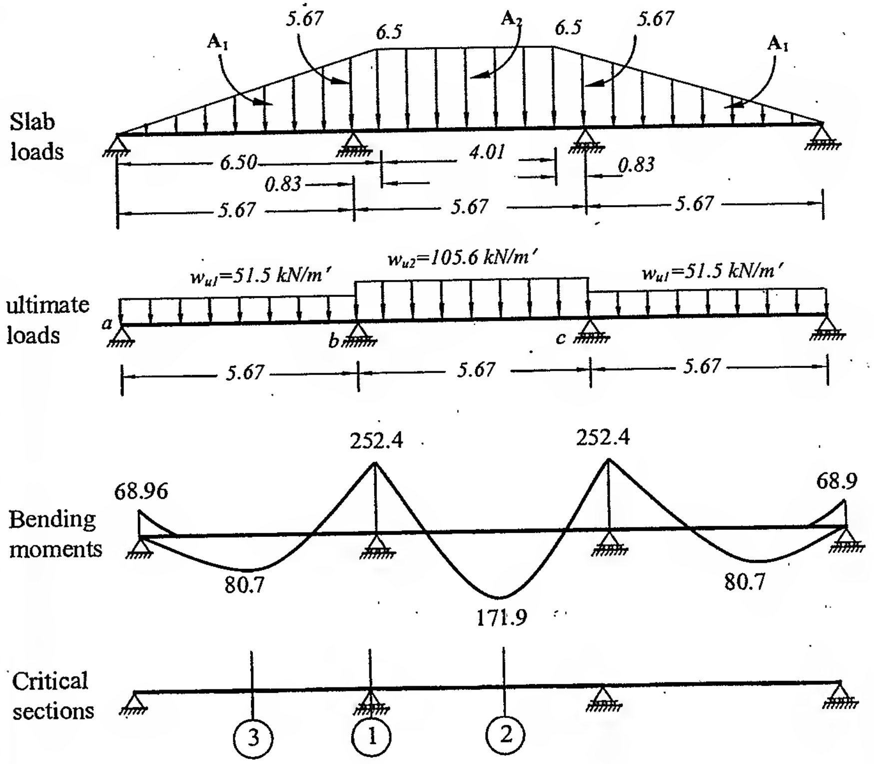
From symmetry $M_b=M_c$ and Ma=0, substituting in the moment equation gives

$0 + 2 M_b (5.67+5.67) + M_b (5.67) = -6 (391.15+802)$

$M_b = -252.4$ kN.m

$$M(+ve)span\,(ab) = \frac{51.5 \times 5.67^2}{8} - \frac{252.4}{2} = 80.7\ kN.m$$

$$M(+ve)span\,(bc) = \frac{105.6 \times 5.67^2}{8} - 252.4 = 171.9\ kN.m$$



Design of beam B6 for flexure

Section 1

M_u=252.4 kN.m (rectangular section)

$$R = \frac{M_u}{f_{cu} b d^2} = \frac{252.4 \times 10^6}{30 \times 250 \times 900^2} = 0.0415$$

From the curve it can be determined that ω=0.050

$$A_s = \omega \frac{f_{cu}}{f_y} b \times d = 0.05 \frac{30}{400} \times 250 \times 900 = 843.75\ mm^2$$

$$A_{s\,\min} = smaller\ of \begin{cases} \dfrac{0.225\sqrt{30}}{400} \times 250 \times 900 = 693\ mm^2 < A_s \quad o.k \\ 1.3 A_s = 1.3 \times 843 = 1095\, mm^2 \end{cases}$$

use (2 Φ 16+2 Φ 18, 911 mm^2)

Section 2

This section has positive bending (171.9 kN.m), the compression flange forms L-section.

$$B = smaller\ of \begin{cases} 6t_s + b & = 6\times100+250 = 850\ mm \\ \frac{L_2}{10}+b & = \frac{0.7\times5.67\times1000}{10}+250 = 646.9\ mm \\ CL\ to\ CL & = 1700\ mm \end{cases}$$

B=646.9 mm

Using the C1-J curve

$$d = C1\sqrt{\frac{M_u}{f_{cu}B}}$$

$$900 = C1\sqrt{\frac{171.9\times10^6}{30\times646.9}} \qquad C1=9.56$$

The point is outside the curve use $c/d)_{min}=0.125$

c=0.125 x 900=112.5 mm

a=0.8 x 112.5 = 90 mm $<t_s$ .....o.k.

use j=0.825

$$A_s = \frac{M_u}{j\times d\times f_y} = \frac{171.9\times10^6}{0.825\times900\times400} = 578\ mm^2$$

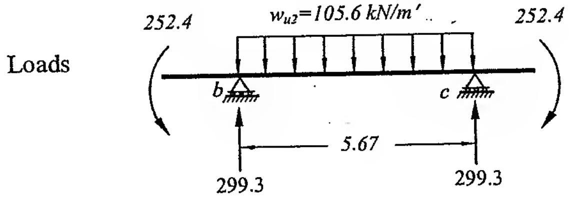
$$A_{s\,min} = smaller\ of \begin{cases} \frac{0.225\sqrt{30}}{400}\times250\times900 = 693\ mm^2 \\ 1.3A_s = 1.3\times578 = 752\ mm^2 \end{cases}$$

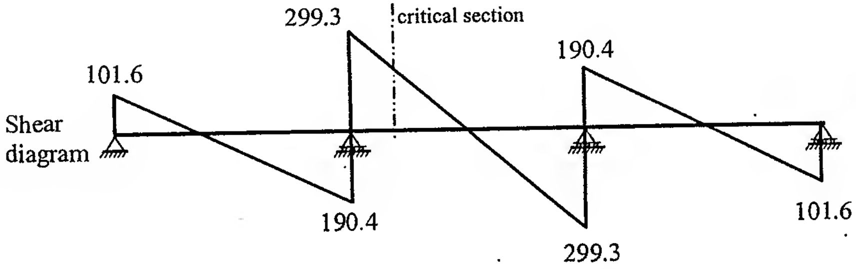
$A_s < A_{s,min}$ use $A_{s,min}$ (693 mm^2)

use (2 Φ 16+2 Φ 18, A_s=911 mm^2)

Design of Beam B6 for shear

Since the loading on the beam is not equal on all spans, code shear coefficients can not be used. The maximum shear is at section middle span (bc) thus, the maximum shear can be obtained from the structural analysis of the beam.





The critical section for shear is at d/2 from the face of the support(b). Assuming that column width (c) is 250 mm

$$Q_u = R_b - w_{ub}\left(\frac{d}{2}+\frac{c}{2}\right) = \frac{105.6\times5.67}{2} - 105.6\left(\frac{0.9}{2}+\frac{0.25}{2}\right) = 238.6\ kN$$

$$q_u = \frac{Q_u}{b\times d} = \frac{238.6\times1000}{250\times900} = 1.06\ N/mm^2$$

concrete shear strength is given by

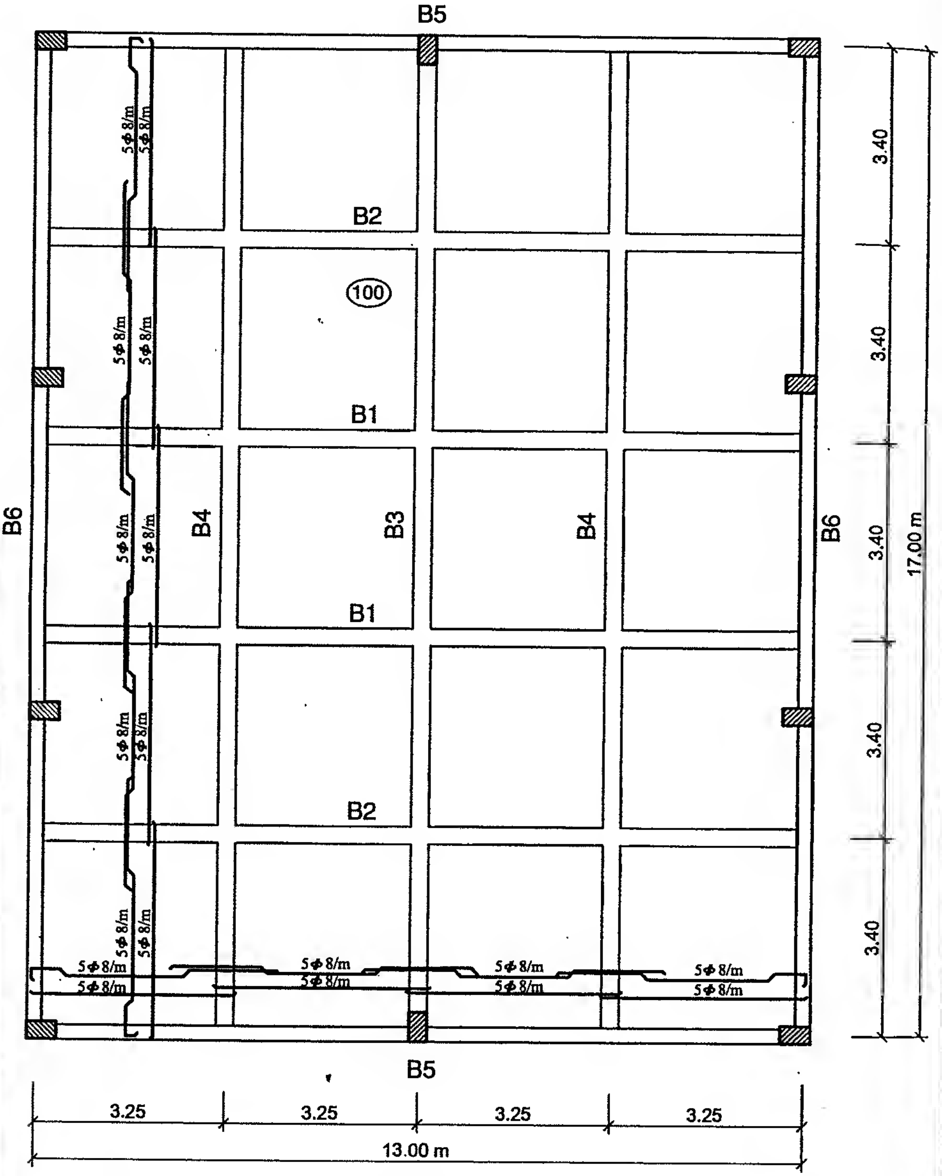
$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{1.5}} = 0.24\sqrt{\frac{30}{1.5}} = 1.07\ N/mm^2$$

Since $q_u<q_{cu}$ the beam is considered safe, provide minimum area of stirrups

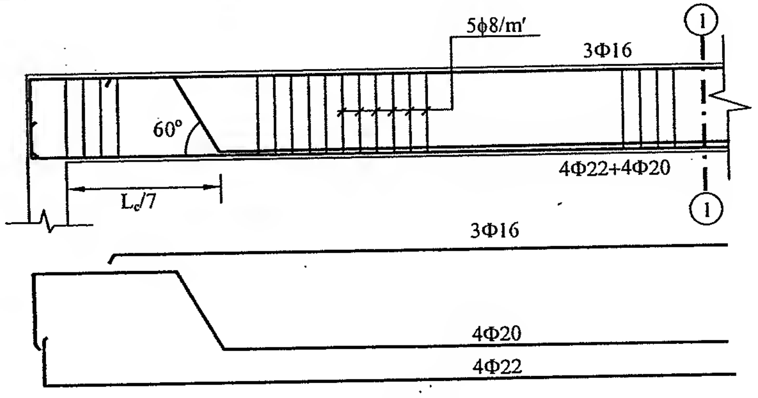
Assuming spacing of 200 mm and using mild steel f_y=240 N/mm^2

$$A_{st,min} = \frac{0.4}{f_y}b\times s = \frac{0.4}{240}\times250\times200 = 83.33\ mm^2$$

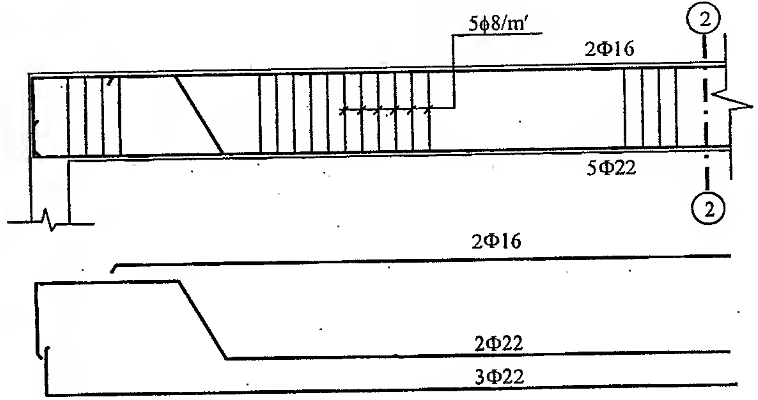
Choose 5 ϕ 8/m'



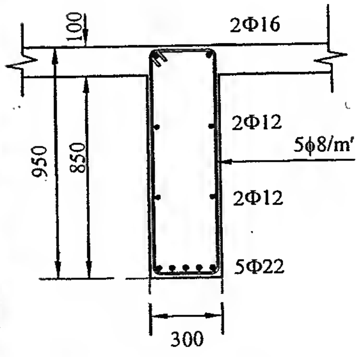
Reinforcement details



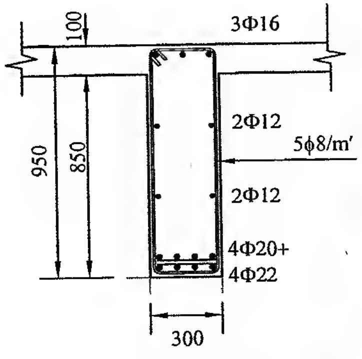
Beam B1



Beam B3

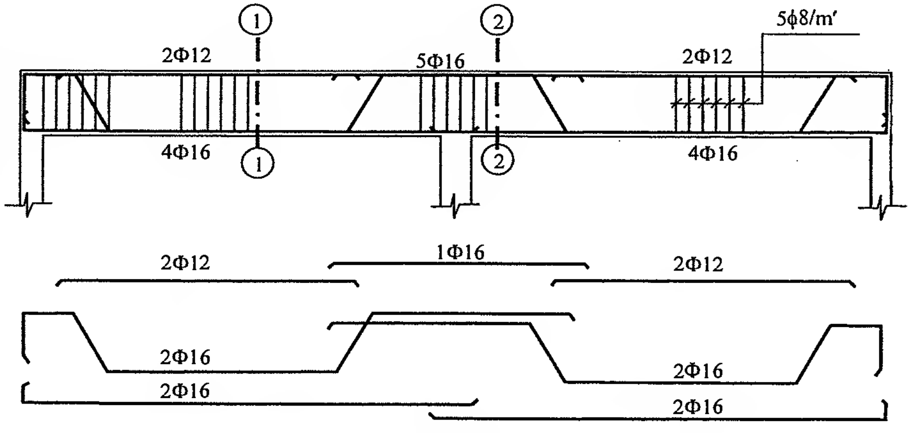


Sec 2 in Beam B3 (longitudinal direction)

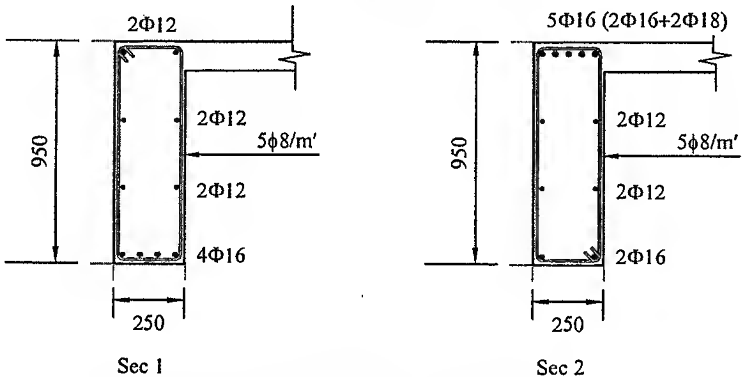


Sec 1 in Beam B1 (short direction)

Reinforcement details for beams B1 and B3



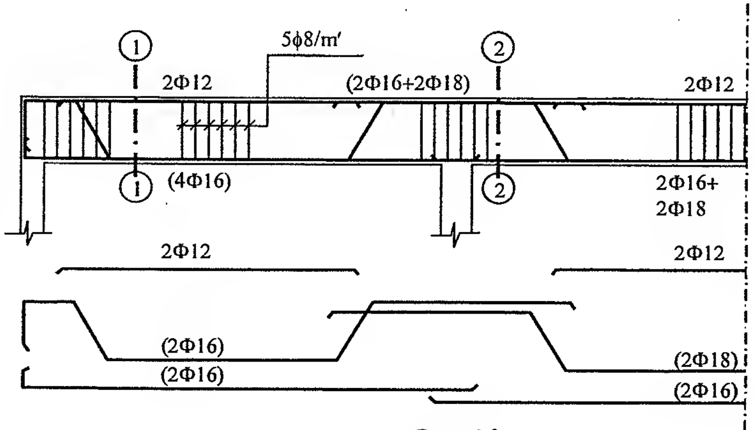
Beam B5



Sec 1

Sec 2

Sections in beams B5 and (B6)



Beam B6

Reinforcement details for beams B5 and B6

4

FLAT SLABS

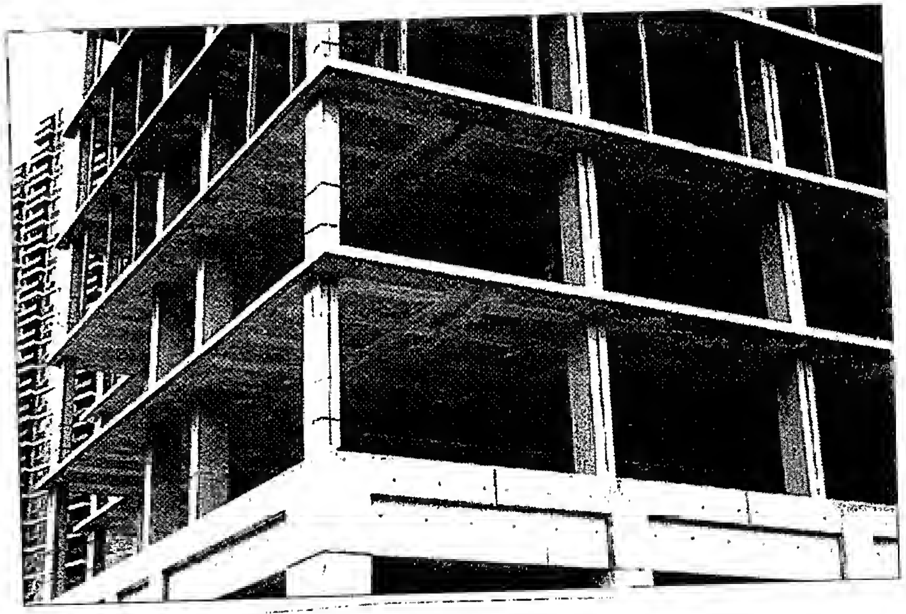
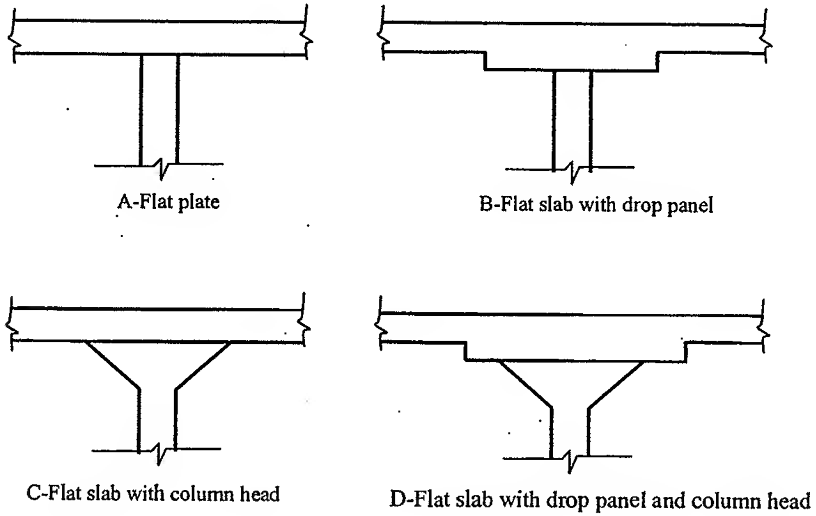


Photo 4.1 Flat slab system

4.1 Introduction

Flat plates are one of the most commonly used structural systems in residential buildings, hotels, commercial buildings, hospitals and office buildings. Flat plates are solid concrete slabs of uniform thickness that transfer the load directly to the columns without the presence of projected beams, drop panels or column capitals as shown in Fig. 4.1.A. The ease of construction is one of the important aspects that make flat slab systems a very attractive solution. Architects prefer this system because the flexibility in the arrangement of columns and partitions with no obstruction of light. In addition, the absence of sharp corners gives greater fire resistance, as there is less danger of the concrete spalling and exposure of the reinforcement.

The major concern when using flat plates is shear transfer from the slab to the columns. In other words, there is a danger that columns may punch through the slab. Therefore, if live loads exceed 3 kN/m^2, it is advisable to use drop panels at the column locations as shown in Fig. 4.1.B. The use of the drop panels increases the negative flexural capacity at locations of high negative bending and reduces the risk of shear failure. Moreover, if the live loads exceed 6 kN/m^2, it is recommend to use column heads as shown in Fig 4.1.C. In case of excessively heavy live loads (>10 kN/m^2) such as in the case of industrial buildings and warehouses, the use of drop panels together with column heads is necessary as shown in Fig. 4.1.D.


A-Flat plate

B-Flat slab with drop panel

C-Flat slab with column head

D-Flat slab with drop panel and column head

Fig. 4.1 Types of flat slabs

The use of drop panels and column heads is more acceptable in parking garages, storage buildings, and similar structures.

4.2 Statical Equilibrium of Flat Slabs

The floor shown in Fig. 4.2 consists of individual concrete strips that form a slab. The two beams support the concrete strips at their edges. Assume that the total load of the strips including self-weight equals w_u (kN/m^2). The moment per meter at mid-span (section A-A) equals

$$M = \frac{w_u \times L_2^2}{8} \quad \text{.......(4.1)}$$

The total moment in X-direction for the entire slab equals

$$M_{A-A} = \frac{(w_u \times L_1) \times L_2^2}{8} \quad \text{.......(4.2)}$$

The reaction of the slab per meter is

$$R = \frac{w_u \times L_2}{2} \quad \text{.......(4.3)}$$

The moment at mid span of the beam in Y-direction is

$$M_{beam} = \frac{R \times L_1^2}{8} = \left(\frac{w_u \times L_2}{2}\right) \times \frac{L_1^2}{8} \quad \text{.......(4.4)}$$

The total moment for both beams equals

$$M_{total} = \frac{(w_u \times L_2) \times L_1^2}{8} \quad \text{.......(4.5)}$$

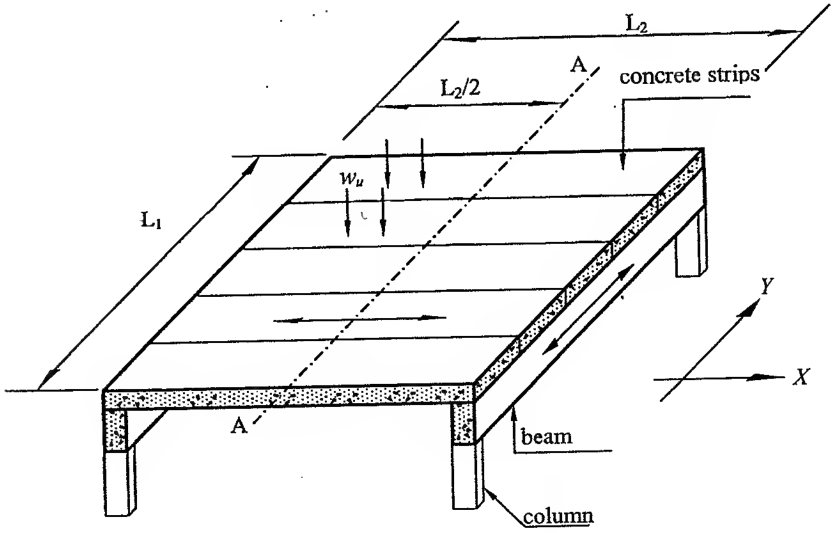


Fig. 4.2 Analysis of individual concrete strips supported on beams

It is clear that the full load transferred by the concrete strips causes a moment in the X-direction of the slab and the full load is transferred again by the beams causing bending moment in Y-direction. This system of transferring the load is similar to a flat slab system supported on four columns. The total moment in X-direction equals

$$\frac{(w_u \times L_1) \times L_2^2}{8} \qquad \text{.. (4.6)}$$

and the total moment in Y-direction equals

$$\frac{(w_u \times L_2) \times L_1^2}{8} \qquad \text{.. (4.7)}$$

It can be concluded that whether the structural system is slab-beam system or flat slabs as shown in Fig.4.3, the load is transferred in both directions. By comparing Equations 4.6 and 4.7, it can be noticed that the largest moment develops in the longer span direction.

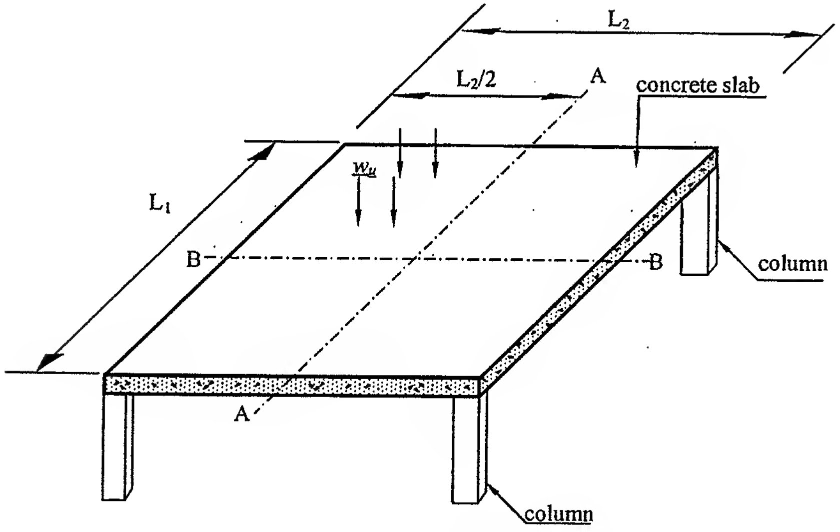


Fig. 4.3 Analysis of a flat plate system

4.3 Minimum Dimensions According to ECP 203

The Egyptian code requires certain minimum dimensions for different elements of the flat slab system. These minimum requirements are summarized in Fig. 4.4.

Slab Thickness(t_s)

$$t_s \text{ the bigger of} \begin{cases} 150\ mm & \\ L/32 & without\ drop \\ L/36 & with\ drop \end{cases}$$

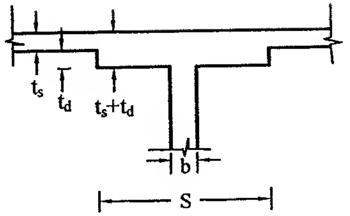
where L is the longer span

- **Drop thickness (t_d)**

$$t_d \geq \frac{t_s}{4}$$

- **Drop width (S)**

$$S \begin{cases} \geq \dfrac{L_{under\ consideration}}{3} \\ \leq \dfrac{L_2 (smaller\ span)}{2} \end{cases}$$



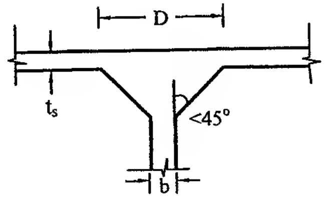
- **Column width (b)**

$$b \text{ the bigger of} \begin{cases} \dfrac{L_{under\ consideration}}{20} \\ \dfrac{H}{15} \quad (H = floor\ height) \\ 300\ mm^{*} \end{cases}$$

* The width of the column may be taken equal to 250 mm provided that detailed column moment transfer calculations were performed

- **Column head width (D)**

$$D \leq \frac{L_2}{4}$$



- **Column Strip width (C_s)**

$$C_s \begin{cases} \dfrac{L_2}{2} & without\ drop \\ Drop\ width\,(S) & with\ drop \end{cases}$$

- **Field Strip width (F_s)**

$$F_s = L_{under\ consideration} - C_s$$

- **Marginal beam thickness (t)**

$$t \geq 3t_s$$

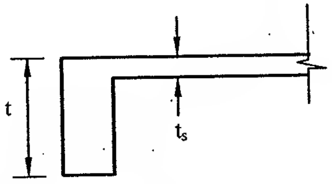
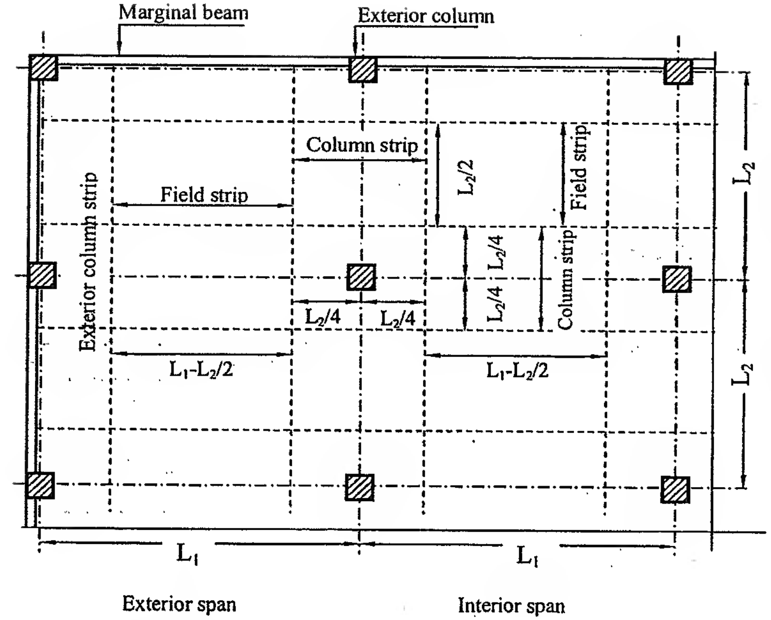
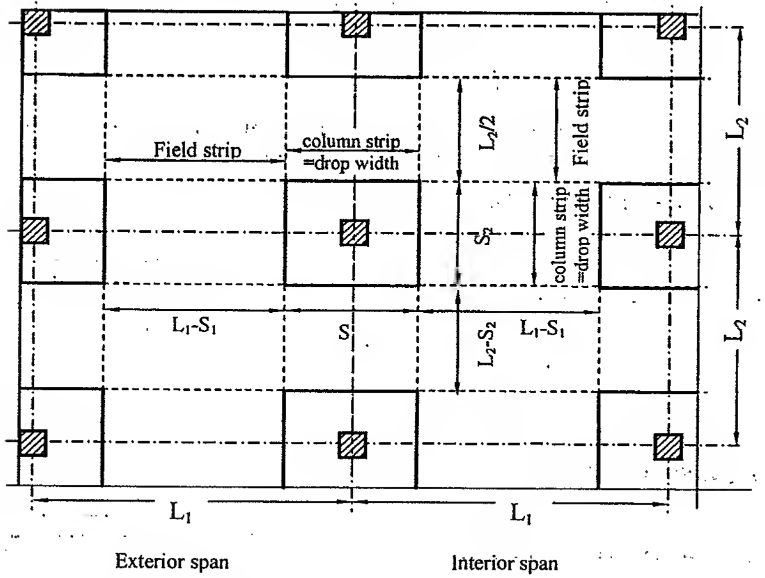


Fig. 4.4 Minimum flat slab dimensions according to ECP 203



Flat slab without drop panel



Flat slab with drop panel

Fig. 4.4 Minimum flat slab dimensions (cont.)

4.4 Analysis of Flat Slabs

Flat Slabs may be analyzed and designed by any method that ensures that all the strength and serviceability requirements of the ECP 203 are satisfied. When slabs are supported on a rectangular grid of columns, the code offers two simplified methods for analyzing the flat slab system. The first method is the direct design method and the second one is the equivalent frame method. These two methods, which are mentioned in section 6-2-5-3 of the ECP 203, will be discussed in detail in the following sections. In addition, if the slabs have unusual geometric configurations or the columns are spaced irregularly, neither of the code methods becomes applicable. For these special cases, the designer may analyze the floor by using finite element analysis (computer model), in which the slab is divided into small finite elements connected by nodes. The stiffness matrix of each element is computed and the global stiffness matrix is constructed. Having determined the deformation at each node, the element internal forces can be evaluated (refer to Section 4.10 of this text).

4.5 Direct Design Method

4.5.1 Limitation of the direct method

To ensure that the moments at the critical sections are adequate, the ECP 203 requires that the following design conditions be satisfied:

1. A minimum of three continuous spans in each direction.
2. The ratio of the longer to the shorter span within a panel should not exceed 1.3
3. Successive span lengths in each direction should not differ by more than 10%
4. Non-Successive span lengths in each direction should not differ by more than 20%

4.5.2 Definition of column strip and field strip

The distribution of moment varies continuously across the width of the slab panel. To simplify the steel arrangement, the design moments are averaged over the width of the column strip.

The column strip width should be taken as ½ the short direction for flat slabs without drop panels. In the case of flat slab with drop panel, the column strip width equals the drop width. The width of the field strip equals the difference between span length and column strip width as shown in Fig. 4.5.

4.5.3 Calculation of slab load

The calculation of the slab load is carried out in a similar manner like solid slabs. However, since the brick walls are often distributed over the plan, an average wall load w_w is added to the design loads as follows:

$$w_w = \frac{\gamma_w \times h_w \times \sum b_w \times wall\ length}{area\ of\ the\ floor} \quad \text{.....................................} \quad (4.8.a)$$

The ultimate loads for flat slabs without drop is

$$w_u = 1.4\times(t_s\times\gamma_c + w_{flooring} + w_w) + 1.6\times w_{LL} \quad \text{..........................} (4.8.b)$$

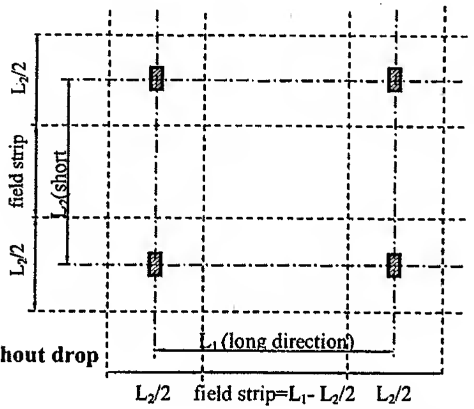
In case of flat slabs with drop, the drop weight is considered by averaging the area of the drop over the total area of the slab then multiplying by concrete weight as follows

$$w_{drop} = \gamma_c \times t_d \times \frac{S_1\times S_2}{L_1\times L_2} \quad \text{..} (4.9)$$

The ultimate loads for flat slabs with drop is

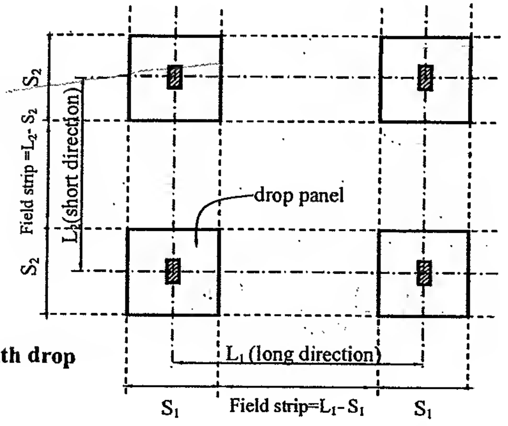
$$w_u = 1.4\times(t_s\times\gamma_c + w_{drop} + w_{flooring} + w_w) + 1.6\times w_{LL} \quad \text{................} (4.10)$$

($\gamma_c = 25\ kN/m^3$ and t_s in meters)



A. Flat slab without drop

S1=column strip width in the short direction.
S2=column strip width in the long direction.
td= drop thickness



B. Flat slab with drop

Fig. 4.5 Definition of column and field strips

4.5.4 Statical Moment M_o

The direct design method is an empirical procedure for establishing the design moments at the critical sections. It is well known that the sum of the negative moment at the supports and positive moment at midspan of a uniformly loaded beam is $w_uL^2/8$. Accordingly, for a uniformly loaded slab with width L2, and span L1, the total statical moment M_o equals

$$M_o = \frac{w_u\times L_2\times L_1^2}{8} \quad \text{..} (4.11)$$

To account for the effect of supports on the value of the bending moment, the code gives the following equation for M_o calculations

$$M_o = \frac{w_u\times L_2}{8}\times\left(L_1 - \frac{2\times D}{3}\right)^2 \quad \text{(long direction) ....} (4.12)$$

$$M_o = \frac{w_u\times L_1}{8}\times\left(L_2 - \frac{2\times D}{3}\right)^2 \quad \text{(short direction) ....} (4.13)$$

where D is the smallest distance at the intersection between the column and the slab. If a supporting element does not have a rectangular cross section or if the sides of the rectangle are not parallel to the span, it is to be treated as a square support having the same area, as illustrated in Fig. 4.6.
The total static moment M_o is divided into a negative moment at the support M_c and a positive moment at midspan M_s as follows

$$M_c + M_s = M_o \quad \text{...} (4.14)$$

Both M_c and M_s shall be distributed again between column and filed strips as described in section 4.5.5.

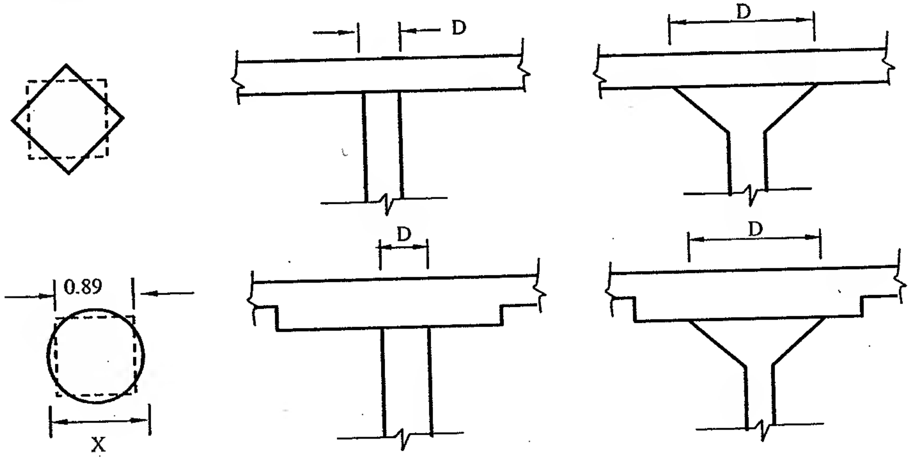


Fig. 4.6 Definition of D for different flat slab systems

4.5.5 Distribution of the Statical Moment

To simplify the analysis of flat slab floors, many design codes, including the ECP-203, approximate the actual distribution of the transverse moments by two regions of constant moment. The center strip, where the moment is the smallest, is called the *field strip* or *middle strip,* and the strip in the column zones, where the moment is the largest, is called the *column strip.*

The distribution of the moments between column strip and field strip can be explained by examining Fig. 4.7. Beam A represents the column strip while beams B and C represent the field strip. Since beam A is rested directly on columns, beams B is supported on beam A. If all of the beams are subjected to uniform load *w*, the developed bending moment in beam A is larger than that in beam B. This is because beam A carries the same uniform load *w* in addition to the reaction *wL/2* from Beam B. For an actual flat slab, the ratio of column strip moment to field strip moment is variable and depends on the rectangularity ratio and the flexural stiffness of the exterior beams (if any) along the building perimeter.

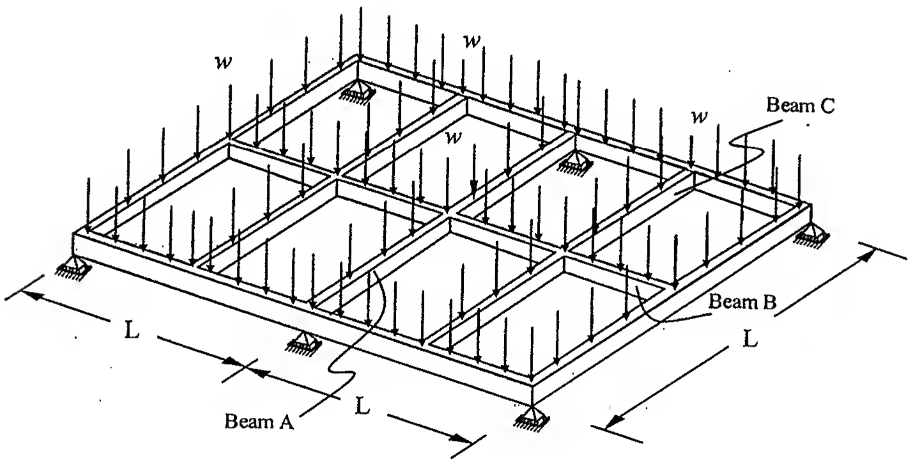


Fig. 4.7 Representation of column and field strips

The total statical moment M_o is divided into positive and negative moments according to the rules given in ECP-203 sec. 6-2-5-5. In the interior spans, 60% of M_o is distributed to the negative moment region and 40% to the positive moment region as shown in Fig. 4.8. This is approximately the case for a beam fixed from both ends and uniformly loaded where the negative moment is $(wL^2/12)$ 67% and the positive moment is $(wL^2/24)$ 33% of the total moment of $wL^2/8$.

The distribution of the negative moment between column strip and field strip varies according to the stiffness of the edge beam. If the edge beam depth is less than three times the slab thickness, only 25% of M_o is assigned to column strip as shown in Fig. 4.8.

In the case of floors that differ in spans (within 20% difference), the negative moment section of the slab is designed for the larger of the two moments unless a moment distribution is carried out. Table 4.1 and Fig 4.9 give the distribution of M_o between column and field strip according to the Egyptian Code.

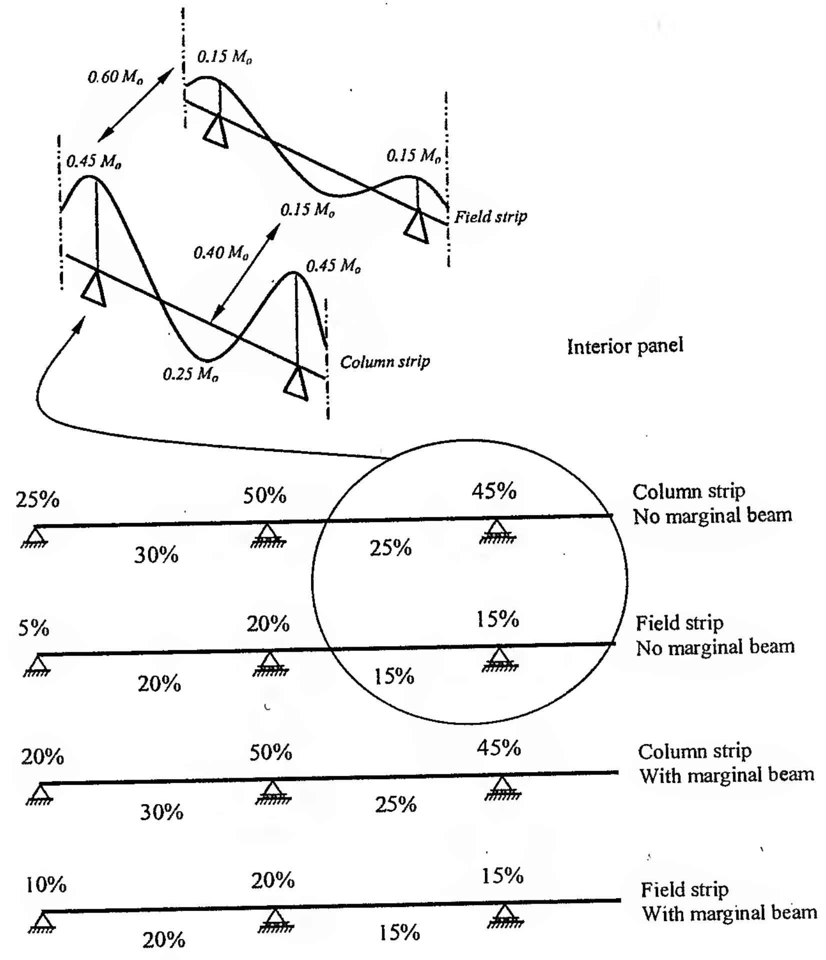
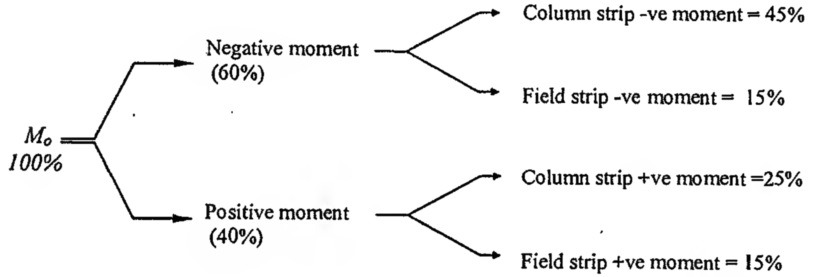


Fig. 4.8 Distribution of M_o between column and field strip

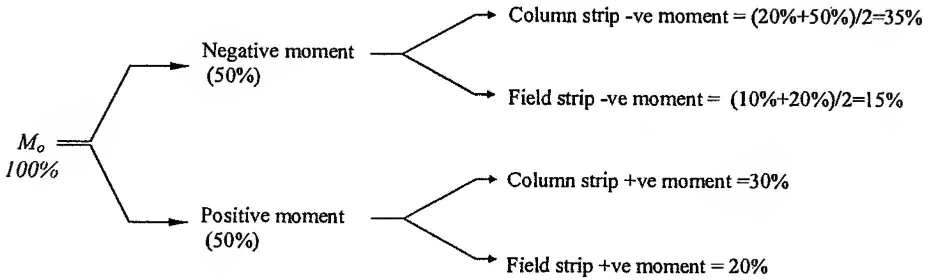
Table 4.1 Distribution of M_o between column and field strip

Strip type	Marginal beam	Exterior bay			Interior bay	
		-ve moment (external)	+ve moment	-ve moment (internal)	-ve moment	+ve moment
Column strip	no beam	25	30	50	45	25
	with beam*	20				
Field strip	no beam	5	20	20	15	15
	with beam*	10				

* the depth of the marginal beam should be at least =3 ts



A: Moment distribution in interior panels (with or without marginal beam)



B: Moment distribution in exterior panels with marginal beam

Fig. 4.9 Distribution Moment in flat slabs using the direct design method

4.5.6 Moment Correction

The distribution of M_o between the field strip and the column strip, as proposed by ECP 203, is based on the assumption that the widths of the column strip and the field strip are as shown in Fig.4.10a. If the field strip width is different from the ideal length as shown in Fig. 4.10b, a correction needs to be made. The moment in the field strip needs to be increased by multiplying their original value by the correction factor given in Table 4.2. In addition, the moment in the column strip is reduced so that the total bending is the same in either case as shown by the following equations.

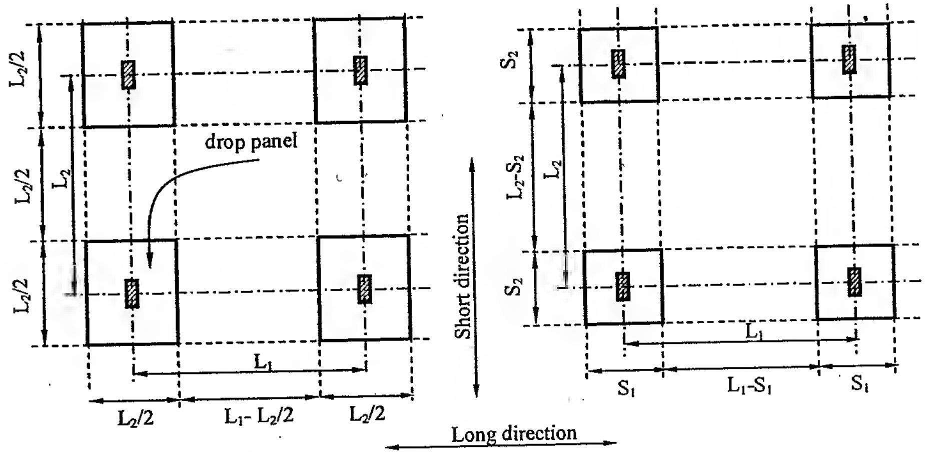
$$M_{F.S.(corrected)} = M_{F.S(ideal)} \times \frac{Actual\ width\ of\ F.S.}{Ideal\ width\ of\ F.S.} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.15)$$

$$M_{C.S.(corrected)} = \{M_{F.S.(ideal)} + M_{C.S.(ideal)}\} - M_{F.S.(corrected)} \quad \ldots\ldots\ldots (4.16)$$

where the actual width and ideal length are given in table 4.2

Table 4.2 Actual width and ideal length in field strip.

Direction	Ideal width	Actual width	Correction factor
Long direction	$L_2/2$	L_2-S_2	$\frac{L_2/2}{(L_2-S_2)}$
Short direction	L_1-$L_2/2$	L_1-S_1	$\frac{L_1-L_2/2}{(L_1-S_1)}$



a. Ideal **b. Actual**

Fig. 4.10 Moment correction for field strip in flat slabs with drop panels

4.5.7 Provision for Pattern Loading

If the slab is subjected to heavy live loads, negative moments shall form at midspan in addition to the positive bending moments. If the live load *(p)* is greater than 1.5 the dead loads *(g)*, the negative bending in column strip in L₁ direction can be estimated as follows

$$M_{-ve}=\left(g-\frac{2\times p}{3}\right)\times\left(\frac{L_2}{40}\right)\times\left(L_1-\frac{2\times D}{3}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.17)$$

and negative moment in the field strip in L₁ direction is

$$M_{-ve}=\left(g-\frac{2\times p}{3}\right)\times\left(\frac{L_2}{100}\right)\times\left(L_1-\frac{2\times D}{3}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.18)$$

where

L_1 = span in direction 1 (refer to Fig. 4.5)
L_2 = span in direction 2
D = width of the column at slab intersection (refer to fig. 4.6)
p = uniform live loads
g = uniform dead loads

4.5.8 Design Steps According to the Direct Design Method

The steps necessary to perform the designs are briefly summarized as follows:

1- Choose the appropriate flat slab system according to the intensity of the live load and the architectural requirements.

2- Estimate the slab thickness according to code requirements

3- Calculate the total static moment to be resisted in the two directions.

4- Distribute the static moment between column strip and field strip.

5- Divide the resulting moments by strip width to obtain the moment per meter.

6- Design the sections to select the reinforcement.

7- Design the slab for punching shear.

Examples 1 and 2 illustrate the use of this method as applied to flat slabs with and without drop panels. If the slab thickness is greater than 160 mm, an upper reinforcement mesh should be provided to satisfy temperature and cracking requirements by the code.

4.6 Reinforcement of Flat Slabs

4.6.1 General

Minimum bar extension requirements are given in Fig. 4.11 and Fig. 4.12. In addition, to ensure the integrity of the flat slab floors, it is recommended that at least two bottom bars in the column strip should run through the core of the column. The minimum area of steel is given by

$$A_{s,\min}=\frac{0.6}{f_y}\times b\times d \quad \text{(for all types of steel)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.20)$$

Photo 4.2 Menara Telekom, Kuala Lumpur, 310 meters, 55 stories, 2001 25th tallest building in the world

Min distances limits			
b	c	d	e
Ln/5	Ln/4.5	Ln/3.3	Ln/3

B : Bottom reinforcement
T : Top reinforcement
L : Center line distance
Ln :Clear distance

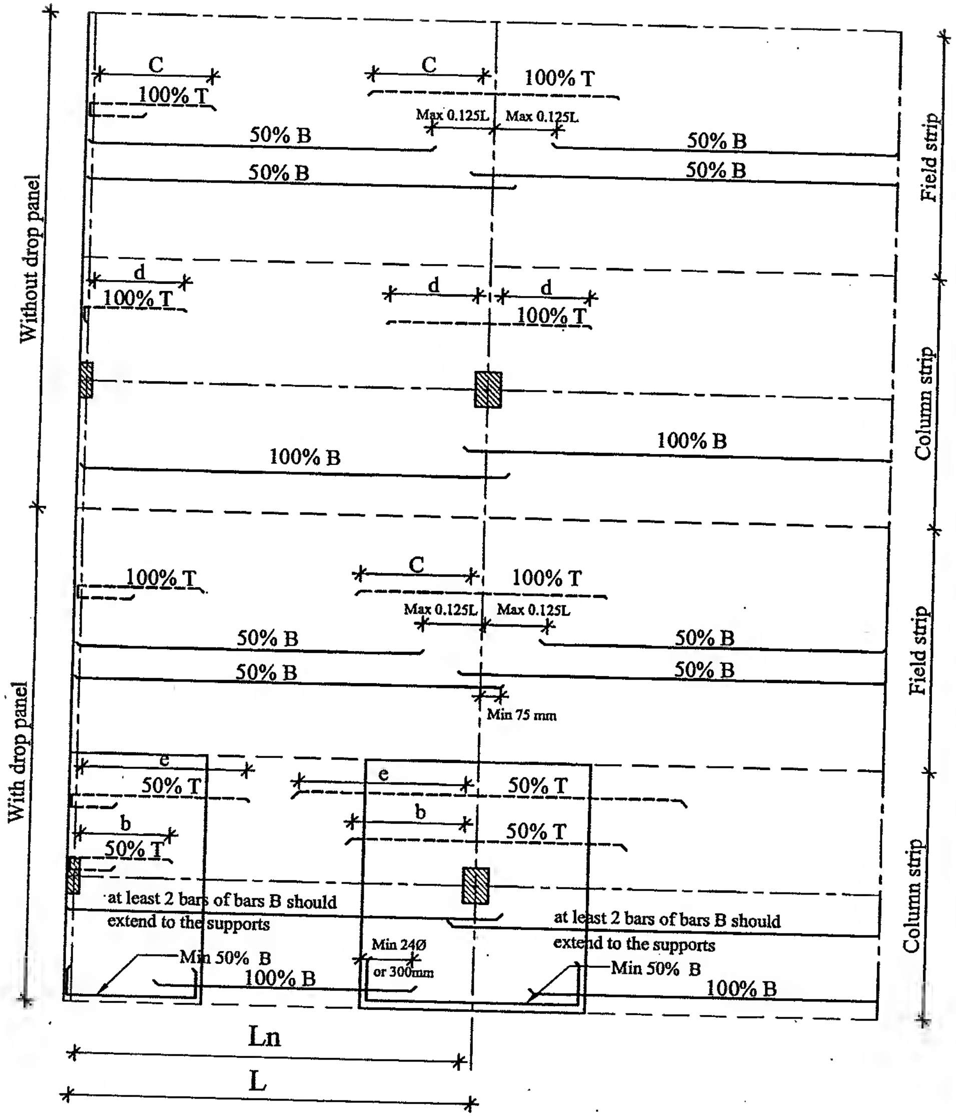


Fig.4-11a Reinforcement layout of flat slabs using bars

Bar length from the face of the support						
	Min length				Max length	
Symbol	b	c	d	e	f	g
Length	0.20Ln	0.22Ln	0.30Ln	0.33Ln	0.20Ln	0.24Ln

B : Bottom reinforcement
T : Top reinforcement
L : Center line distance
Ln :Clear distance

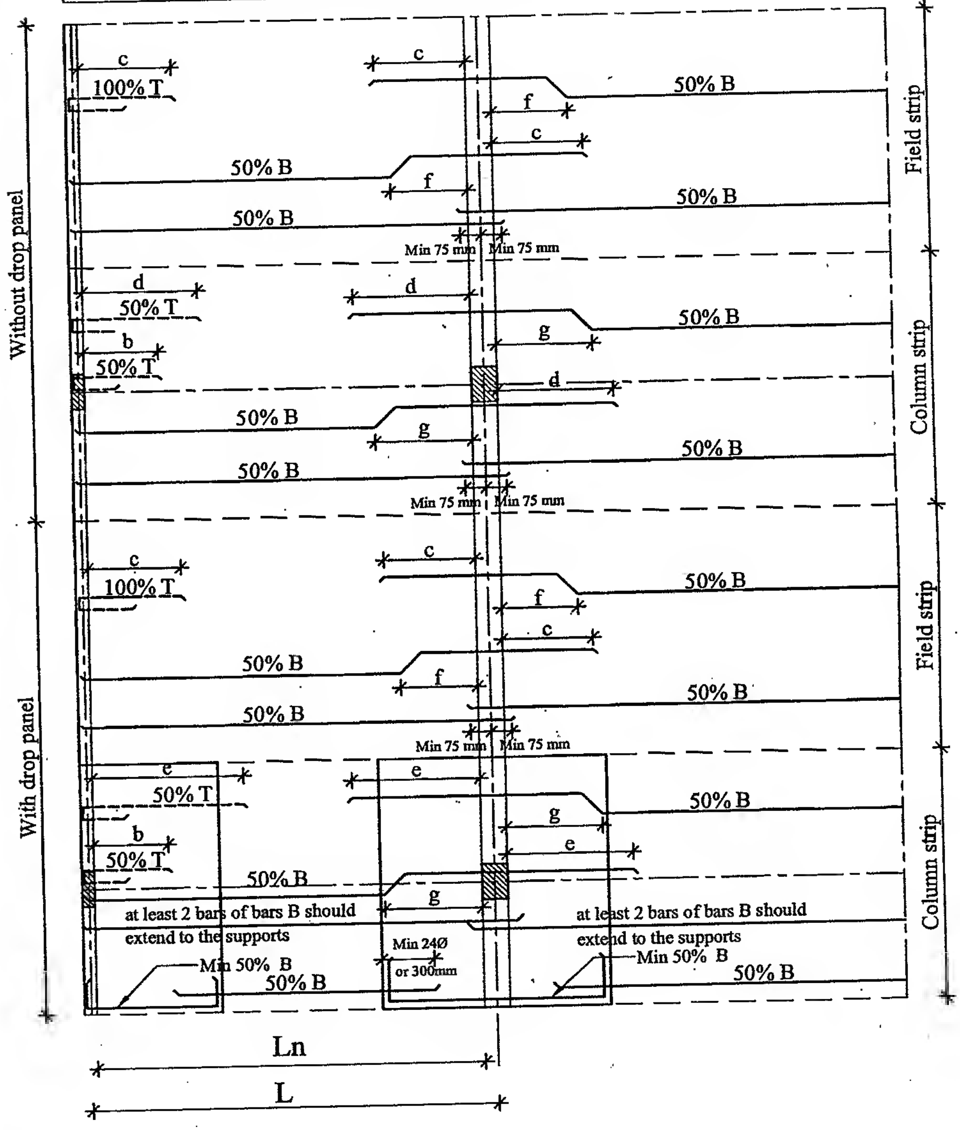


Fig.4-11b Reinforcement layout of flat slabs using bent bars

A	Top reinforcement mesh
B	Bottom reinforcement mesh
C	Additional top reinforcement for column strip
D	Additional bottom reinforcement for column strip

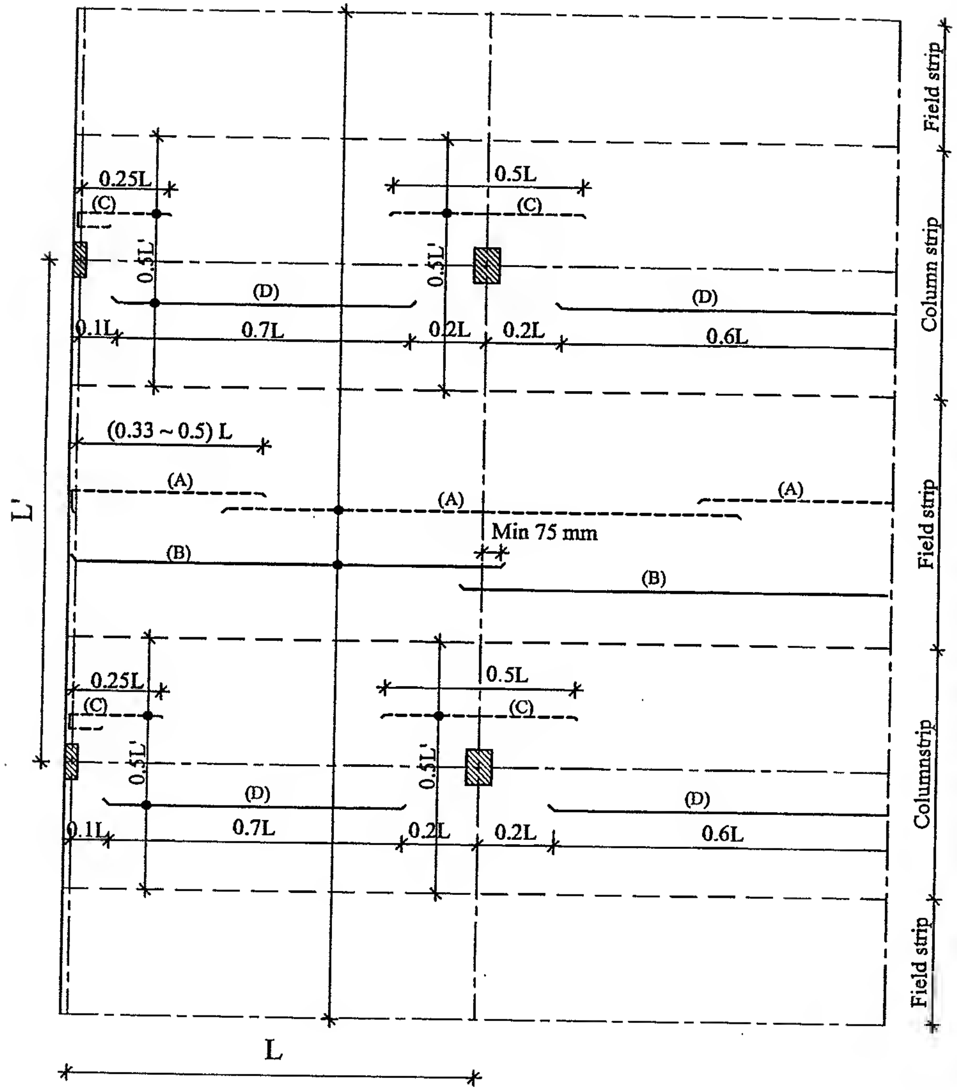


Fig. 4-12 Reinforcement layout of flat slabs using mesh

4.6.2 Column Head Reinforcement

Column heads should be reinforced with bars in the directions 1 and 2 as shown in Fig. 4.13. Closed stirrups should be provided to secure the reinforcement of the column head in place. Column head reinforcement should be designed to resist flexural moments resulting from either equivalent frame analysis or bending transferred to columns. This reinforcement should not be less than 4% of the column strip negative reinforcement per meter multiplied by the perpendicular span. In the case of circular columns, the required reinforcement is the sum of the two directions and should be uniformly distributed along the perimeter.

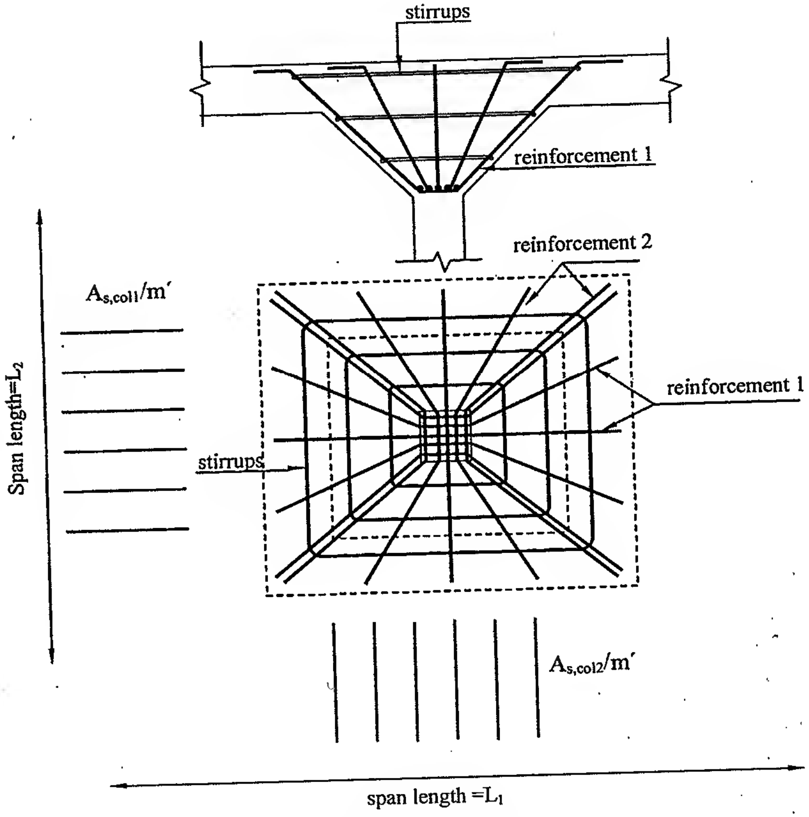


Fig. 4.13 Reinforcement in column head

$$\text{Area of reinforcement 1} = \frac{4}{100} \times A_{s,col1}/m' \times L_2 \qquad (4.20)$$

$$\text{Area of reinforcement 2} = \frac{4}{100} \times A_{s,col2}/m' \times L_1 \qquad (4.21)$$

4.6.3 Reinforcement at Openings in Flat Slabs

An opening of a size not more than 0.4 span length may be formed in the intersection of the two middle strips as shown in Fig. 4.14 (zone A), provided that the total amount of reinforcement required for the slab without the opening is maintained. However, at the intersection of the two column strips, no more than one-tenth of the width of the column strip in either span shall be interrupted by openings (zone C). Equivalent amounts of reinforcement shall be added on the sides of the opening. In zone B, no more than one-quarter of the reinforcement in either strip shall be interrupted by openings. It should be noted that the ultimate shear capacity is reduced due to the presence of the opening.

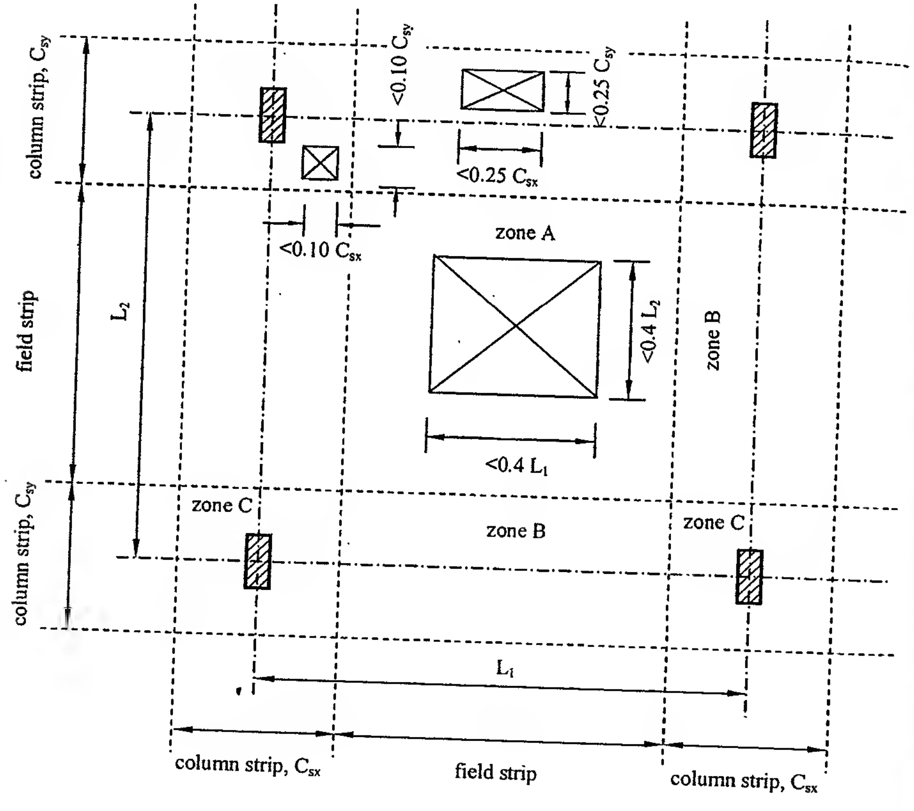


Fig. 4.14 Allowed opening dimensions and locations in flat slab systems

4.7 Punching Shear Strength of Flat Slabs

4.7.1 General

Punching shear strength is one of the most critical design aspects in determining the flat slab thickness. There are two shear failure mechanisms that may be encountered in a flat slab system. The first is the one-way shear similar to that in beams. This type rarely controls the design of flat slab floors. The second is the two-way shear, in which the failure surrounds the column forming a pyramid shape. Normally, the stresses resulting from the two-way shear are much higher than that resulting from the one-way shear.

Two-way shear failure mechanism is usually encountered in flat slab and footings. Interior columns are generally subjected to shear with negligible moment transfer from the slab to the columns. However, to ensure adequate shear strength, the Egyptian code requires that part of the connecting moment be transferred between the slab and the columns, resulting in additional punching shear.

The combination of shear and unbalanced moment is unavoidable at edge and corner column locations and occurs at interior columns because of unequal spans and lateral loads. Prevention of punching failure of column-slab connections transferring moment depends on an accurate calculation of shear stresses produced by moment transfer.

One of the most widely used analysis methods is based on summing the stresses developed by vertical shear and the stresses developed by the unbalanced moments. This *detailed analysis* is adopted by the ACI and the Egyptian code of practice. However, the computational time required for such analysis is still costly and is not suitable for routine design computations. The Egyptian code also offers a *simplified analysis* method for calculating punching shear stress due to both gravity and moment transferred to columns due to torsion. In this method, a magnification factor is used to account for the portion transferred by the unbalanced moment

4.7.2 Critical Sections

For both design methods, the critical section for punching shear is at d/2 from the face of the column. Fig. 4.15 shows several examples of internal, external and corner columns. If the openings are located less than 10 times the slab thickness, the code requires that the critical perimeter be reduced as illustrated in Fig 4.16. In floors with drop panels, two critical sections should be investigated as shown in Fig. 4.17.

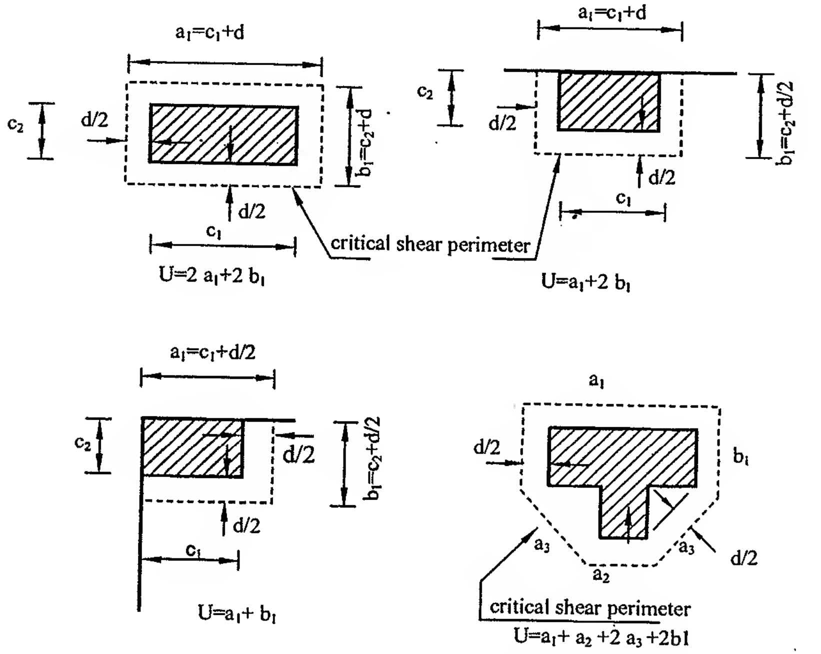


Fig. 4.15 Critical shear perimeter for internal, exterior and corner columns

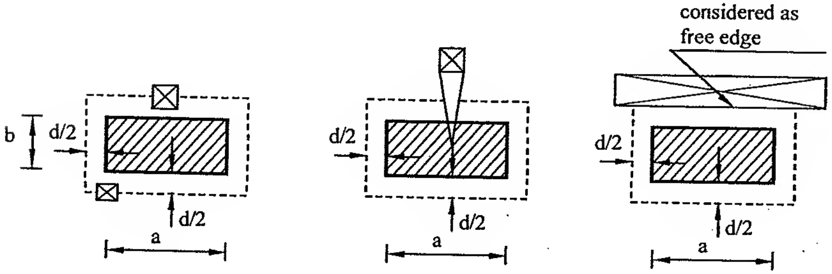
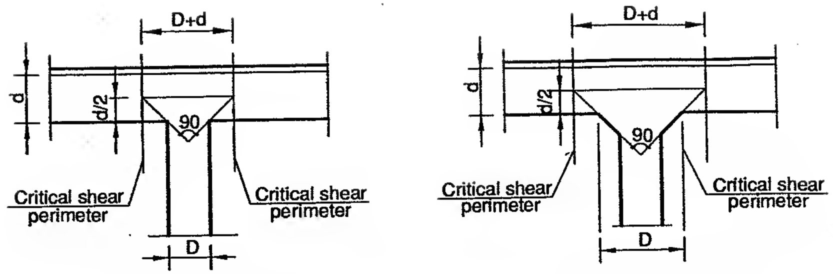
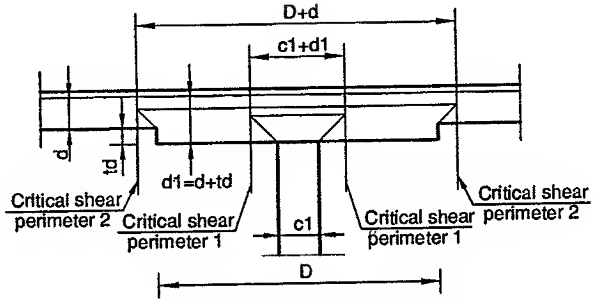


Fig.4.16. Critical sections for flat slabs with openings

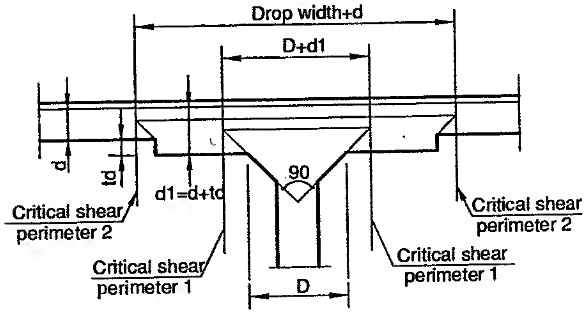


a)Flat slab without drop panel or column head

b)Flat slab with column head&without drop panel



c)Flat slab with drop panel &without column head



d)Flat slab with drop panel & column head

Fig. 4.17 Critical shear perimeter in flat slab

4.7.3 Concrete Punching Shear Strength

The Egyptian code states that the smallest of the following three values represents the concrete punching shear strength q_{cup}

$$q_{cup} = 0.316\,(0.5 + \frac{a}{b}) \times \sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{..........(4.22.A)}$$

$$q_{cup} = 0.8\,(\frac{\alpha\, d}{b_o} + 0.2) \times \sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{..........(4.22.B)}$$

$$q_{cup} = 0.316\sqrt{\frac{f_{cu}}{\gamma_c}} \le 1.6\,N/mm^2 \quad \text{..........(4.22.C)}$$

where

a is the column dimension in the analysis direction.

b is the column dimension in the perpendicular direction.

α equals 4,3,2 for interior, exterior, corner column respectively.

b_o is the critical shear perimeter.

The applied shear stress, calculated using either the detailed analysis described in section 4.7.4 or the simplified method described in section 4.7.5, should be less than concrete punching shear strength q_{cup}.

Photo 4.3 Flat slab and solid slabs during construction in Dubi

4.7.4 Detailed Analysis

4.7.4.1 Introduction

Concentric loading on flat slab systems as shown in Fig. 4.18, produces uniform punching shear stress that can be calculated from Eq. 4.23

$$q_{up} = \frac{Q_{up}}{b_o\, d} \quad \text{..........(4.23)}$$

where

Q_{up} is the ultimate design shear force.

b_o is the critical shear perimeter b_o=2 [(a+d)+(b+d)]

d is the effective flat slab depth.

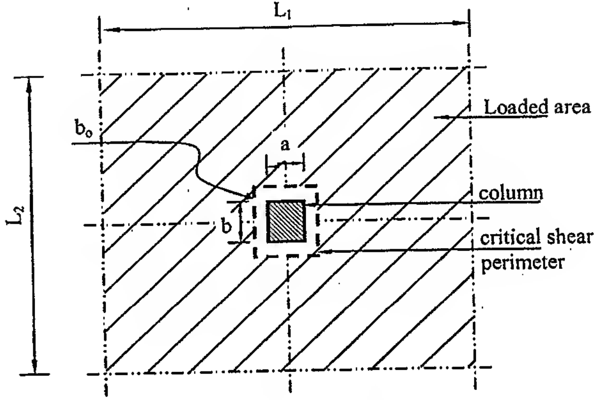


Fig. 4.18 Concentric shear stress calculations for interior column

The case of concentric loading is rarely encountered in real structures. Therefore, the Egyptian code imposes a minimum amount of unbalanced moment that should be transferred to columns. This amount varies according to the location of the column. The moment transferred between slab and column, produce a complex behavior involving flexure, shear and torsion.

4.7.4.2 Calculations of the Punching Stresses

Figure 4.19 illustrate the moment and shear transfer at interior and exterior columns, where a shear and an unbalanced moment are transferred from the slab to column. A fraction of the unbalanced moment transferred by flexure ($\gamma_f\ M_f$), where γ_f is calculated from

$$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{c_1 + d}{c_2 + d}}} \quad \text{..........(4.24)}$$

in which

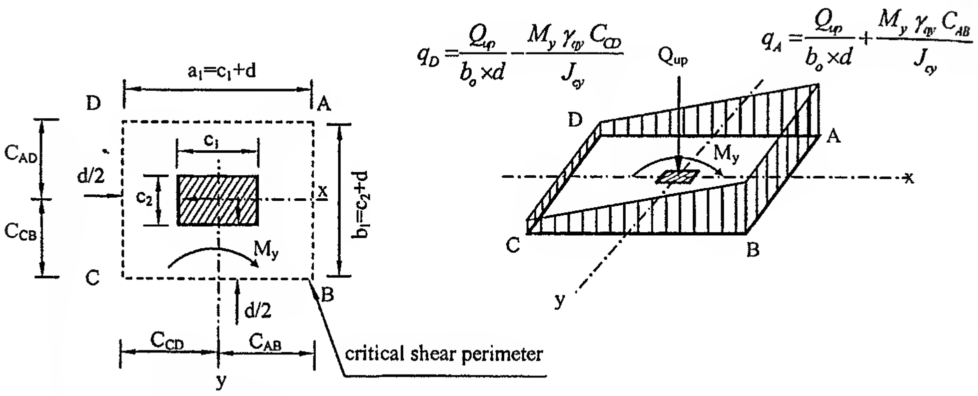
γ_f= fraction of the unbalanced moment transferred by flexure
c_1= is the column dimension in the analysis direction
c_2= is the column dimension in the perpendicular direction
d= depth of the flat slab
The moment transferred to the column by eccentric shear stress is ($\gamma_q\ M_f$) where

$$\gamma_q = 1 - \gamma_f \tag{4.25a}$$

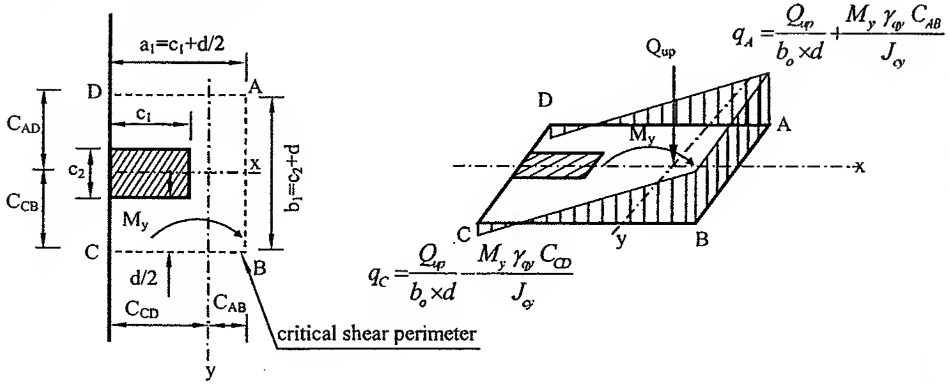
$$\gamma_{qx} = 1 - \gamma_{fx} \tag{4.25b}$$

$$\gamma_{qy} = 1 - \gamma_{fy} \tag{4.25c}$$

This moment causes additional shear stresses that have to be combined with those resulting from the vertical shear. The code requires that the moment transferred be taken as 50% and 90% of the negative bending of the column strip for the interior and exterior columns, respectively. For an edge bay, the amount of the negative bending transferred is reduced by half.



a-Internal column



b-Edge column

Fig. 4.19 Punching stress distribution due to an unbalanced moment

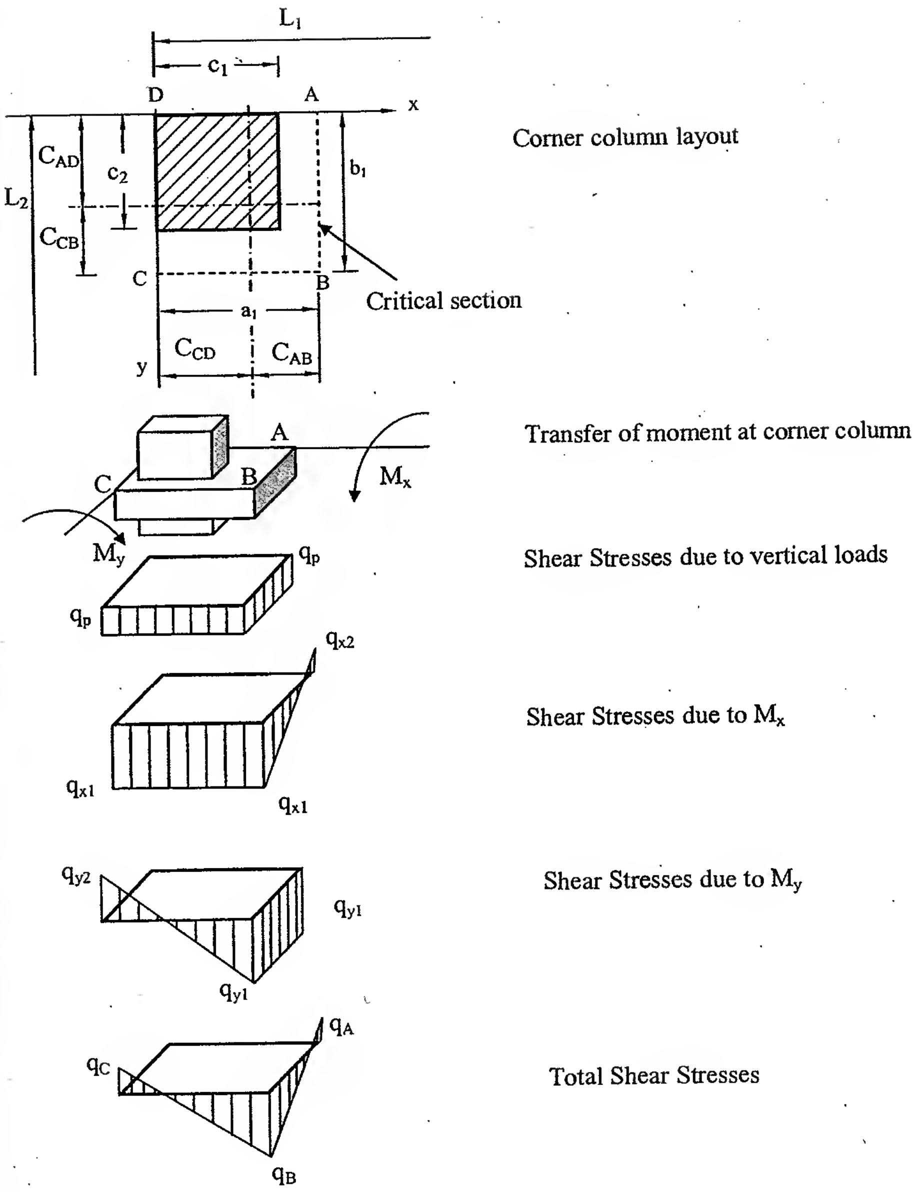


Fig. 4.20: Punching shear stresses for a corner column

The case of corner column subjected to eccentric punching stress is illustrated in Fig. 4.20. The centroid of the critical perimeter lies closer to the inside face of the column. Hence, shear stresses due to moment transfer are larger at the outside perimeter. If the shear stress due to transferred moment is larger than shear stress due to gravity load q_p as defined by Eq. 4.26, a negative shear stress may occur at these points (corners A, C). The punching shear stresses calculated using the code-detailed method is given in Eqs. (4.27-4.30). These stresses should be added to the punching stresses caused by the vertical loads. The final shear stress at each point for the corner column is computed using Eqs. (4.31-4.33).

The shear stress due to gravity loads is given by:

$$q_p = \frac{Q_{up}}{b_o\ d} \quad \text{.........(4.26)}$$

The shear stresses due to unbalanced moment M_y are given by:

$$q_{y1} = \frac{M_y\ \gamma_{qy}\ C_{AB}}{J_{cy}} \quad \text{.........(4.27)}$$

$$q_{y2} = \frac{M_y\ \gamma_{qy}\ C_{CD}}{J_{cy}} \quad \text{.........(4.28)}$$

The shear stresses due to unbalanced moment M_x are given by:

$$q_{x1} = \frac{M_x\ \gamma_{qx}\ C_{CB}}{J_{cx}} \quad \text{.........(4.29)}$$

$$q_{x2} = \frac{M_x\ \gamma_{qx}\ C_{AD}}{J_{cx}} \quad \text{.........(4.30)}$$

The total stresses at column corner points equal to:

$$q_A = q_p + q_{y1} - q_{x2} \quad \text{.........(4.31)}$$

$$q_B = q_p + q_{x1} + q_{y1} \quad \text{.........(4.32)}$$

$$q_C = q_p + q_{x1} - q_{y2} \quad \text{.........(4.33)}$$

These stresses should be checked against concrete shear strength given in Eq. 4.22. where the properties C_{AB}, C_{CD}, C_{CB},C_{AD}, J_{cy}, and J_{cx} for different column locations are given by:

Case of interior column (Fig. 4.19)

$$C_{AB} = C_{CD} = \frac{a_1}{2} \qquad C_{CB} = C_{AD} = \frac{b_1}{2}$$

$$J_{cy} = \frac{d \times a_1^3}{6} + \frac{a_1 \times d^3}{6} + \frac{d \times a_1^2 \times b_1}{2} \quad \text{.........(4.34)}$$

$$J_{cx} = \frac{d \times b_1^3}{6} + \frac{b_1 \times d^3}{6} + \frac{d \times b_1^2 \times a_1}{2} \quad \text{.........(4.35)}$$

Case of exterior column (Fig. 4.19)

$$C_{AB} = \frac{a_1^2}{(b_1 + 2 \times a_1)} \qquad C_{CB} = C_{AD} = \frac{b_1}{2}$$

$$C_{CD} = a_1 - C_{AB}$$

$$J_{cy} = d \times b_1 \times C_{AB}^2 + \frac{2 \times d \times C_{CD}^3}{3} + \frac{2 \times d \times C_{AB}^3}{3} + \frac{a_1 \times d^3}{6} \quad \text{.........(4.36)}$$

$$J_{cx} = \frac{d \times b_1^3}{6} + \frac{b_1 \times d^3}{6} + \frac{d \times b_1^2 \times a_1}{2} \quad \text{.........(4.37)}$$

Case of corner column (Fig. 4.20)

$$C_{AB} = \frac{a_1^2}{(2 \times b_1 + 2 \times a_1)} \qquad C_{CB} = \frac{b_1^2}{(2 \times b_1 + 2 \times a_1)}$$

$$C_{CD} = a_1 - C_{AB} \qquad C_{AD} = b_1 - C_{CB}$$

$$J_{cy} = d \times b_1 \times C_{AB}^2 + \frac{d \times C_{CD}^3}{3} + \frac{d \times C_{AB}^3}{3} + \frac{a_1 \times d^3}{12} \quad \text{.........(4.38)}$$

$$J_{cx} = d \times a_1 \times C_{CB}^2 + \frac{d \times C_{AD}^3}{3} + \frac{d \times C_{CB}^3}{3} + \frac{b_1 \times d^3}{12} \quad \text{.........(4.39)}$$

4.7.5 Simplified Method

The Egyptian code offers a simplified design method for calculating the total punching shear stress including shear stresses due to moment transferred to columns. In this simplified method, the shear stress due to vertical gravity loads is magnified by the factor β to account for unbalanced moment transferred to columns. The method implies that the estimated additional increase in shear stresses due to moment transferred from slab to column is 15%, 30%, 50% for the interior, exterior and corner column, respectively.
The shear stress is given by

$$q_{up} = \frac{Q_{up}\ \beta}{b_o\ d} \quad \text{...(4.40)}$$

where
Q_{up} is the ultimate design shear force
$\beta = 1.15$ for interior column
$\beta = 1.30$ for exterior column
$\beta = 1.50$ for corner column
b_o is the critical shear perimeter
d is the effective slab depth

Fig. 4.21 shows the critical shear perimeter and the loaded area for an interior column in flat slab floor. According to this figure, the calculation procedure is as follows:

$$Q_{up} = w_u \times \{L_1 \times L_2 - a_1 \times b_1\} \quad \text{...(4.41)}$$

$$q_{up} = \frac{Q_{up} \times 1.15}{b_o\ d} \quad \text{...(4.42)}$$

$$a_1 = a + d \qquad b_1 = b + d \qquad b_o = 2a_1 + 2b_1$$

Table 4.3 summarizes the calculations of the critical shear perimeter and design shear force for interior, exterior and corner columns.

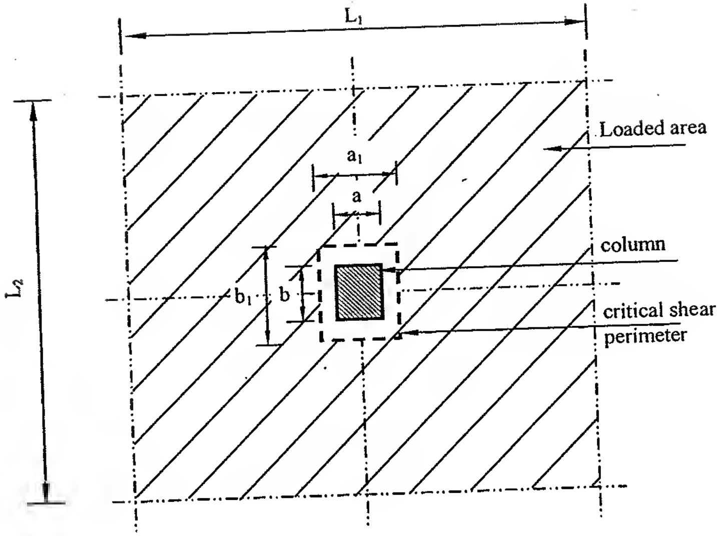
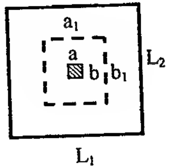
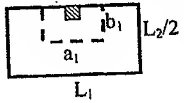
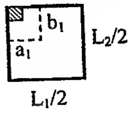


Fig. 4.21 loaded area and critical shear perimeter for an interior column

Table 4.3 Calculation of critical shear perimeter and design shear force

Type of column	**Interior Column**	**Exterior Column**	**Corner column**
Shape			
a_1	a + d	a + d	a +d/2
b_1	b + d	b +d/2	b +d/2
perimeter (b_o)	$2(a_1+b_1)$	$a_1+2\,b_1$	a_1+b_1
β	1.15	1.3	1.5
Q_{up}	$w_u(L_1 \times L_2 - a_1 \times b_1)$	$w_u(\frac{L_1 \times L_2}{2} - a_1 \times b_1)$	$w_u(\frac{L_1 \times L_2}{4} - a_1 \times b_1)$

4.8 One-Way Shear Strength

Stresses resulting from one-way shear are normally low, and usually do not control the design. One-way shear stresses must be resisted by concrete strength only and without any reinforcement contribution. The Egyptian code gives the following equation for one-way concrete shear strength:

$$q_{cu} = 0.16\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{..........}(4.43)$$

The critical section for one-way shear is taken at d/2 from the face of the column as shown in Fig. 4.22. The calculated shear stress should be less than concrete shear strength. For example, shear stresses for the interior column shown in Fig. 4.22 are given by

$$q_{u1} = \frac{w_u \times L_2 \times L_x}{L_2 \times d} \le q_{cu} \quad \text{..........}(4.44)$$

$$q_{u2} = \frac{w_u \times L_1 \times L_y}{L_1 \times d} \le q_{cu} \quad \text{..........}(4.45)$$

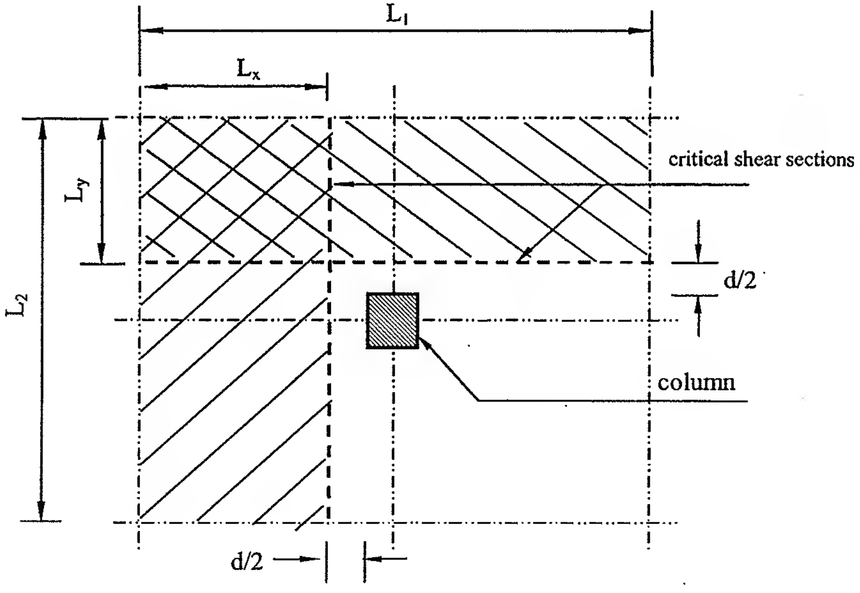


Fig. 4.22 Critical sections for one-way shear in interior columns

Example 4.1

It is required to design the Flat Slab Roof shown in Fig. EX4.1. Columns (500x 500 mm) are only allowed as shown. For architectural purposes, it is required that no drop panels or columns' heads to be used.

Data:-

Concrete Characteristic Strength	=	25	N/mm^2
Steel Yield Stress	=	360	N/mm^2
Live Load	=	4.0	kN/m^2
Flooring	=	2.0	kN/m^2
Equivalent wall loads	=	1.5	kN/m^2
Floor Height	=	3.50	m

Solution:

The floor consists of three equal spans in each direction, the span in the long direction equals $L_1 = 6.0$ m and the span in the short direction L_2=5.0 m

The system satisfies the requirements of the empirical method specified in the Code-article (6-2-5-5)

The average length L_{avg}=5.5 m

Step 1: Dimensioning

- **Slab thickness (t_s):-**

$$t_s = bigger\ of \begin{cases} 150\ mm \\ \frac{L_{long}}{32} = \frac{6000}{32} = 187.5\ mm \end{cases}$$

Take t_s = 200 mm

- **Column Dimensions (bxb)**

$$b = bigger\ of \begin{cases} 300\ mm \\ \frac{h_{clear}}{15} = \frac{(3500-200)}{15} = 220\ mm \\ \frac{L_1}{20} = \frac{6000}{20} = 300\ mm \end{cases}$$

Thus b=500 mm is satisfactory

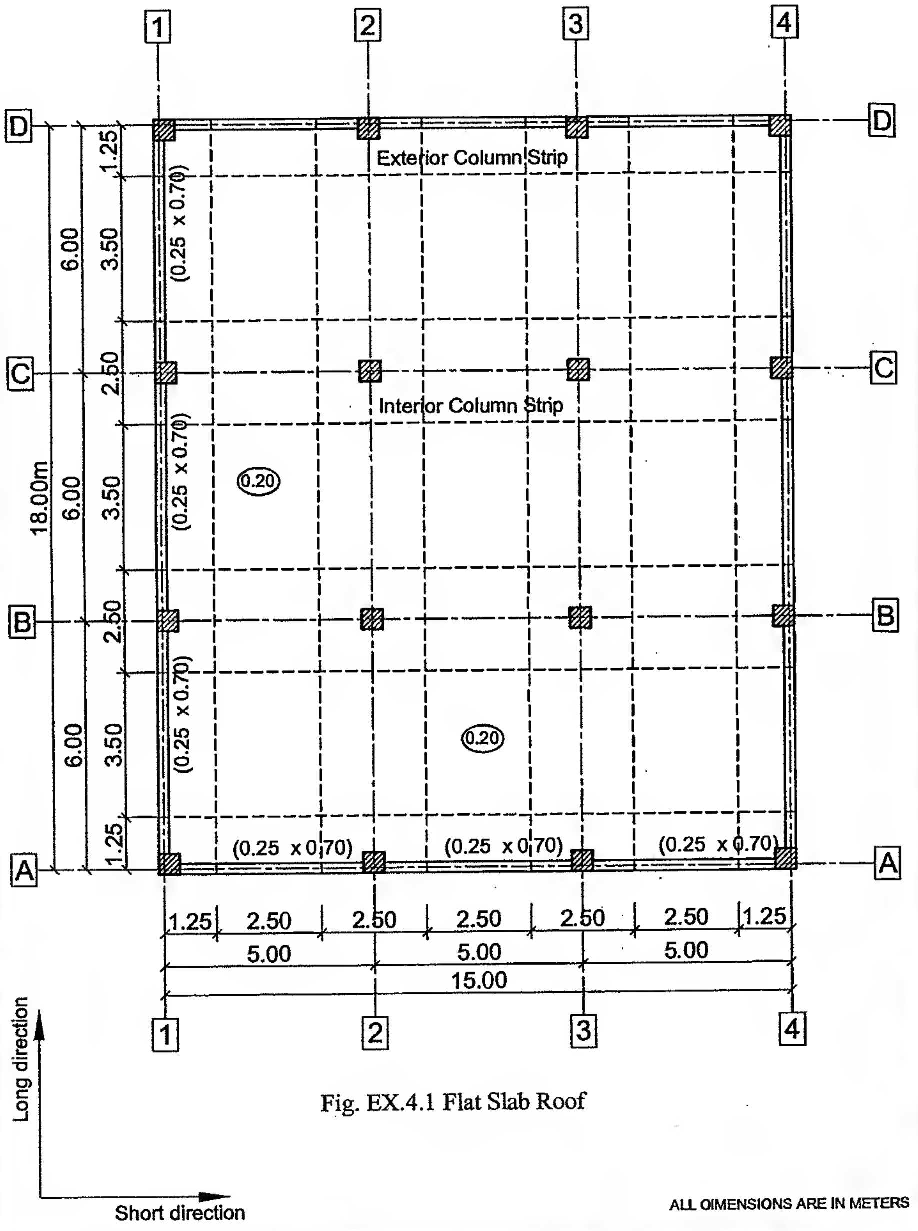


Fig. EX.4.1 Flat Slab Roof

- **Marginal Beams ($b_b x t_b$)**

Punching stresses are usually high in exterior and corner columns. Hence, a marginal beam is considered with thickness $t_b \geq 3t_s \geq 600$ mm

Take $t_b = 700$ mm & $b_b = 250$ mm

- **Column and Field Strips:-**

Assume that width of Column Strip = ½ Smaller side = 1/2x5.0 = 2.50 m

Width of Field Strip:-

For short direction = 6.00-2.50 = 3.50 m

For long direction = 5.00-2.50 = 2.50 m

Step 2: Minimum steel requirements:-

For $f_y = 360$ N/mm^2

$$A_{s,min}(long) = \frac{0.6}{360}\times 1000\times(200-20) = 300\ mm^2/m'$$

$$A_{s,min}(short) = \frac{0.6}{360}\times 1000\times(200-30) = 283\ mm^2/m'$$

Step 3: Load calculations

Dead Load, g_s = Own weight + Flooring + Equivalent Wall Loads

$$= 25\times 0.20 + 2.0 + 1.5 = 8.5\ kN/m^2$$

Live Load, p_s = 4.0 kN/m^2

Since the live loads is less than 0.75 the dead loads

$$w_{su} = 1.50\ (g_s + p_s) = 1.5 \times (8.50 + 4.0) = 18.75\ \text{kN/m}^2$$

Step 4: Design of Strips

Step 4-a: Long Direction

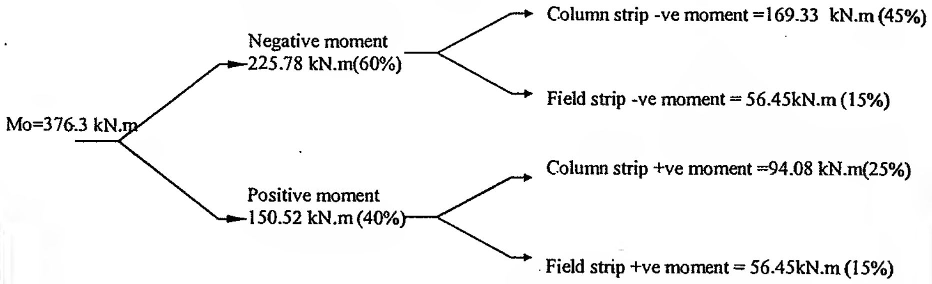
Step 4-a-i: Statical system and bending moment

$$M_o = \frac{w_{su}\times L_2}{8}\left(L_1 - \frac{2\times D}{3}\right)^2$$

As no column head is used D=b=0.50 m

$$M_o = \frac{18.75\times5}{8}\left(6.0-\frac{2\times0.5}{3}\right)^2 = 376.3\ kN.m$$

Percentage of moments is taken from Table 4.1



Distribution of M_o for an interior bay

Step 4-a-ii: Design of sections

Since the width of the column strip is 2.5 m, the maximum negative moment at the interior panel per meter equals

$$M_u = \frac{169.33}{2.5} = 67.73\ kN.m/m'$$

$$R = \frac{M_u}{f_{cu}\times b\times d^2} = \frac{67.73\times10^6}{25\times1000\times180^2} = 0.084$$

For $\alpha = 0.0$ $\quad \omega = 0.107$

$$A_s = \omega\times\frac{f_{cu}}{f_y}\times b\times d = 0.107\times\frac{25}{360}\times1000\times180 = 1337\ mm^2/m'$$

use (9Φ 14 /m') as negative column strip reinforcement in the long direction

The design for the rest of the long direction critical section is given in the figure

Step 4-b: Short Direction:-

Step 4-b-i: Statical system and bending moment:-

$$M_o = \frac{w_{su}\times L_1}{8}\left(L_2-\frac{2\times D}{3}\right)^2$$

As no column head is used D=b=0.50 m

$$M_o = \frac{18.75\times6}{8}\left(5.0-\frac{2\times0.5}{3}\right)^2 = 306.25\ kN.m$$

Percentage of moments is taken from Table 4.1

Step 4-b-ii: Design of sections

Since the width of the column strip is 2.5 m, the maximum negative moment at the interior panel per meter equals

$$M_u = \frac{137.81}{2.5} = 55.12\ kN.m/m'$$

since this is the secondary direction d=200-30=170 mm

$$R = \frac{M_u}{f_{cu}\times b\times d^2} = \frac{55.12\times10^6}{25\times1000\times170^2} = 0.076$$

For $\alpha = 0.0$ $\quad \omega = 0.097$

$$A_s = \omega\times\frac{f_{cu}}{f_y}\times b\times d = 0.097\times\frac{25}{360}\times1000\times170 = 1144\ mm^2/m'$$

use (6Φ12/m'+3Φ14 /m') as negative column strip reinforcement of the short direction

The design for the rest of the short direction critical sections is given in the figure

Step 5: Design of edge column strip

Due to the presence of the marginal beam, the moment in the exterior strip/m' equal half the moment in the interior strip/m'.

So, the reinforcement in the exterior strip/m equals half the reinforcement in the interior strip/m'. (While considering the minimum steel requirements).

Step 6: Check for negative reinforcement in the field span

As $g_s > 2/3\ p_s$, no top reinforcement is required. However, since the slab thickness is greater than 160 mm, shrinkage top mat is needed (use 6 Φ 10/m' in the long direction and 5 Φ 10/m' in the short direction)

Step 7: Design for Punching Shear for Interior Column

Assume concrete cover of 20 mm

d = 200-20 = 180 mm

$a_1 = b_1 = 500 + 180 = 680$ mm

$Q_{up} = 18.75 \times 5.0 \times 6 - 18.75 \times 0.68 \times 0.68 = 554$ kN

$\beta = 1.15$ (case of interior column)

$b_o = 2 \times (680+680) = 2720$ mm

$$q_u = \frac{Q_{up}\ \beta}{b_o\ d} = \frac{554 \times 1000 \times 1.15}{2720 \times 180} = 1.3\ N/mm^2$$

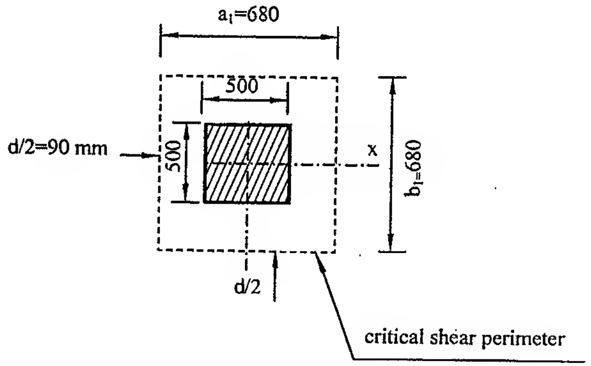
q_{cup} is the smallest of

1. $q_{cup} = 0.316\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\sqrt{\frac{25}{1.5}} = 1.29\ N/mm^2 < 1.6.....o.k$
2. $q_{cup} = 0.316\ (0.5+\frac{a}{b}) \times \sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\ (0.5+\frac{500}{500}) \times \sqrt{\frac{25}{1.5}} = 1.93\ N/mm^2$
3. $q_{cup} = 0.8\ (\frac{\alpha\ d}{b_o}+0.2) \times \sqrt{\frac{f_{cu}}{\gamma_c}} = 0.8\ (\frac{4 \times 180}{2720}+0.2) \times \sqrt{\frac{25}{1.5}} = 1.51\ N/mm^2$

Note : α =4.0 for interior columns

q_{cup}=1.29 N/mm^2

Since $q_u \cong q_{cup}$, the slab is considered safe regarding to punching

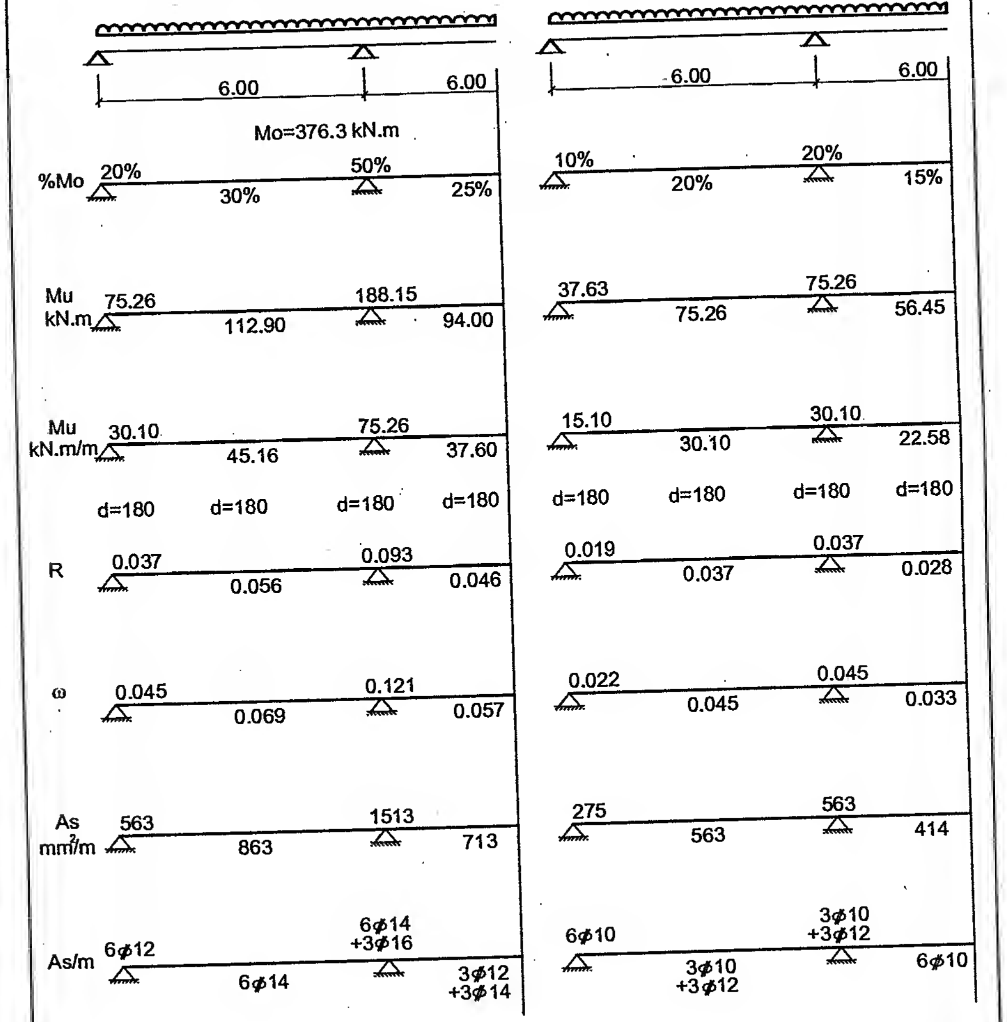


The punching strength of the exterior and corner columns can be checked in similar manner.

Design of Long Direction

Column Strip (Width=2.50 m)

Field Strip (Width=2.50 m)



Column Strip (Width=2.50 m)

%Mo	20%	30%	50%	25%
Mu kN.m	75.26	112.90	188.15	94.00
Mu kN.m/m	30.10	45.16	75.26	37.60
	d=180	d=180	d=180	d=180
R	0.037	0.056	0.093	0.046
ω	0.045	0.069	0.121	0.057
As mm²/m	563	863	1513	713
As/m	6ϕ12	6ϕ14	6ϕ14 +3ϕ16	3ϕ12 +3ϕ14

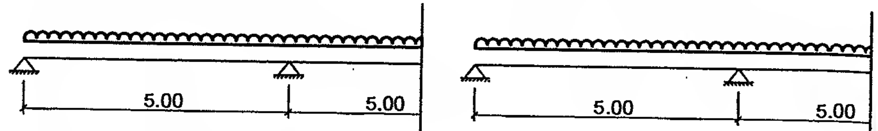
Field Strip (Width=2.50 m)

%Mo	10%	20%	20%	15%
Mu kN.m	37.63	75.26	75.26	56.45
Mu kN.m/m	15.10	30.10	30.10	22.58
	d=180	d=180	d=180	d=180
R	0.019	0.037	0.037	0.028
ω	0.022	0.045	0.045	0.033
As mm²/m	275	563	563	414
As/m	6ϕ10	3ϕ10 +3ϕ12	3ϕ10 +3ϕ12	6ϕ10

Design of Short Direction

Column Strip (Width=2.5m)

Field Strip (Width=3.50 m)



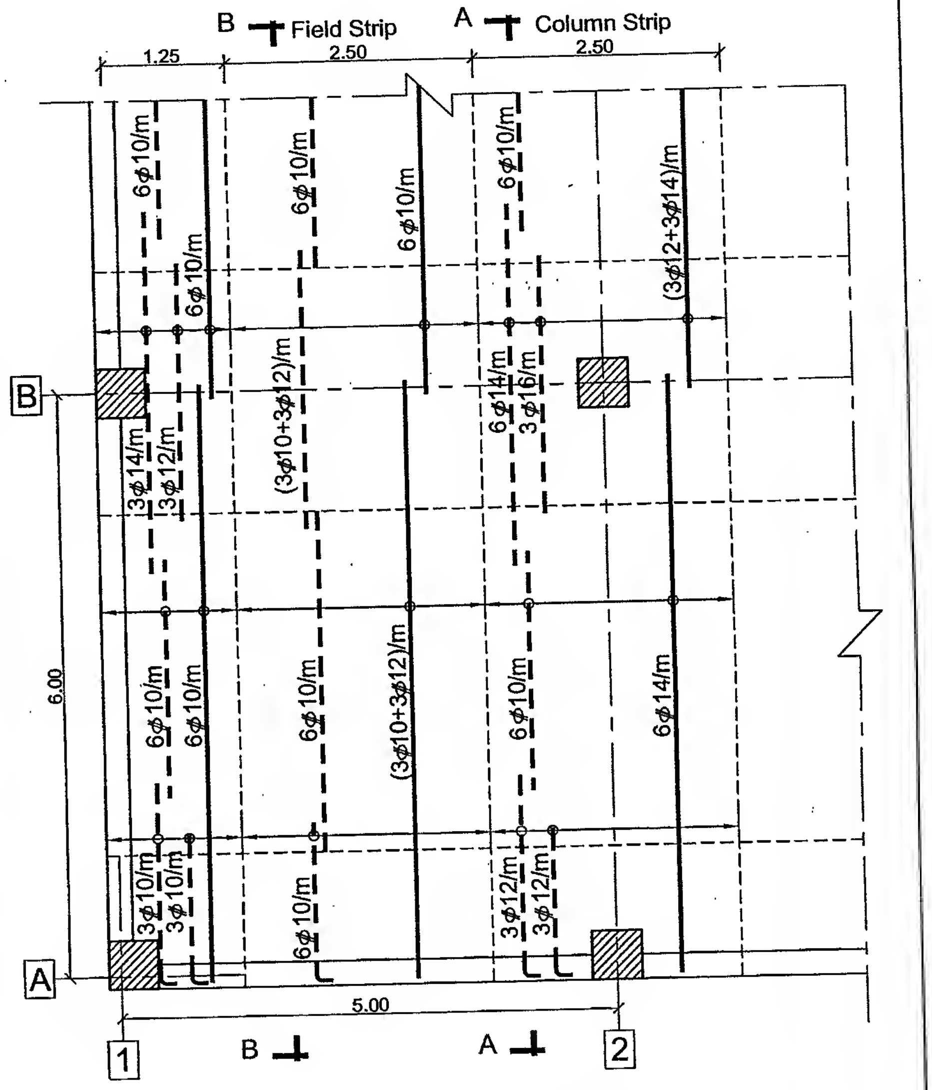
Mo=306.3 kN.m

Column Strip (Width=2.5m)

%Mo	20%	30%	50%	25%
Mu kN.m	61.25	91.88	153.13	76.56
Mu kN.m/m	24.50	36.75	61.25	30.62
	d=170	d=170	d=170	d=170
R	0.034	0.051	0.085	0.042
ω	0.041	0.062	0.109	0.051
As mm²/m	481	737	1291	600
As/m	6ϕ12	3ϕ12 +3ϕ14	9ϕ14	6ϕ12

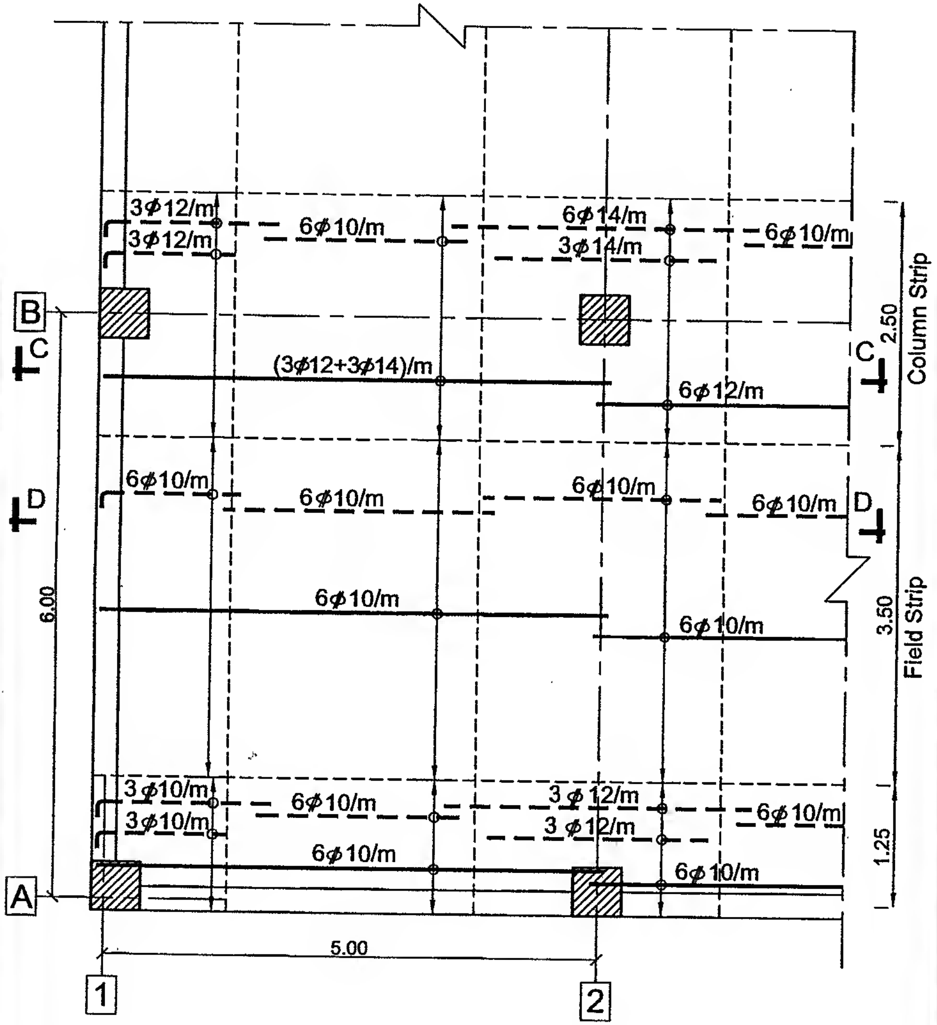
Field Strip (Width=3.50 m)

%Mo	10%	20%	20%	15%
Mu kN.m	30.63	61.25	61.25	45.94
Mu kN.m/m	8.75	17.50	17.50	13.13
	d=170	d=170	d=170	d=170
R	0.012	0.024	0.024	0.018
ω	0.014	0.028	0.028	0.021
As mm²/m	165	335	335	250
As/m	6ϕ10	6ϕ10	6ϕ10	6ϕ10



Reinforcement Details
Long Direction

LEGEND
- - - - - TOP
——— BOTTOM

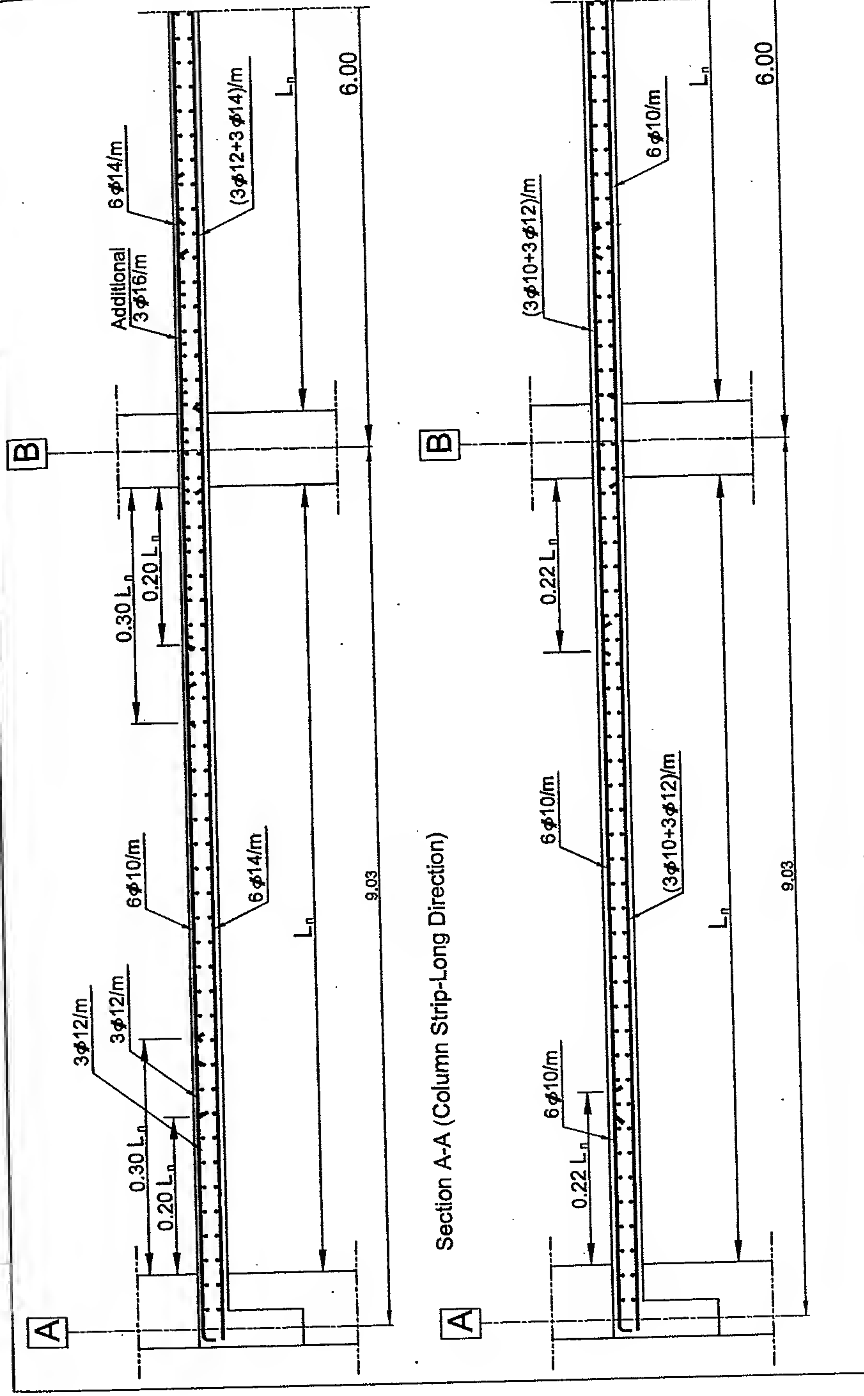


Reinforcement Details
Short Direction

LEGEND

- - - - - - TOP

———— BOTTOM

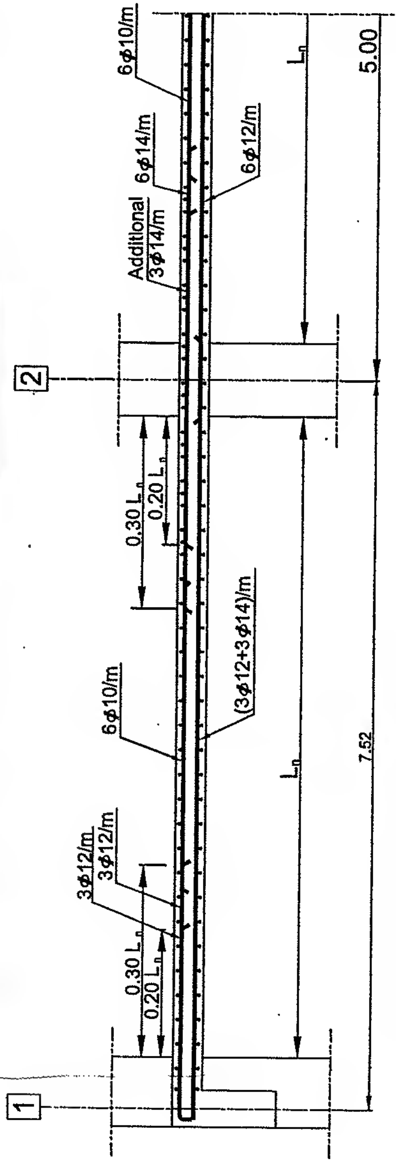


Section A-A (Column Strip-Long Direction)

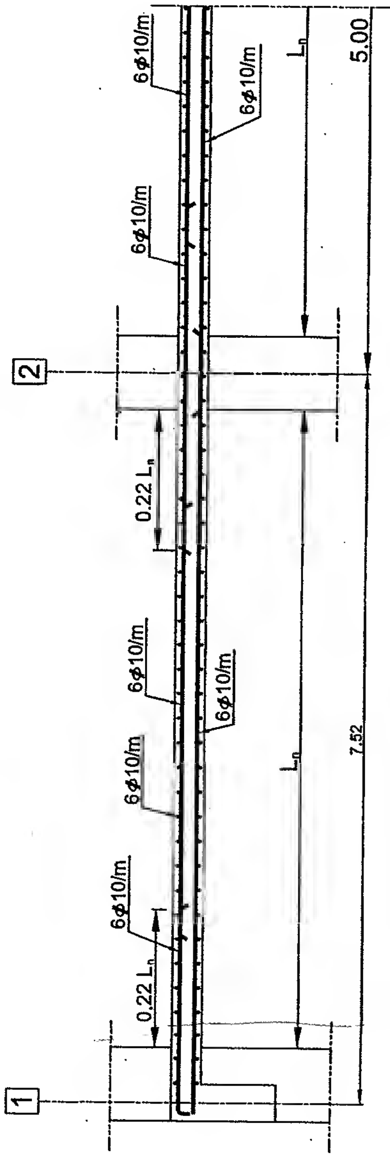
Section B-B (Field Strip-Long Direction)

Cross sections in the long direction

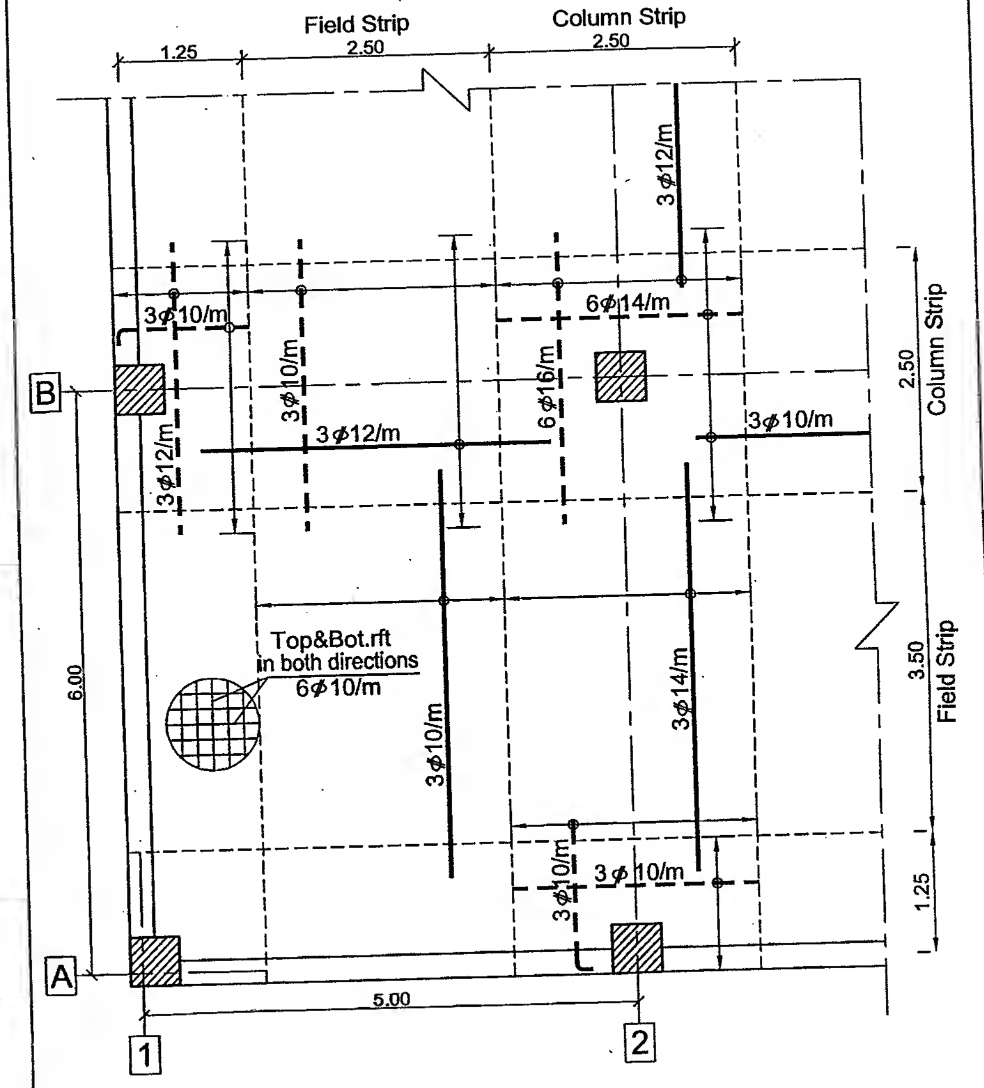
Cross sections in the short direction



Section C-C (Column Strip-Short Direction)



Section D-D (Field Strip-Short Direction)



Alternative reinforcement using mesh

LEGEND

- - - - Top

—— Bottom

▦ Reinforcement mesh

Example 4.2

It is required to design the flat slab Roof shown in Fig. EX4.2. Columns (500x 500 mm) are only allowed as shown.

Data:-

Concrete Characteristic Strength	=	25	N/mm^2
Steel Yield Stress	=	360	N/mm^2
Live Load	=	10.0	kN/m^2
Flooring	=	2.0	kN/m^2
Equivalent wall loads	=	1.5	kN/m^2
Floor Height	=	3.50	m

Solution:-

Since live load is relatively high and there is no architecture restriction, use flat slab with drop panels.

The floor consists of three equal spans in each direction, the span in the long direction equals $L_1 = 6.0$ m and the span in the short direction $L_2=5.0$ m.

The system satisfies the requirements of the empirical method specified in the Code-article (6-2-5-5).

Step 1: Dimensioning

- **Slab thickness (t_s):-**

$$t_s = bigger\ of \begin{cases} 150\ mm \\ \dfrac{L_{long}}{36} = \dfrac{6000}{36} = 166.67\ mm \end{cases}$$

Take t_s = 200 mm

- **Column Dimensions (bxb)**

$$b = bigger\ of \begin{cases} 300\ mm \\ \dfrac{h_{clear}}{15} = \dfrac{(3500-200)}{15} = 220\ mm \\ \dfrac{L_1}{20} = \dfrac{600}{20} = 300\ mm \end{cases}$$

Take b=500 mm

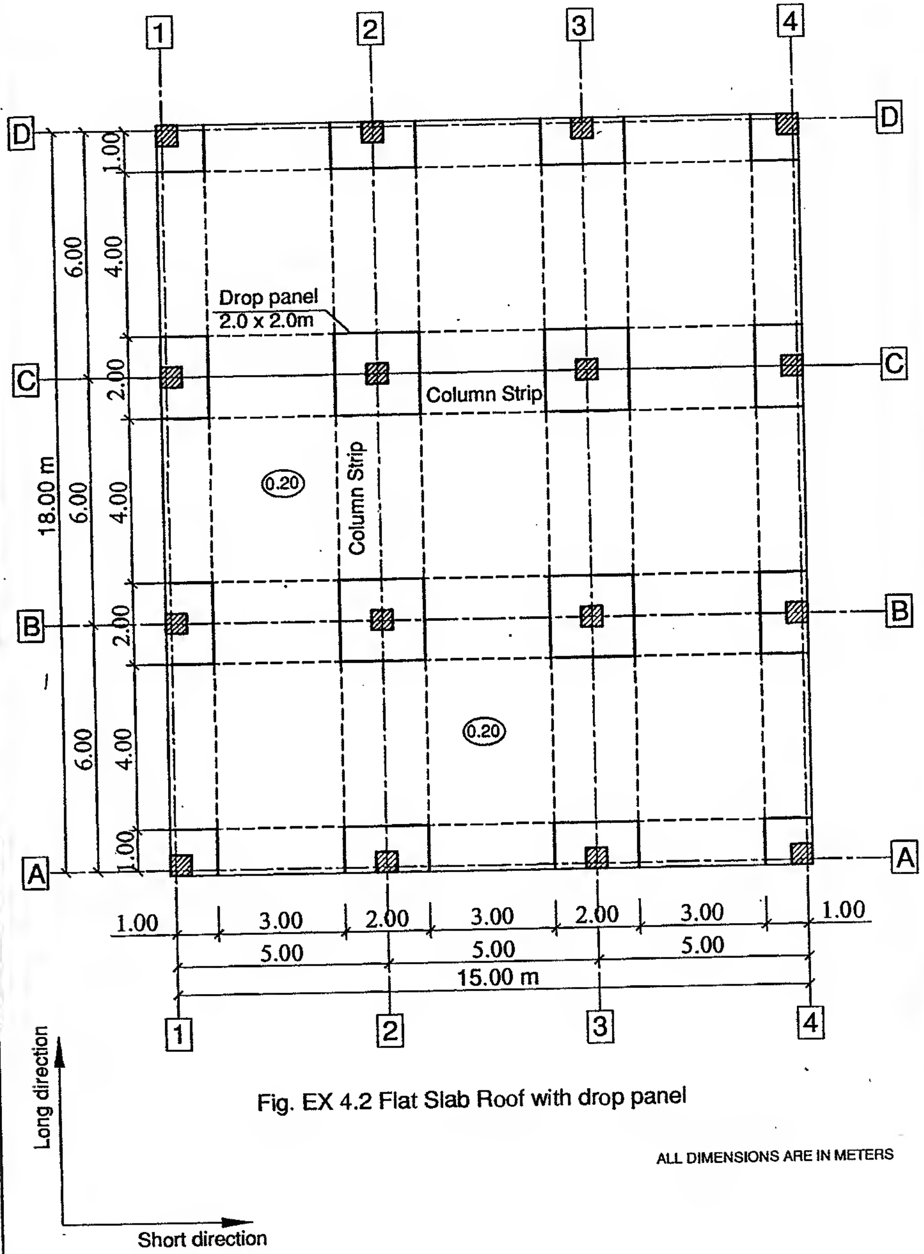


Fig. EX 4.2 Flat Slab Roof with drop panel

ALL DIMENSIONS ARE IN METERS

- **Drop Panel**

-Dimensions of Drop panels S

$\geq \quad L_1/3$

$\leq \quad L_2/2$

Taking the Drop Panel Dimensions 2.00 x 2.00 (refer to roof layout)

- **Thickness of Drop Panel**

Thickness of drop panel under the slab $\geq t_s/4 = 200/4 = 50$ mm

Taking the total depth at drop panel = 200+50 = 250 mm

- **Marginal Beams ($b_b x t_b$)**

No marginal beam is provided

- **Column and Field Strips:-**

Width of Column Strip = width of drop panel = 2.0 m (in each direction)

Width of Field Strip:-

For Long Direction = 5.00-2.00 = 3.0 m

For Short Direction = 6.00-2.00 = 4.0 m

Step 2: Minimum Steel Requirement

For $f_y = 360$ N/mm^2

$$A_{s,\min}(long) = \frac{0.6}{360} \times 1000 \times (200-20) = 300\ mm^2/m'$$

$$A_{s,\min}(short) = \frac{0.6}{f_y} \times 1000 \times (200-30) = 283\ mm^2/m'$$

Minimum Steel Reinforcement at the location of drop panel

$$A_{s,\min}(long) = \frac{0.60}{360} \times 1000 \times (250-20) = 383\ mm^2/m'$$

$$A_{s,\min}(short) = \frac{0.60}{360} \times 1000 \times (250-30) = 367\ mm^2/m'$$

Step 3: Calculation of Loads

There is additional weight due to the presence of 50 mm drop in the column strip

$$w_{drop} = 25 \times t_d \times \frac{S \times S}{L_1 \times L_2} = 25 \times 0.05 \times \frac{2 \times 2}{5 \times 6} = 0.1667\ kN/m^2$$

Dead load, g_s = Own weight + Flooring + Drop + Equivalent Wall Loads

$$= 25 \times 0.20 + 2.0 + 0.167 + 1.5 = 8.667\ kN/m^2$$

Live load, p_s = 10.0 kN/m^2

Since the live loads is greater than 0.75 the dead loads

$w_{su} \quad = \quad 1.4g_s + 1.6p_s$

$\quad\quad = \quad 1.4 \times 8.667 + 1.6 \times 10$

$\quad\quad = \quad 28.13$ kN/m^2

Step 4: Design of Strips

Step 4-a: Long Direction

Step 4-a-i: Statical system and Bending Moment

$$M_o = \frac{w_{su} \times L_2}{8}\left(L_1 - \frac{2 \times D}{3}\right)^2$$

As no column head is used D=b=0.50 m

$$M_o = \frac{28.13 \times 5}{8}\left(6.0 - \frac{2 \times 0.5}{3}\right)^2 = 564.6\ kN.m$$

Percentage of moments is taken from Table 4.1

Step 4-a-ii: Moment Correction

Since the width of the column strip is less than ½ the short span and the flat slab is with drop panel, moment correction needs to be carried out. The correction is applied to the field strip and then the column strip moment is adjusted accordingly. The correction factor equals to the ratio of the actual width of the filed strip to the width of the filed strip in case of no drop panel is used (ideal width). The ideal column strip width equals 2.5 m and the ideal field strip width is given in the following table:

Long direction		
Actual width of filed strip	Ideal width of filed strip	correction factor
3	5-2.5=2.5	1.2

The calculation of bending moments in the longitudinal direction of an interior bay is given in the following table:

	Negative moment (kN.m)		Positive moment (kN.m)	
	field strip (20%)	column strip (%50)	field strip (15%)	column strip (25%)
before correction	112.92	282.31	84.7	141.16
After correction	1.2x112.92 =135.5	(112.92+282.31)-135.5 =259.73	1.2x84.7 =101.6	225.86-101.6 =124.2

Step 4-a-iii: Design of Sections

Since the width of the column strip is 2.0 m, the maximum negative moment at the interior panel per meter equals:

$$M_u = \frac{259.73}{2.0} = 129.86 \ kN.m/m'$$

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{129.86 \times 10^6}{25 \times 1000 \times 230^2} = 0.098$$

For α =0.0   ω=0.129

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.129 \times \frac{25}{360} \times 1000 \times 230 = 2063 \ mm^2/m'$$

Step 4-b: Short Direction

Step 4-b-i: Statical system and Bending Moment

$$M_o = \frac{w_{su} \times L_1}{8}\left(L_2 - \frac{2 \times D}{3}\right)^2$$

As no column head is used D=b=0.50 m

$$M_o = \frac{28.13 \times 6}{8}\left(5.0 - \frac{2 \times 0.5}{3}\right)^2 = 459.5 \ kN.m$$

Percentage of moments is taken from Table 4.1

Step 4-b-ii: Moment Correction

Since the width of the column strip is less than ½ the short span and the flat slab is with drop panel, moment correction needs to be carried out. The correction is applied to the field strip and then the column strip moment is adjusted accordingly. The correction factor equals to the ratio of the actual width of the filed strip to the width of the filed strip in case of no drop panel is used (ideal width). The ideal column strip width equals 2.5 m and the ideal field strip width is given in the following table:

Short direction		
Actual width of filed strip	Ideal width of filed strip	correction factor
4	6-2.5=3.5	1.143

Step 4-b-iii: Design of Sections

Since the width of the column strip is 2.0 m, the maximum negative moment at the interior panel per meter equals

$$M_u = \frac{216.6}{2.0} = 108.3 \ kN.m/m'$$

The depth in the short direction=250-30=220 mm

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{108.3 \times 10^6}{25 \times 1000 \times 220^2} = 0.09$$

For α =0.0   ω=0.116

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.116 \times \frac{25}{360} \times 1000 \times 220 = 1773 \ mm^2/m'$$

Step 5: Design of edge column strip

Due to the presence of the marginal beam, the value of the bending moments in the exterior strip/m' equal to half the value of the bending moments in the interior strip/m'.

Accordingly, the reinforcement in the exterior strip/m equals half the reinforcement in the interior strip/m'. (While considering the minimum steel requirements).

Step 6: Negative reinforcement in the midspan

Since g_s >2/3 p_s (8.6>2/3 x 10=6.67), no negative moments will be developed at midspan and no top reinforcement is required for such a reason. However, since the slab thickness is greater than 160 mm, top reinforcement is needed to resist shrinkage and temperature stresses.

Provide 6 Φ 10 /m' in both directions.

Step 7: Design for Punching Shear for Interior Column

Since the floor is flat slab with drop panels, two critical sections should be investigated as shown in figure.

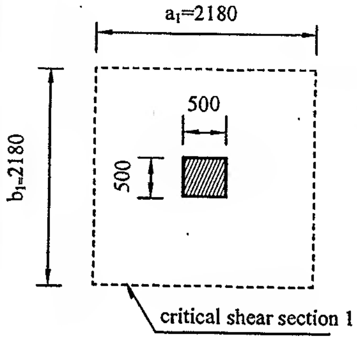
Step 7.1: Section 1

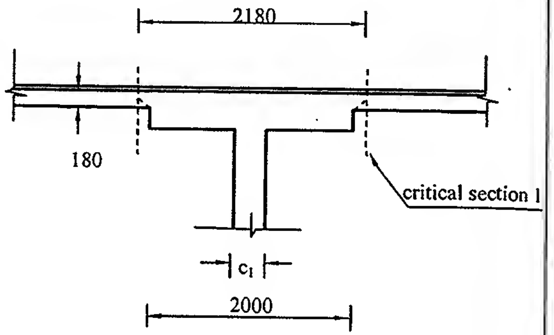
d = 200-20 = 180 mm

$a_1 = b_1 = 2000 + 180 = 2180$ mm

$Q_{up} = 28.13 \times 5.0 \times 6 - 28.13 \times 2.18 \times 2.18 = 710$ kN

β =1.15 (case of interior column)





$b_o = 2 \times (2180+2180) = 8720$ mm

$$q_u = \frac{Q_{up}\,\beta}{b_o\,d} = \frac{710 \times 1000 \times 1.15}{8720 \times 180} = 0.52\ N/mm^2$$

q_{cup} is the smallest of

1. $q_{cup} = 0.316\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\sqrt{\frac{25}{1.5}} = 1.29\ N/mm^2 < 1.6 \ldots\ldots o.k$
2. $q_{cup} = 0.316\,(0.5 + \frac{a}{b}) \times \sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\,(0.5 + \frac{500}{500}) \times \sqrt{\frac{25}{1.5}} = 1.93\ N/mm^2$
3. $q_{cup} = 0.8\,(\frac{\alpha\, d}{b_o} + 0.2) \times \sqrt{\frac{f_{cu}}{\gamma_c}} = 0.8\,(\frac{4 \times 180}{8720} + 0.2) \times \sqrt{\frac{25}{1.5}} = 0.92\ N/mm^2$

Note : α =4.0 for interior columns

$q_{cup} = 0.92\ N/mm^2$

since $q_u < q_{cup}$, section 1 is considered safe

Step 7.2: Section 2

Assume concrete cover of 20 mm

d = 250-20 = 230 mm

$a_1 = b_1 = 500 + 230 = 730$ mm

$Q_{up} = 28.13 \times 5.0 \times 6 - 28.13 \times 0.73 \times 0.73 = 829$ kN

β =1.15 (case of interior column)

$b_o = 2 \times (730+730) = 2920$ mm

$$q_u = \frac{Q_{up}\,\beta}{b_o\,d} = \frac{829 \times 1000 \times 1.15}{2920 \times 230} = 1.42\ N/mm^2$$

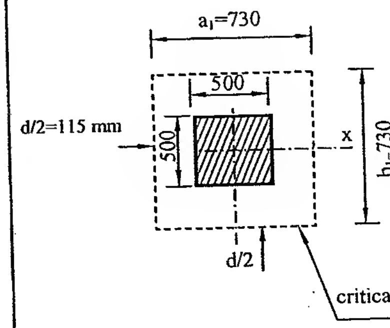
q_{cup} is the smallest of

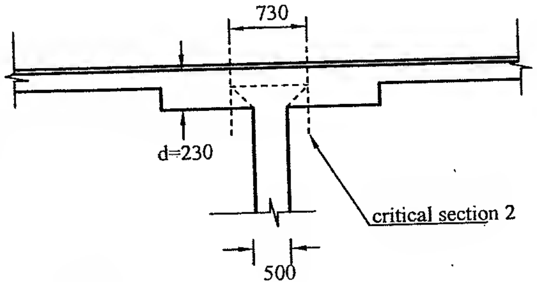
1. $q_{cup} = 0.316\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\sqrt{\frac{25}{1.5}} = 1.29\ N/mm^2$
2. $q_{cup} = 0.316\,(0.5 + \frac{a}{b}) \times \sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\,(0.5 + \frac{500}{500}) \times \sqrt{\frac{25}{1.5}} = 1.93\ N/mm^2$
3. $q_{cup} = 0.8\,(\frac{\alpha\, d}{b_o} + 0.2) \times \sqrt{\frac{f_{cu}}{\gamma_c}} = 0.8\,(\frac{4 \times 230}{2920} + 0.2) \times \sqrt{\frac{25}{1.5}} = 1.68\ N/mm^2$

Note : α =4.0 for interior columns

$q_{cup} = 1.29\ N/mm^2$

Since $q_u > q_{cup}$, the slab is considered *unsafe*

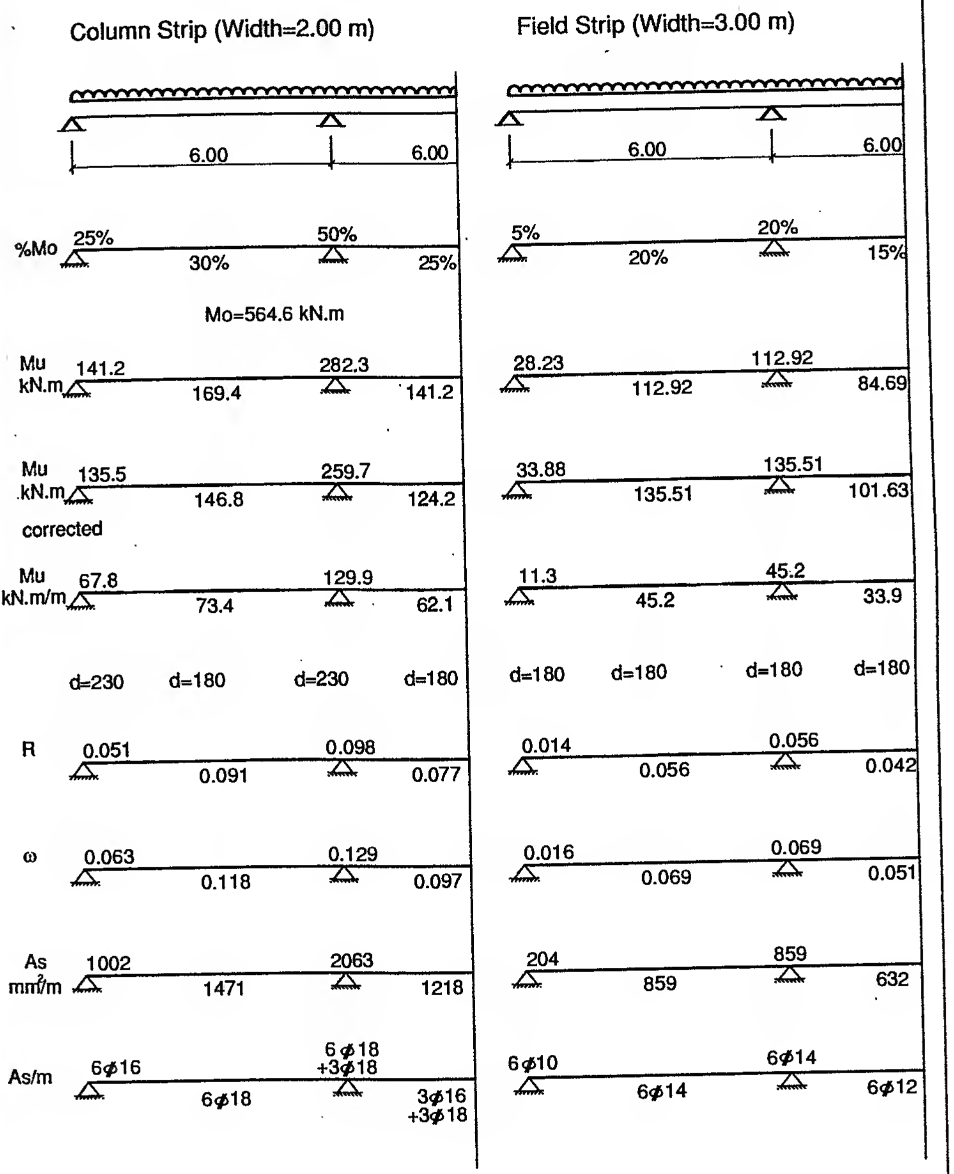




One of following solutions can be followed

1. Increasing concrete compressive strength to 30 N/mm^2 will increase q_{cup} to 1.42 N/mm^2
2. Increasing column dimensions to 600x600 will decrease q_u to 1.24 N/mm^2
3. Increasing drop panel thickness to 70 mm will decrease q_u to 1.27 N/mm^2

Design of Long Direction



Column Strip (Width=2.00 m)

Span		6.00		6.00
%Mo	25%	30%	50%	25%
Mu kN.m	141.2	169.4	282.3	141.2
Mu kN.m corrected	135.5	146.8	259.7	124.2
Mu kN.m/m	67.8	73.4	129.9	62.1
d	d=230	d=180	d=230	d=180
R	0.051	0.091	0.098	0.077
ω	0.063	0.118	0.129	0.097
As mm^2/m	1002	1471	2063	1218
As/m	6ϕ16	6ϕ18	6ϕ18 +3ϕ18	3ϕ16 +3ϕ18

Mo=564.6 kN.m

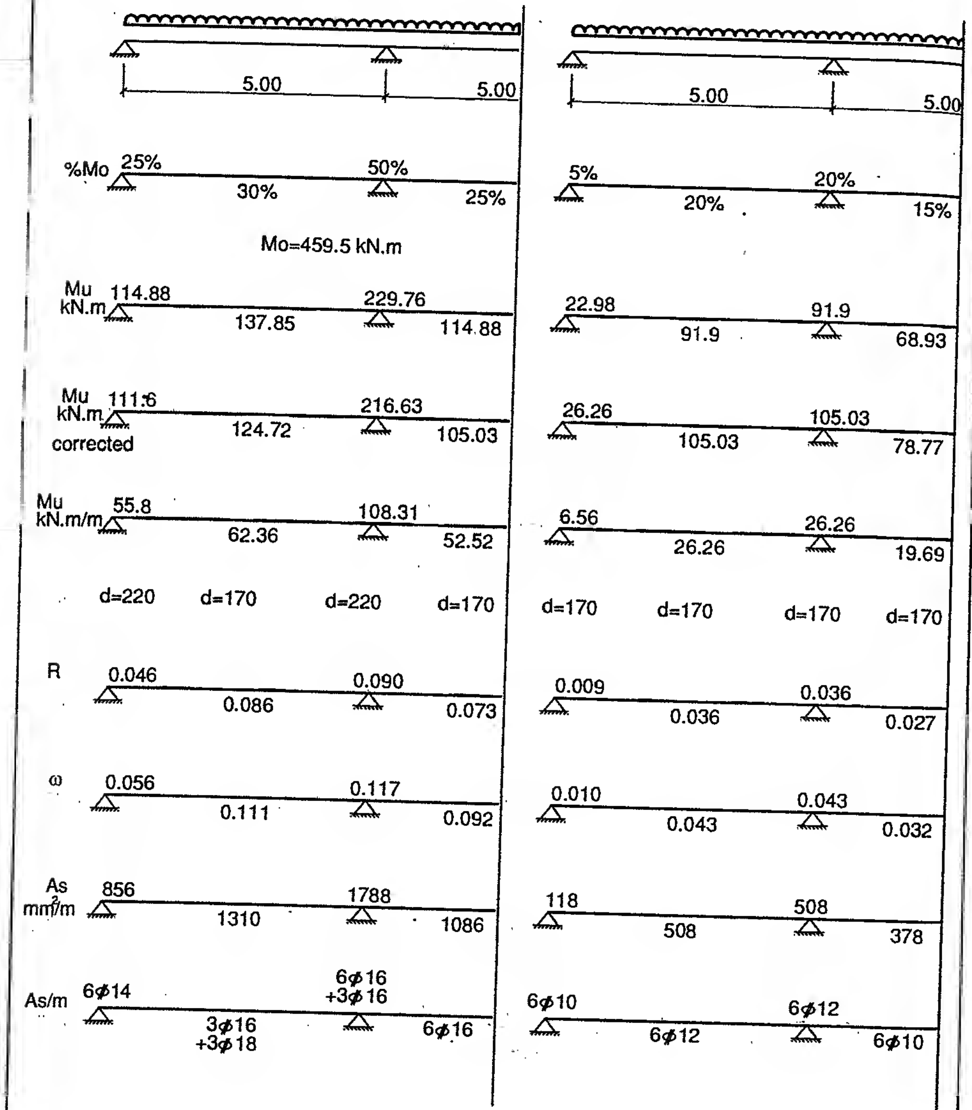
Field Strip (Width=3.00 m)

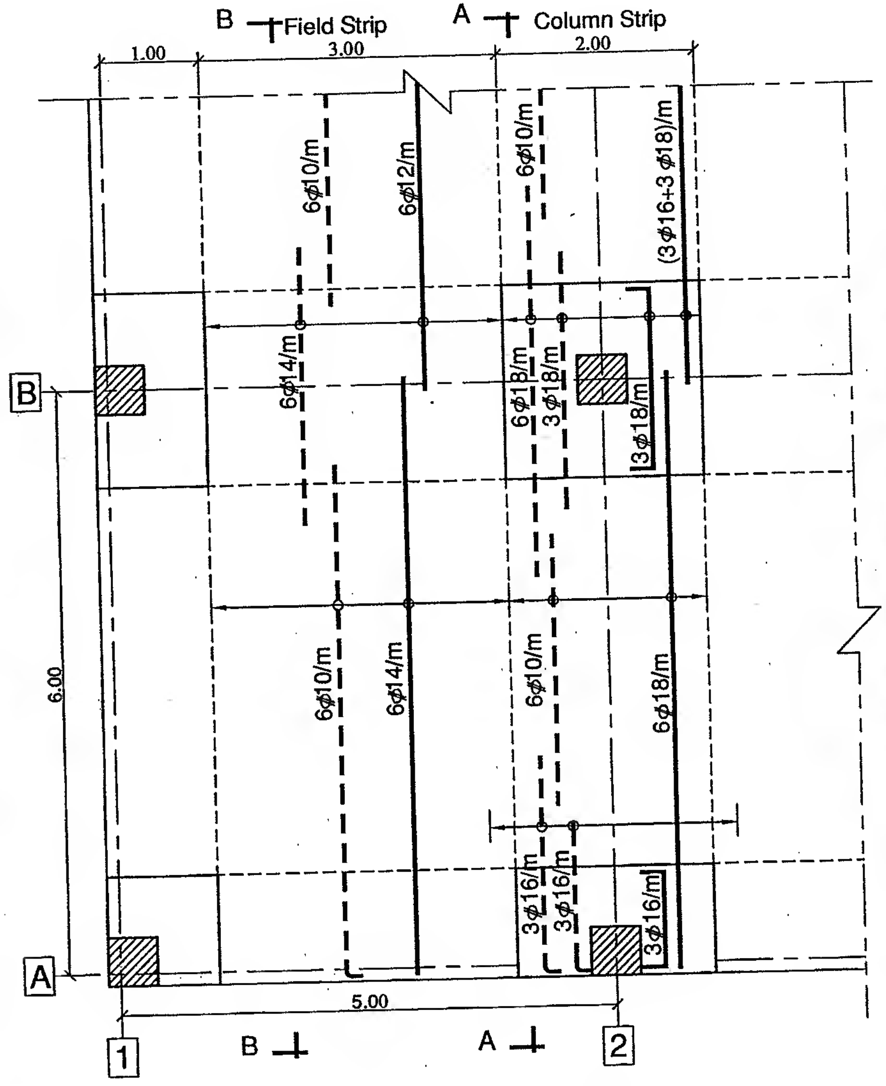
Span		6.00		6.00
%Mo	5%	20%	20%	15%
Mu kN.m	28.23	112.92	112.92	84.69
Mu kN.m corrected	33.88	135.51	135.51	101.63
Mu kN.m/m	11.3	45.2	45.2	33.9
d	d=180	d=180	d=180	d=180
R	0.014	0.056	0.056	0.042
ω	0.016	0.069	0.069	0.051
As mm^2/m	204	859	859	632
As/m	6ϕ10	6ϕ14	6ϕ14	6ϕ12

Design of Short Direction

Column Strip (Width=2.0m)

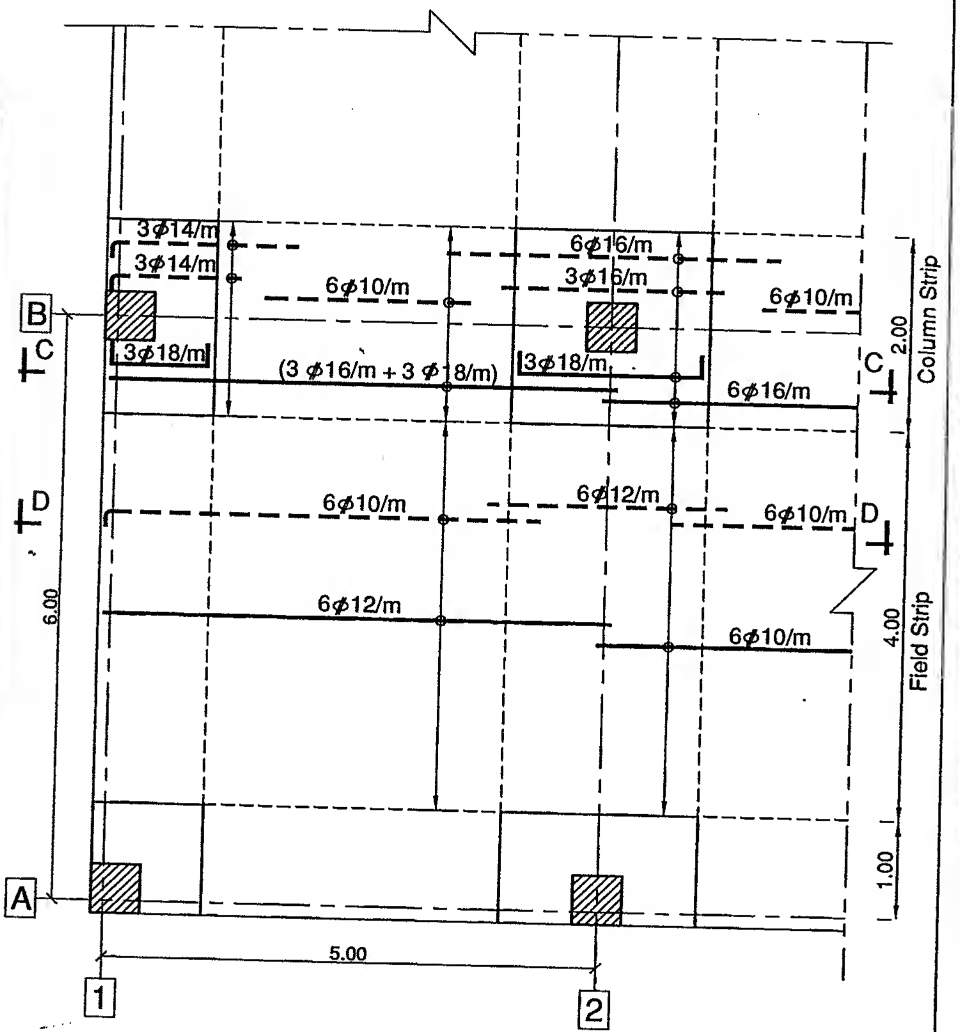
Field Strip (Width=4.00 m)





Reinforcement Details
Long Direction

LEGEND
– – – – TOP
——— BOTTOM

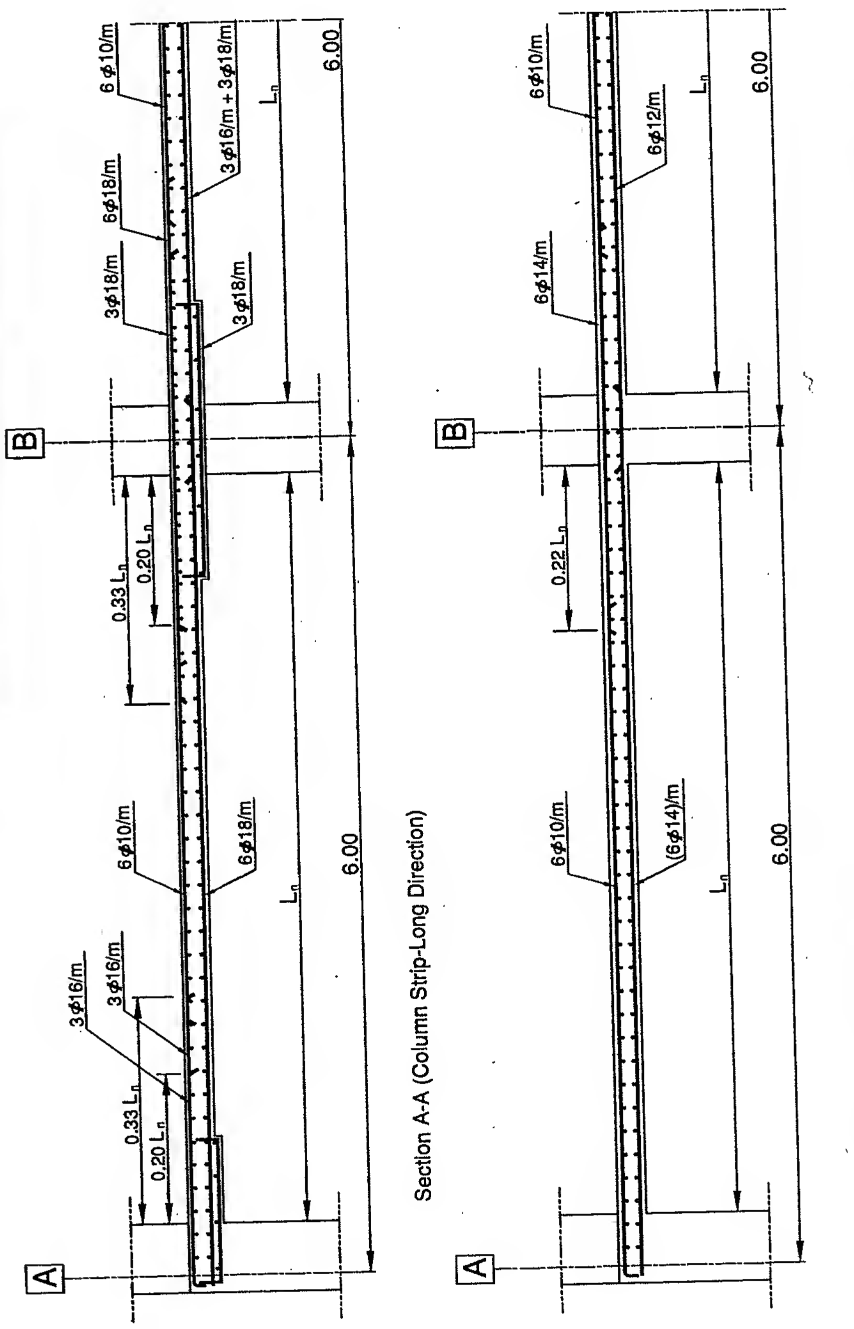


Reinforcement Details
Short Direction

LEGEND

- - - - - - TOP

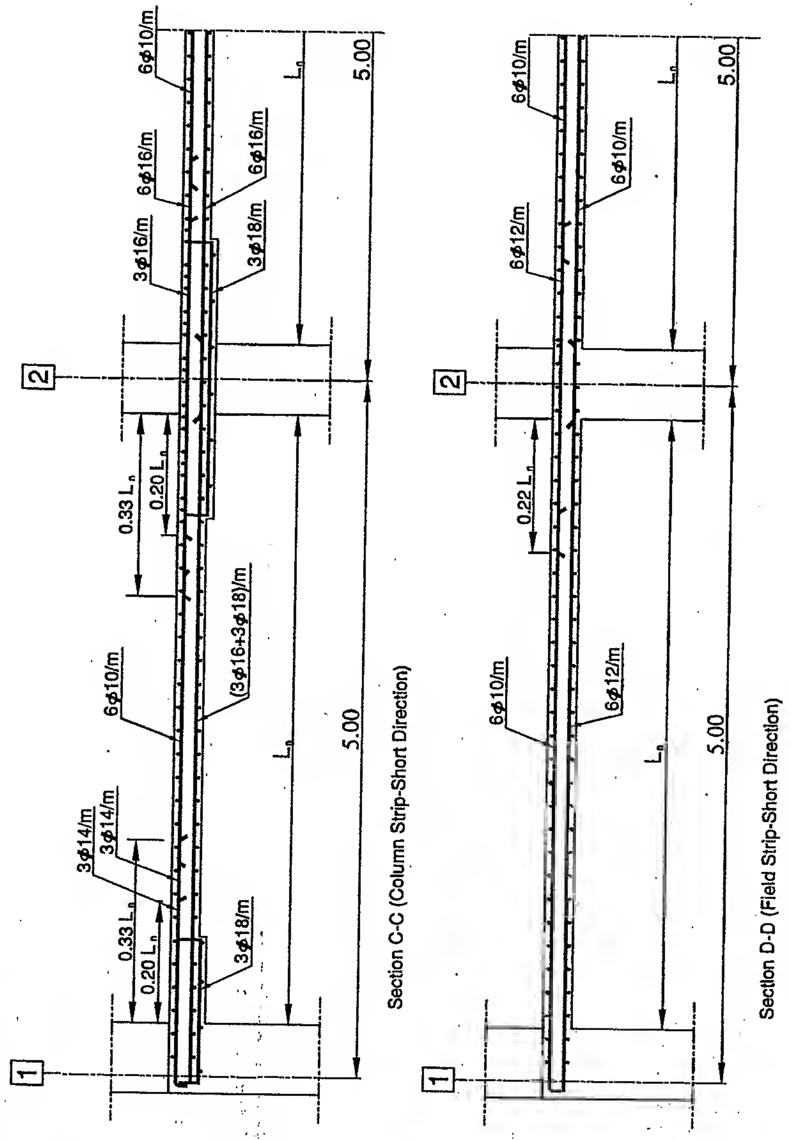
——————— BOTTOM



Section A-A (Column Strip-Long Direction)

Section B-B (Field Strip-Long Direction)

Cross sections in the long direction



Section C-C (Column Strip-Short Direction)

Section D-D (Field Strip-Short Direction)

Cross sections in the short direction

Example 4.3

Using the code simplified method, check the safety against punching failure of a corner column (0.4 x 0.4 m) in a flat slab system without marginal beams of a typical panel (6.5m x 6m). The total ultimate loads is 17 kN/m^2, t_s=200 mm and f_{cu}=25 N/mm^2 .

Solution

Assume concrete cover of 20 mm

$d = t_s - 20 = 180$ mm

$a_1 = b_1 = 400 + 180/2 = 490$ mm

$Q_{up} = 17 \times 6.5/2 \times 6/2 - 17 \times 0.49 \times 0.49 = 161.7$ kN

$\beta = 1.5$ (case of corner column)

$b_o = 2 \times 490 = 980$ mm

$$q_{up} = \frac{Q_{up}\ \beta}{b_o\ d} = \frac{161.7 \times 1000 \times 1.5}{980 \times 180} = 1.374\ N/mm^2$$

q_{cup} is the smallest of

1. $q_{cup} = 0.316\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\sqrt{\frac{25}{1.5}} = 1.29\ N/mm^2 < 1.6.....o.k$

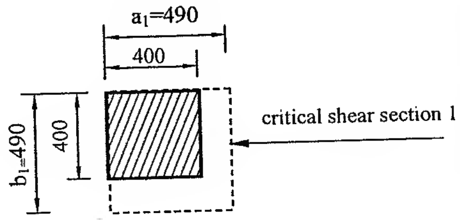
2. $q_{cup} = 0.316\ (0.5 + \frac{a}{b}) \times \sqrt{\frac{f_{cu}}{\gamma_c}} = 0.316\ (0.5 + \frac{400}{400}) \times \sqrt{\frac{25}{1.5}} = 1.93\ N/mm^2$

3. $q_{cup} = 0.8\ (\frac{\alpha\ d}{b_o} + 0.2) \times \sqrt{\frac{f_{cu}}{\gamma_c}} = 0.8\ (\frac{2 \times 180}{980} + 0.2) \times \sqrt{\frac{25}{1.5}} = 1.85\ N/mm^2$

q_{cup}=1.29 N/mm^2

$q_{up} > q_{cup}$, punching strength of the corner column is **unsafe** . To increase the punching strength of the slab one of the following solution may be adopted:

1. Increase the compressive strength of concrete
2. Increase the dimensions of the corner column
3. Increase the thickness of the slab
4. Use drop panel or column head if such a solution meets the acceptance of the architect.



Example 4.4

Redesign the previous example using the code-detailed method

Solution

Step1: Calculate the unbalanced moments

$M_{ox} = 17 \times 6 \times (6.5 - 2/3 \times 0.4)^2 / 8 = 495$ kN.m

$M_{oy} = 17 \times 6.5 \times (6.0 - 2/3 \times 0.4)^2 /8 = 454$ kN.m

% of column strip moment = 40% (no edge beams)

M-ve column strip in x-direction (M_y) = 0.4 x 495 = 198.2 kN.m

M-ve column strip in y-direction (M_x) = 0.4 x 454 = 181.6 kN.m

Moment transferred to column in x-direction (M_y)= 0.9 x 0.5 x 198.2 = 89.2 kN.m

Moment transferred to column in y-direction (M_x)= 0.9 x 0.5 x 181.6 = 81.7 kN.m

$$\gamma_{fx} = \gamma_{fy} = \frac{1}{1+\frac{2}{3}\sqrt{\frac{c_1+d}{c_2+d}}} = \frac{1}{1+\frac{2}{3}} = 0.6$$

$\gamma_{qx}=\gamma_{qy}= 1- 0.6 = 0.4$

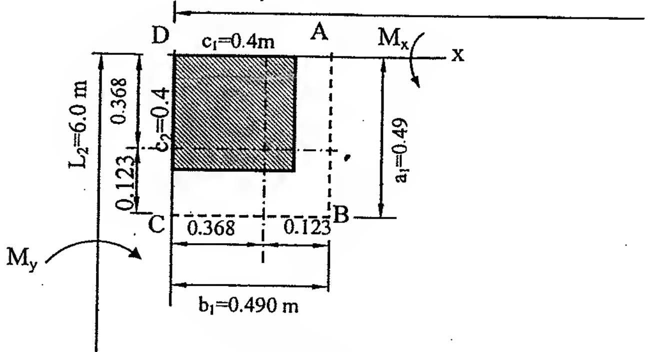
Moment transferred by torsion in x-direction M_y= 0.4 x 89.2 = 35.7 kN.m
Moment transferred by torsion in y-direction M_x= 0.4 x 81.7 = 32.7 kN.m

Step 2: Calculate section properties

$a_1=c_1+d/2 = 0.4 +0.18/2 = 0.49$ m

$b_1=c_2+d/2 = 0.4 +0.18/2 = 0.49$ m

$$C_{AB} = C_{CB} = \frac{a_1^2}{(2\times b_1 + 2\times a_1)} = \frac{0.49^2}{2\times 0.49 + 2\times 0.49} = 0.123\,m$$



Corner column notations

$C_{CD}=C_{AD}= 0.49 -0.123 =0.368$ m

$$J_{cy} = d\times b_1\times C_{AB}^2 + \frac{d\times C_{CD}^3}{3} + \frac{d\times C_{AB}^3}{3} + \frac{a_1\times d^3}{12}$$

$$J_{cx} = J_{cy} = 0.18\ \times 0.49\times 0.123^2 + \frac{0.18\times 0.368^3}{3} + \frac{0.18\times 0.123^3}{3} + \frac{0.49\times 0.18^3}{12} = 0.00465 m^4$$

Step 3: Calculate shear stresses

The ultimate shear force equals

$Q_{up} = 17 \times 6.5/2 \times 6/2 - 17 \times 0.49 \times 0.49 = 161.7$ kN

The shear stress due to punching of the gravity loads equals to:

$$q_p = \frac{Q_{up}}{b_o\,d} = \frac{161.7\times 1000}{980\times 180} = 0.917 \quad N/mm^2$$

$$q_{y1} = \frac{(M_y\,\gamma_{qy})C_{AB}}{J_{cy}} = \frac{35.7\times\ 0.123}{0.00465\times 1000} = 0.944\,N/mm^2$$

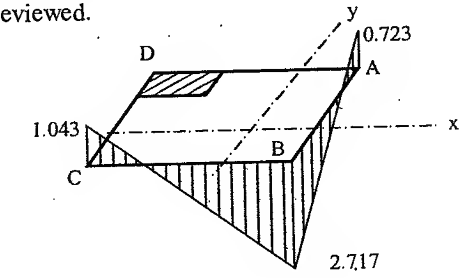
$$q_{y2} = \frac{(M_y\,\gamma_{qy})C_{CD}}{J_{yx}} = \frac{35.7\times\ 0.368}{0.00465\times 1000} = 2.82\,N/mm^2$$

$$q_{x1} = \frac{(M_x\,\gamma_{qx})C_{CB}}{J_{cx}} = \frac{32.7\times\ 0.123}{0.00465\times 1000} = 0.86\,N/mm^2$$

$$q_{x2} = \frac{(M_x\,\gamma_{qx})C_{AD}}{J_{cx}} = \frac{32.7\times\ 0.368}{0.00465\times 1000} = 2.58\,N/mm^2$$

$q_A = q_p + q_{y1} - q_{x2}= 0.917 + 0.944 - 2.58 = -0.723$ N/mm^2
$q_B = q_p + q_{x1} + q_{y1} = 0.917 + 0.86 + 0.944 = 2.717$ N/mm$^2 \longrightarrow\longrightarrow > q_{cup}$ (unsaf)
$q_C = q_p + q_{x1} - q_{y2}= 0.917 + 0.86 - 2.82 = -1.043$ N/mm^2

Therefore, the maximum shear stress is at corner B with a value of 2.717 N/mm^2. The value of the shear stress obtained using the simplified method (1.375 N/mm^2) is extremely low when compared to the value obtained using the detailed method. This leads to a conclusion that the values of β given in the simplified method of the Egyptian code should reviewed.



4.9 The Equivalent Frame Method

4.9.1 Introduction

The Equivalent frame method was first introduced in 1948. It is intended for use in analyzing moments in any practical frame building. The method is more general than the direct design method, which is subjected to limitations described in section 6-2-7-5-A in the ECP. Thus, if the buildings do not satisfy the loading and geometrical conditions required by the code or if lateral loads due to wind or earthquake exist, the ECP 203 requires the building to be analyzed by the *equivalent frame method.*
The equivalent frame method for design of the flat slab system is a more rigorous method of analysis when compared with the direct design method. The building is represented by a series of two-dimensional frames, which are then analyzed for loads acting in the planes of the frames. In the direct method, the statical moment, M_o is calculated for each span and divided between positive and negative moment regions according to the code coefficients given in Table 4-4.

The main features of this method can be summarized as follows:

1. Moments are distributed among the critical sections by employing an elastic analysis. Cases of loading have to be considered for the most critical loading conditions.
2. There are no limitations on loading or dimensions.
3. The variations in the moment of inertia such as drop panel have to be considered in the analysis.
4. Lateral analysis can be performed during the computations of the equivalent frame.
5. The total statical moment calculated using this method need not exceed the moment M_o required by the direct design method.

4.9.2 Structural Analysis

The slab is divided into a series of equivalent frames running in two perpendicular directions. These frames consist of the slab, drop panel, projected beams, and the columns above and below the considered floor. These frames are in turn divided into column and middle strips. The live loads should be placed at the locations that produce maximum moment for the members.

In the case of analyzing vertical loads, the moment of inertia for the slab is calculated using the total strip width. While in the case of performing lateral load analyses, the moment of inertia for the slab is calculated using column width plus three times the slab thickness from each side (not exceeding span/3).

When a lateral analysis is carried out, the full height of the building should be modeled to account for the variation of wind or earthquake forces at each level. If the analysis is limited to gravity loads, calculations can be greatly simplified by analyzing each floor and its attached column separately. The column ends are assumed to be fixed at the intersection with the floor above and below as shown in Fig. 4.23.

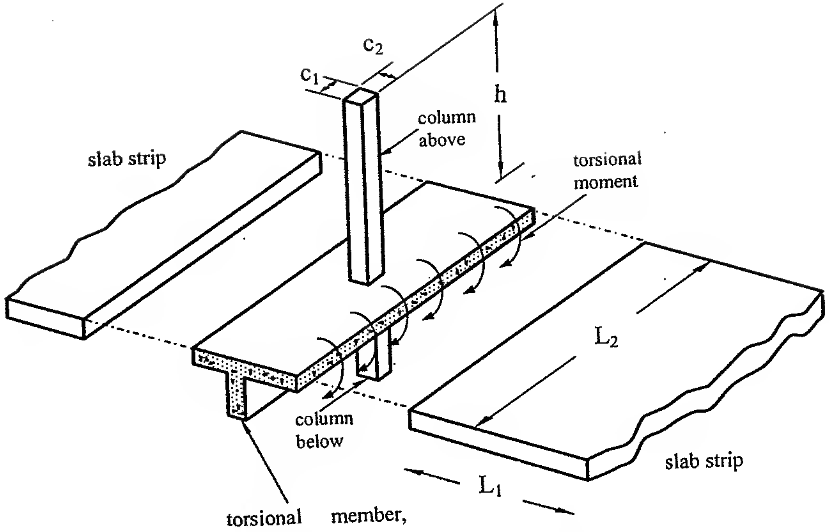


Fig. 4.23 Elements of the equivalent frame method

The moment of inertia of columns may be based on the gross area uncracked concrete, allowing for variations due to changes in column cross section along the length of the column.

Fig. 4.23 illustrates the different elements of the equivalent frame for a flat slab system. The column provides a resisting moment M_T equivalent to the applied torsional intensity M_t. Therefore, the exterior ends of the slab strip rotate more than the central section because of the torsional deformation. To account for the deformation and rotation, an equivalent column replaces the actual column and the transverse slab strip. The flexibility of the equivalent column is equal to the sum of the flexibilities of the actual column and the slab strip.

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_t} \quad \text{...(4.46)}$$

Recalling that the stiffness of a member is the inverse of the flexibility of that member, the previous equation can be rewritten as

$$K_{ec} = \frac{\sum K_c}{\left(1 + \frac{\sum K_c}{K_t}\right)} \quad \text{...(4.47)}$$

Where:

$$K_c = \frac{4 \times E_c \times I_g}{h}$$

The form of the previous equation can be explained by making an analogy between the equivalent column and the system of two springs. The total deformation of both systems equals the sum of the two individual displacements. If the two springs are replaced by a single spring with an equivalent stiffness K_{ec}, the second system must deflect similar to the original system when identical load P is applied at the end. Equating the deflection of system 1 to that of the equivalent system 2 gives

$$\Delta = \Delta_1 + \Delta_2 \quad \text{...(4.48)}$$

Since the relation between force and displacement for a spring is P=KΔ, where K equals spring stiffness, Eq. 4.48 can be expressed in term of the applied load P and the stiffness as

$$\frac{P}{K_c} + \frac{P}{K_t} = \frac{P}{K_{ec}} \quad \text{...(4.49)}$$

dividing both sides with P gives

$$\frac{1}{K_c} + \frac{1}{K_t} = \frac{1}{K_{ec}} \quad \text{...(4.50)}$$

Code equation 6.16 is similar to Eq. 4.50 but includes a summation sign to account for the possibility of contributions from columns above and below the slab.
An approximate expression for the stiffness of the torsional member, based on the results of three-dimensional analysis of various slab configurations is given by

$$K_t = \sum \left(\frac{9 E_c \cdot C}{L_2 \cdot \left(1 - \left(\frac{c_2}{L_2} \right) \right)^3} \right) \quad \text{...(4.51)}$$

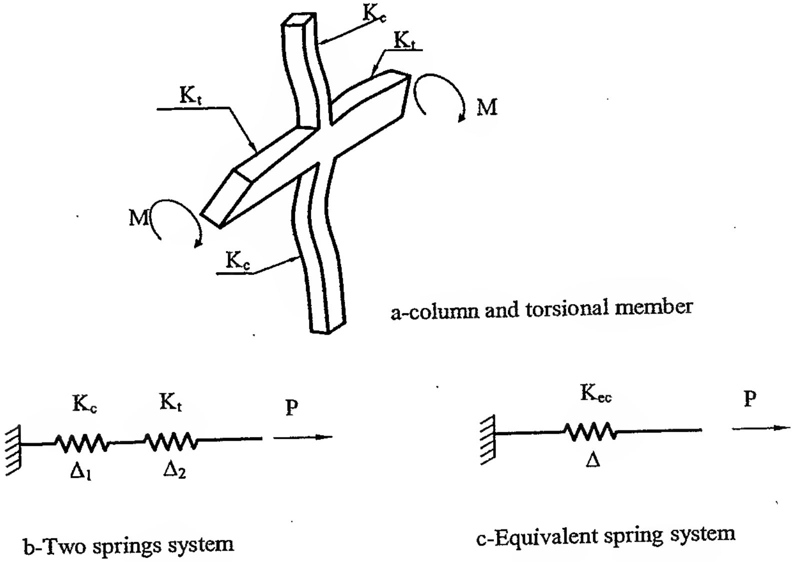
E_c is the modulus of elasticity of concrete.
c_2 is the transverse dimension of the column, equivalent column, capital or bracket.
L_2 is the center to center distance measured perpendicular to the analysis direction as shown in Fig. 4.23
An expression for C, which is a cross sectional constant to define torsional properties, is given by:

$$C = \sum \left[\left(1 - 0.63 \frac{b}{t} \right) \times \frac{b^3 \times t}{3} \right] \quad \text{...(4.52)}$$

where b and t are the shorter and the longer dimensions respectively for the member. It the torsional member consists of beam and slab, the section can be divided into a number of rectangles.

Table 4.4 Distribution of column and field strip in equivalent frame method

Moment type	Percentage of moment form total moments	
	Column strip	Field strip
negative moment in interior panel	75	25
negative moment in exterior panel	80	20
positive moment	55	45



a-column and torsional member

b-Two springs system

c-Equivalent spring system

Fig. 4.24 Equivalent column and analogous spring system

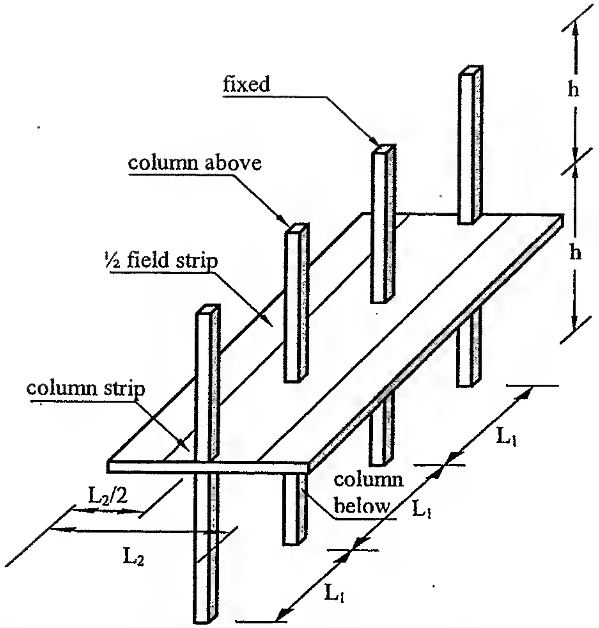


Fig. 4.25 Column and field strips in the equivalent frame method.

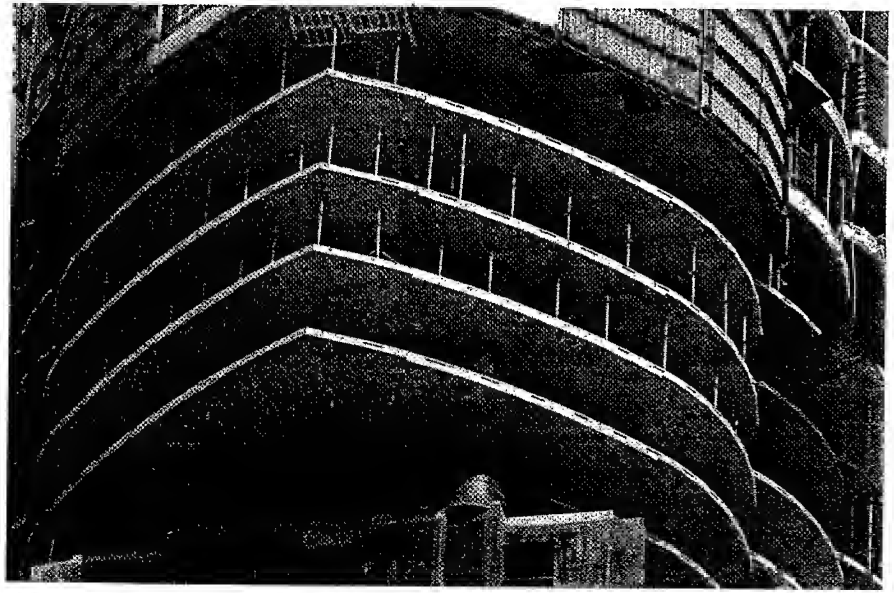
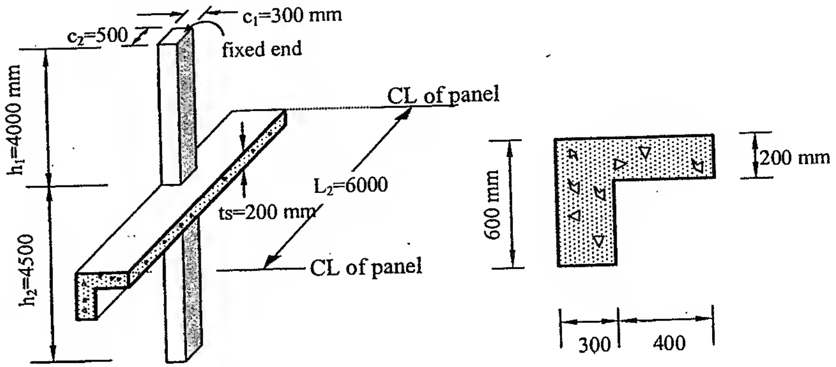


Photo 4.4 Flat slab system during construction

Example 4.5

Evaluate the effective stiffness for exterior column shown in figure below. The floor consists of an 200 mm thick slab supported on a marginal beams that are 300 mm wide and 600 mm deep.



Solution

Step 1: Compute column stiffness

The equivalent column stiffness equals

$$K_{ec} = \frac{\sum K_c}{\left(1 + \frac{\sum K_c}{K_t}\right)}$$

The stiffness of each column equals

$$K_c = \frac{4 \times E_c \times I_g}{h}$$

The height center to center of the column varies from one floor to the other. $h_1=4500\ mm$ and $h_2=4000\ mm$. The moment of inertia for the column is taken about axis parallel to the edge of the slab

$$I_g = \frac{500 \times 300^3}{12} = 1.13 \times 10^9\ mm$$

$$\sum K = \frac{4 \times E_c \times I_g}{h_1} + \frac{4 \times E_c \times I_g}{h_2} = \frac{4 \times E_c \times 1.13 \times 10^9}{4000} + \frac{4 \times E_c \times 1.13 \times 10^9}{4500}$$

$$\sum K = 2.125 \times 10^6\ E_c$$

Step 2: Compute the torsional member properties

According to the ECP 203 the attached torsional member is divided in two parts. The dimension of the slab portion equals (b_1=200, t_1=400) and that for the beam equals (b_2=300, t_2=600). The torsional constant C equals

$$C = \sum\left[\left(1-0.63\,\frac{b}{t}\right)\times\frac{b^3\times t}{3}\right]$$

$$C = \left[\left(1-0.63\,\frac{200}{400}\right)\times\frac{200^3\times 400}{3}\right]+\left[\left(1-0.63\,\frac{300}{600}\right)\times\frac{300^3\times 600}{3}\right] = 4.43\times10^9\ mm^4$$

Since torsion arms of the proportions exists on both sides of the column, two identical terms are included in the determination of K_t as follows

$$K_t = \sum\left(\frac{9\,E_c\cdot C}{L_2\cdot\left(1-(c_2/L_2)\right)^3}\right)$$

$$K_t = 2\times\left(\frac{9\,E_c\cdot 4.43\times10^9}{6000\cdot\left(1-\left(\frac{500}{6000}\right)\right)^3}\right) = 17.25\times10^6\ E_c$$

Step 3 Compute the Equivalent column stiffness K_{ec}

$$K_{ec} = \frac{\sum K_c}{\left(1+\frac{\sum K_c}{K_t}\right)} = \frac{2.125\times10^6\ E_c}{\left(1+\frac{2.125\times10^6\ E_c}{17.25\times10^6\ E_c}\right)} \approx 1.89\times10^6\ E_c$$

4.10 Computer Model of Flat Slabs

Flat slab can be modeled using shell or plate bending elements, while the beams are modeled using frame elements.

When flat slabs are modeled using thin shell elements and beams are modeled using frame elements, the eccentricity of the slab from the c.g. of the member should be considered. However, the common practice is to neglect such an effect.

The effect of the torsional moment m_{xy} must be considered when analyzing the results of finite element programs as shown in Fig. 4.26. In case of using uniform reinforcement arranged in two perpendicular directions, the bending moment in any strip can be obtained using the following equations

$$\bar{m}_x = |m_x| + |m_{xy}| \qquad \text{...(4.53)}$$

$$\bar{m}_y = |m_y| + |m_{xy}| \qquad \text{...(4.54)}$$

where

$\bar{m}_x$ and $\bar{m}_y$ are the maximum bending moments per meter in x and y directions, respectively. The design moment $\bar{m}_x$ or $\bar{m}_y$ need not exceed 1.5 the average moment in the strip.

$|m_x|$ and $|m_y|$ are the absolute value of the bending moment in the strip

$|m_{xy}|$ is the absolute value of the torsional bending moment in the strip

In the case of modeling the columns as points restrained in the vertical direction, the design moment is taken at the perimeter of the columns. It should be noted that the finite element method usually overestimates the negative bending moment over the supports and underestimates the positive bending moment at midspan.

It should be mentioned also that deflections obtained using analysis that based on linear elastic finite elements should be modified to take into account the effect of cracking.

According to the ECP 203, the main reinforcement can be arranged in the direction of the principal tensile stresses with a maximum deviation of ± 15 degrees. Otherwise, the reinforcement should placed in two perpendicular directions.

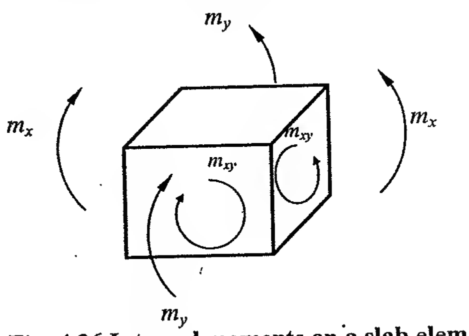


Fig. 4.26 Internal moments on a slab element.

An example of the results obtained from modeling the flat slab using plate elements is shown in Fig. 4.27. One can notice the intensity of the moment contours over the column locations.

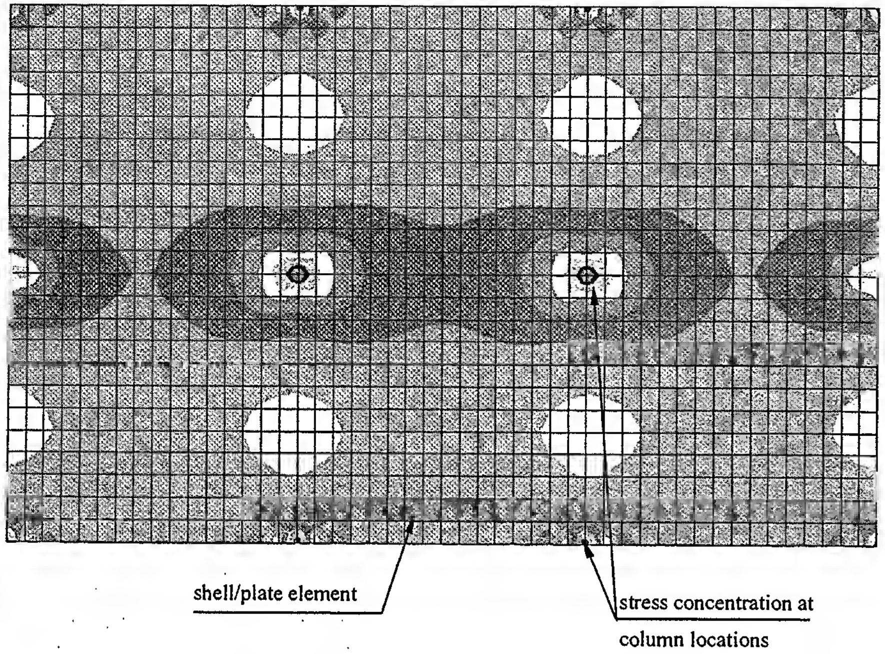


Fig. 4.27 Computer model of flat slab system

When designing the top reinforcement of the flat slab one should use the value of the bending moment at the face of the column (Sec. 1-1 and Sec. 2-2) as shown in Fig. 4.28. In other words, the contour line located inside the columns should be ignored.

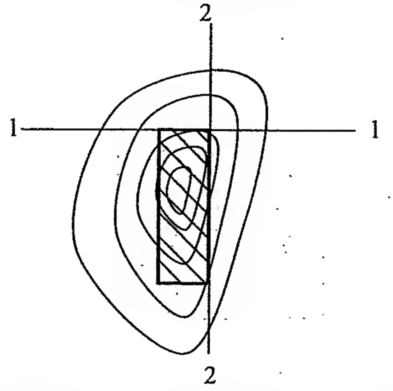


Fig. 4.28 Moment contours at column locations

5

REINFORCED CONCRETE STAIRS

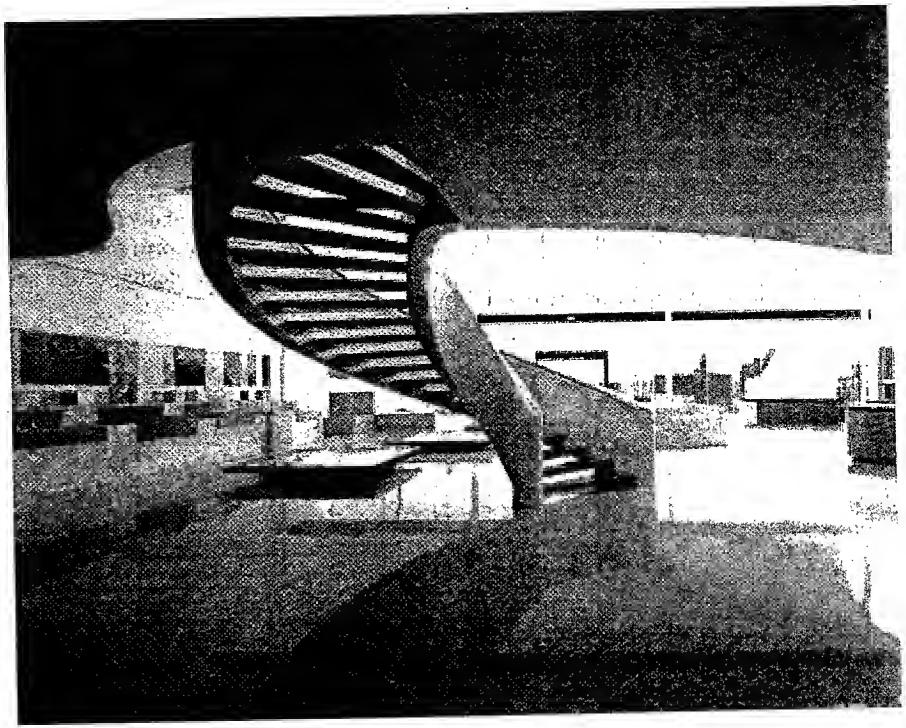


Photo 5.1 Spiral staircase at Arca showroom, USA.

5.1 Introduction

Reinforced concrete stairs are essential elements in buildings to transfer people from one level to another. The staircase consists of landings and flights. The flight is an inclined slab that consists of risers and treads (going), while the landing is a horizontal slab. The flights and the landings are supported on broken, inclined or horizontal beams and columns as shown in Fig. 5.1.

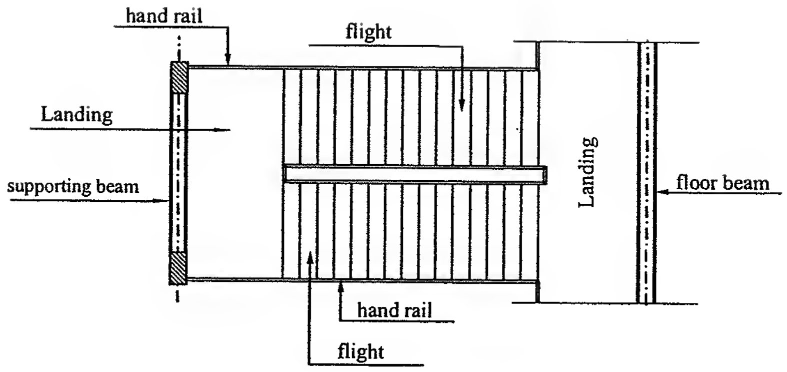


Fig. 5.1 Details of landings and flights for a staircase

The stair consists of risers and treads (*going*). The height of the riser (r) is about 150-200 mm, while the width of the going (t_r) is about 250-300 mm as shown in Fig. 5.2. The higher the riser the shorter the going. It is common practice to form the height of the riser at 150 mm and the width of the going at 300 mm. A good design of the stair should comply with the following role of thumb

$$2r + t_r = 600\text{-}620 \text{ mm} \quad \ldots\ldots (5.1)$$

To achieve comfort, landing has to be formed every 10-14 steps. Landing may be also needed when a change in the direction of the stairs is required.

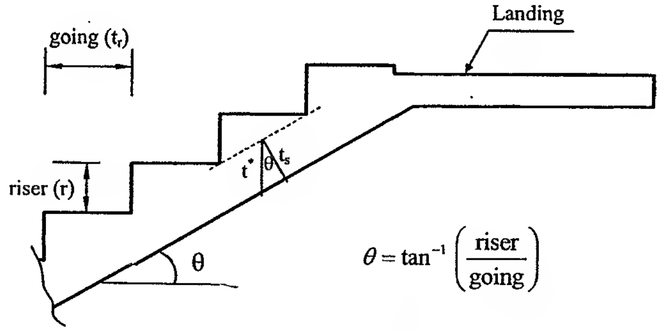


Fig. 5.2 Geometric design of the stairs

The width of the stair in each direction usually ranges from 0.9-1.5 m and the space (*stairwell*) between each flight ranges from 0.3-0.6 m. Thus, for residential building with a height of no more than 3 m, the total space for the staircase is approximately 3x6 m as shown in Fig 5.3. For proper geometric design, care should be given to the details C and D in Fig. 5.3.

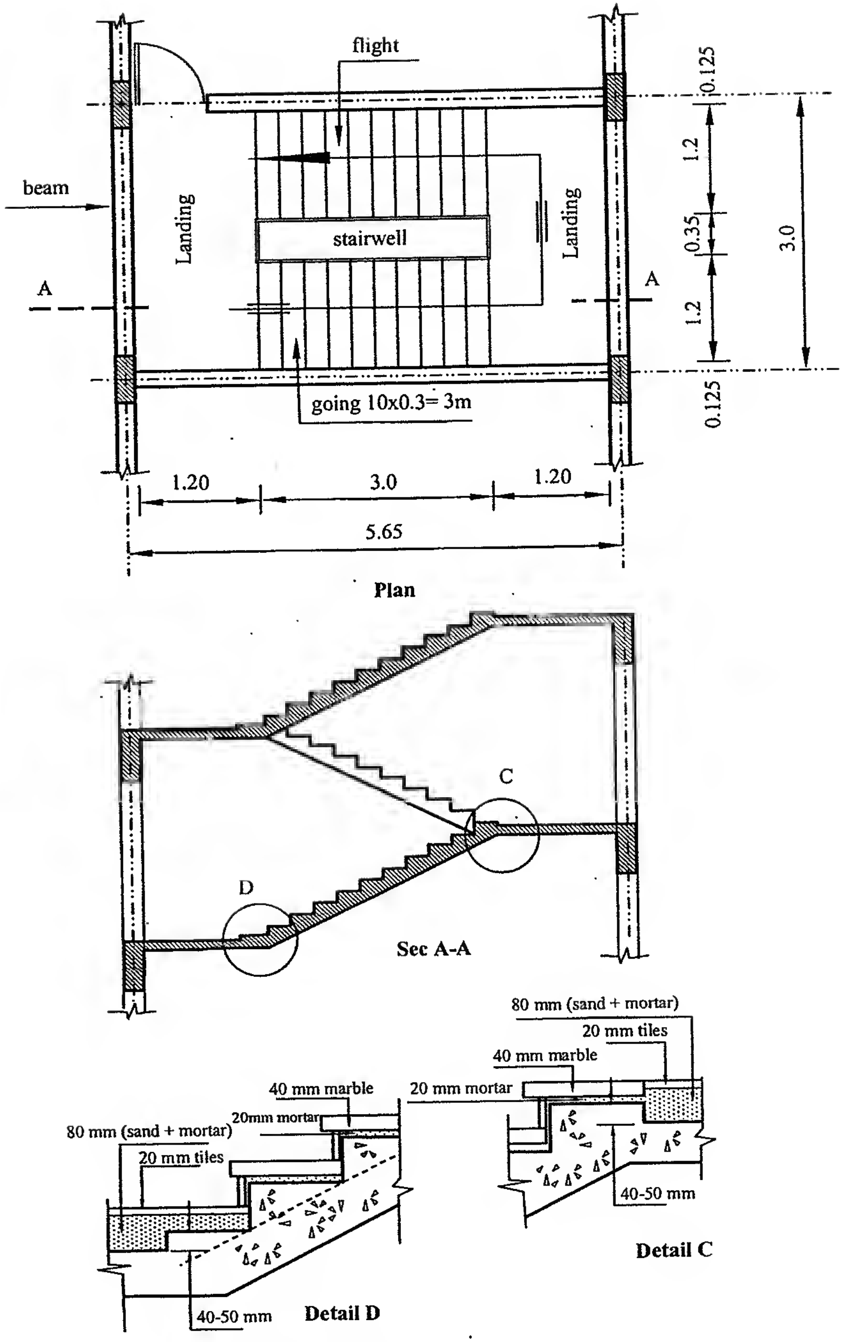


Fig. 5.3 Typical 2-flight staircase in residential buildings

5.2 Structural Systems of Stairs

From the structural point of view, the types of stairs can be classified into five categories as follows:

1. Cantilever type.
2. Slab type.
3. Slab-beam type.
4. Spiral type
5. Free-stranding type

The main differences among these types are the way through which the flights and the landings transfer their loads to the supporting beams and columns.

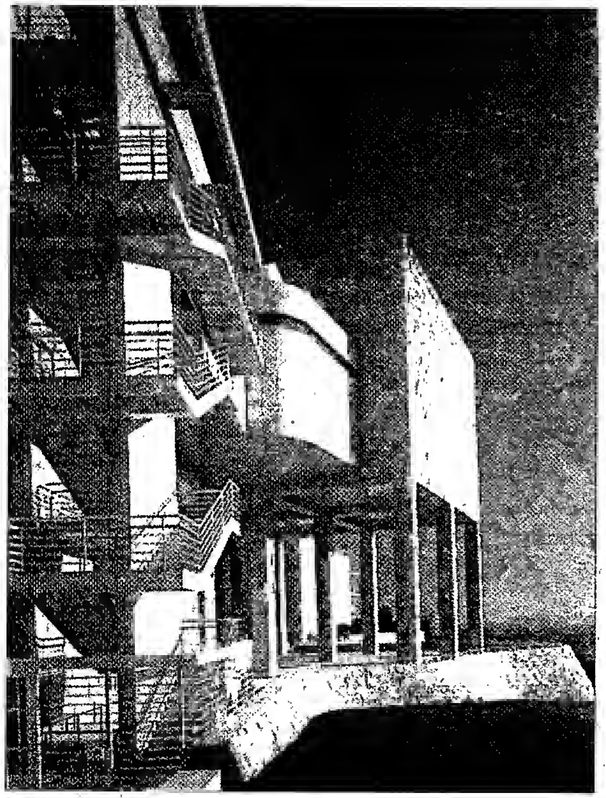
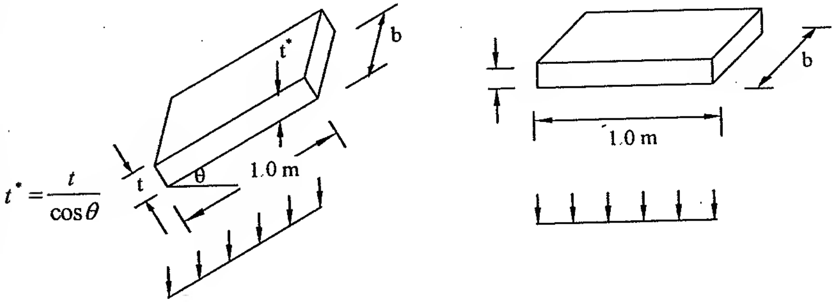


Photo 5.2: Cantilever reinforced concrete stair

Since the flight is inclined, the self-weight of the flight is normally calculated in the horizontal projection. Thus, the vertical distance t^* is used in the weight calculations of the loads as shown in Fig. 5.4.



$w = \gamma_c \times (b \times t \times 1)$ $\qquad$ $w = \gamma_c \times (b \times t \times 1)$

$$w\,(H.P) = \frac{\gamma_c \times (b \times t \times 1)}{\cos\ \theta} = \gamma_c \times (b \times t^* \times 1)$$

Fig. 5.4 Calculation of the flight self weight

5.2 Cantilever Type

In this type, the flights, the landings and the stairs act as a cantilever slab supported on a beam or a wall as shown in Fig. 5.5. Care should be taken to the correct placing of the main reinforcement in the stairs and inside the beam during construction time.

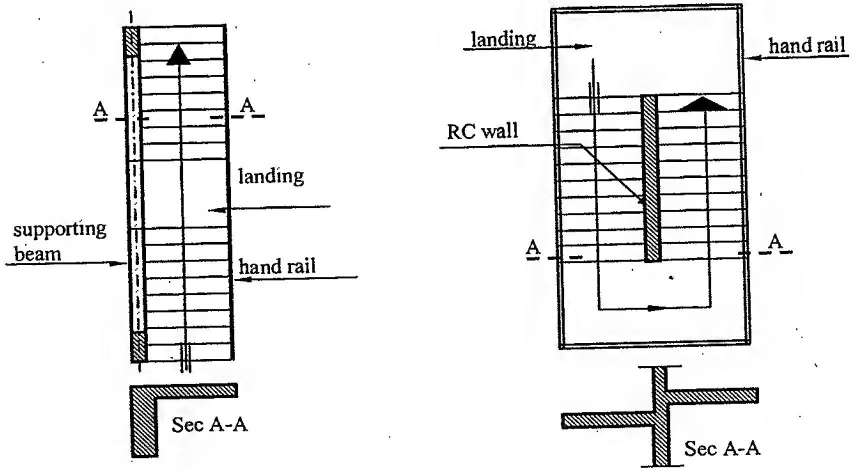


Fig. 5.5 Examples of cantilever stairs

The staircase is designed by taking one-meter width of the flight or the landing with an effective depth d_{avg}. The effective depth is taken as the average depth of the section. The main reinforcement is placed in the top of the stairs and anchored to the supporting beam as shown in Fig.5.6. The selected reinforcement is normally two bars preferably placed at the middle of the stair and at the stair edge as shown in Fig. 5.7. In this case, an additional bar is placed at the other edge. The bending moment developed in the stairs is transformed into torsion on the beam. The amount of the developed torsion is quite large and should be investigated. A light reinforcement mesh is placed in the bottom face to resist cracking and shrinkage.

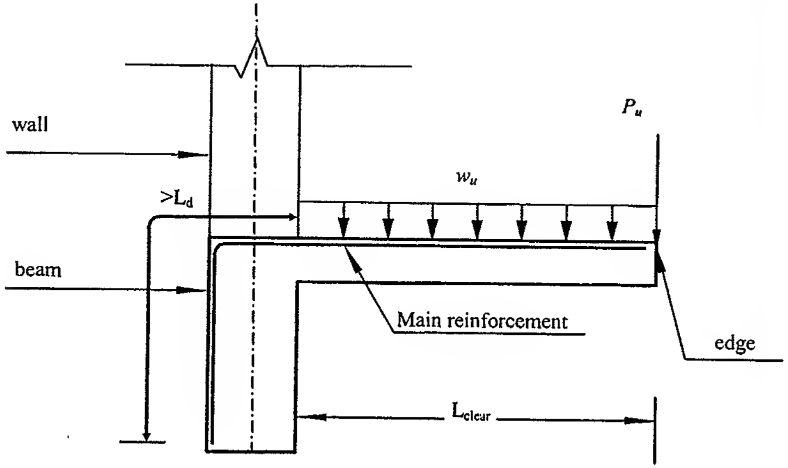


Fig. 5.6 Cross section in a cantilever stair

Live loads (3-4 kN/m^2) are applied in form of uniform loads on the stairs in the horizontal direction and a concentrated line load (P_u=1-1.5 kN) on the free edge as shown in Fig. 5.6. The uniform loads w_u is the summation of the (slab+stair) weight(using t_{avg}), covering material (0.8→1 kN/m^2), and live loads. The effective span L_{eff} is taken as

$$L_{eff} = \min \begin{cases} L_{clear} + t_{avg} \\ \text{edge to CL} \end{cases} \quad \text{.......... (5.2)}$$

$$t^* = \frac{t}{\cos\theta} \quad \text{.......... (5.3)}$$

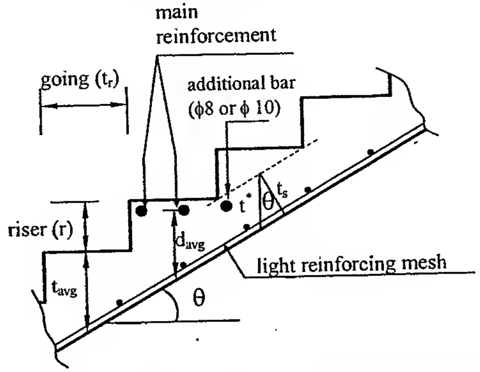
$$t_{avg} = t^* + \frac{riser(r)}{2} \quad \text{.......... (5.4)}$$

$$\text{Stair total self weight} = t_{avg} \times \gamma_c \quad \text{.......... (5.5)}$$

$$w_u = 1.4\, w_{DL} + 1.6\, w_{LL} \quad \text{.......... (5.6)}$$

$$w_u = 1.4\, [t_{avg} \times \gamma_c + \text{covering material}(0.8 \rightarrow 1)] + 1.6\, w_{LL}(3 \rightarrow 4) \quad \text{.......... (5.7)}$$

The main reinforcement should securely anchored in the supporting beam with a minimum distance equal to tension development length (L_d) as shown in Fig. 5.6.



$$t^* = \frac{t}{\cos\theta}$$

$$t_{avg} = t^* + \frac{riser(r)}{2}$$

Fig. 5.7 Reinforcement placement in a cantilever stair

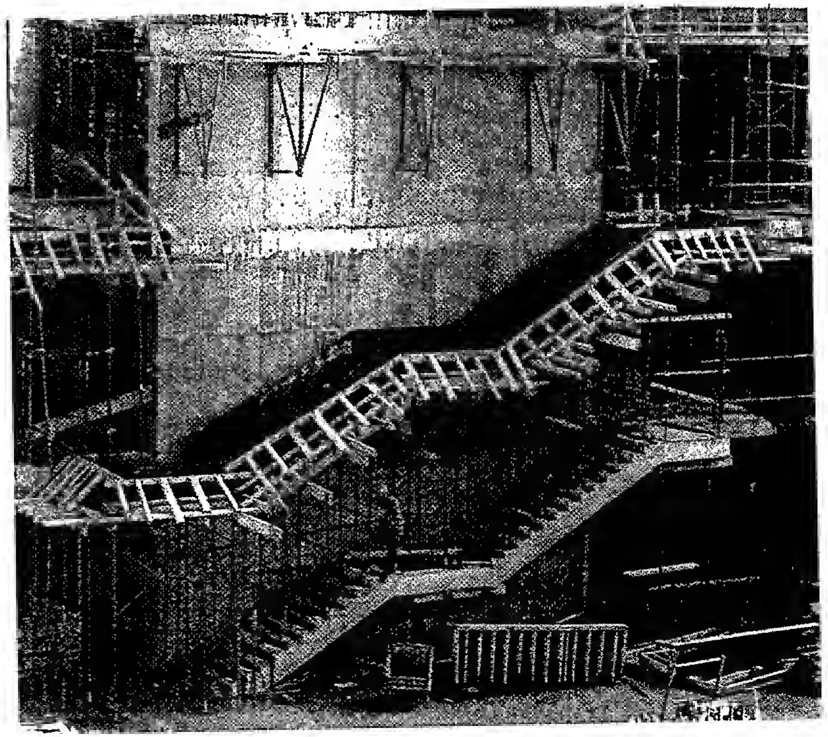
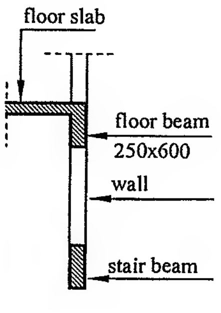


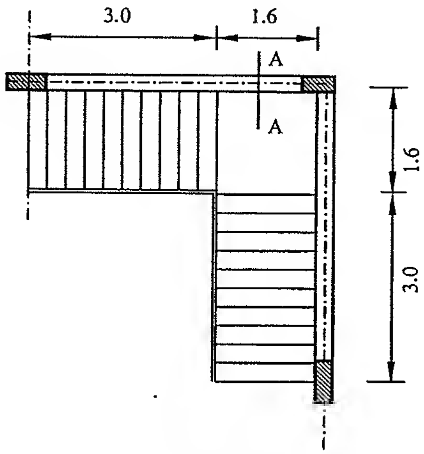
Photo 5.3 Cantilever stair during construction

Example 5.1

The staircase shown in figure below is a cantilever type for a floor height of 3.0 m. The live load is 3 kN/m^2, the characteristic strength of concrete is f_{cu} =35 N/mm^2 and f_y=400 N/mm^2. The weight of the covering material is 0.8 kN/m^2. Design and draw reinforcement details for the staircase and the supporting beams.



Sec A-A



Architecture Plan

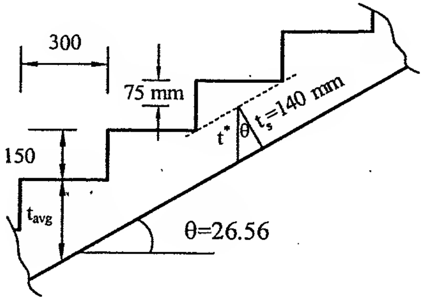
Solution

Step 1: Design of the flight

Step 1.1: Load calculations

Assume the riser height is 150 mm and the going width is 300 mm, the slope of the stair equals

$$\theta = \tan^{-1}\left(\frac{riser}{going}\right) = \tan^{-1}\left(\frac{150}{300}\right) = 26.56^{o}$$



Assume t_s=140 mm

To calculate the loads on the horizontal projection, use the vertical thickness t^*

$$t^* = \frac{t_s}{\cos(\theta)} = \frac{140}{\cos(26.56)} = 156.5\ mm$$

To include the weight of the stairs (goings and risers) in the dead load calculation (in the horizontal direction), one has to calculate the average vertical thickness

$$t_{avg} = t^* + \frac{riser}{2} = 156.5 + \frac{150}{2} = 231.5\ mm$$

Stair total self weight =$t_{avg} \times \gamma_c$ = 0.2315 x 25=5.7875 kN/m^2

w_u= 1.4 w_{DL} + 1.6 w_{LL}=1.4 (self weight +covering material)+ 1.6 w_{LL}

w_u=1.4 × (5.78 + 0.8) + 1.6 × 3 = 14.02 kN/m^2

Taking a one-meter strip then w_u= 14.02 kN/m′

An additional concentrated live load (P_{edge})of 1.5 kN/m′ is assumed at the free edge.

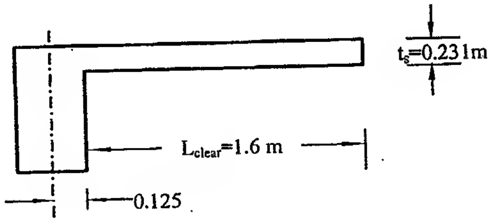
$$P_u = 1.6P_{edge} = 1.6\times1.5 = 2.4\ kN/m'$$

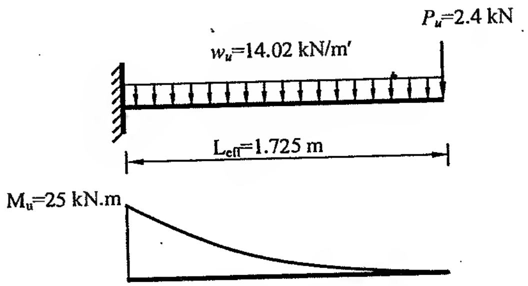
Step 1.2 Bending moments

The effective span is given by

$$L_{eff} = \min\begin{cases} L_{clear} + t_{avg} \\ \text{edge to CL}\end{cases} = \min\begin{cases}1.6+0.231=1.831\ m \\ 1.6+0.125=1.725\ \ m\end{cases}$$

L_{eff}= 1.725 m





The maximum bending moment on a 1.0 m strip of the stairs is

$$M_u = \frac{w_u \times L_{eff}^2}{2} + P_u \times L_{eff} = \frac{14.02 \times 1.725^2}{2} + 2.4 \times 1.725 = 25 \ kN.m$$

Step 1.3: Design of stairs

Assuming 20 mm concrete cover, the effective depth equals

$d = t_{avg}$ - cover = 231.5 – 20 = 211.5 mm

Recalling that we base the design on a strip of 1.0 m (b=1000 mm)

$$R = \frac{M_u}{f_{cu}\ b\ d^2} = \frac{25 \times 10^6}{35 \times 1000 \times 211.5^2} = 0.016$$

Using the R-ω curve →→→ ω=0.019

$$A_s = \omega \frac{f_{cu}}{f_y} b\ d = 0.019 \times \frac{35}{400} \times 1000 \times 211.5 = 346\ mm^2$$

$$A_{s,\min} = \frac{0.6}{f_y} b d = \frac{0.6}{400} \times 1000 \times 211.5 = 317\ mm^2 < As$$

A_s/step = A_s × step width(going)=346 × 0.3 =104 mm^2

Choose 2 Φ 10 (157 mm^2)/step

Step 2: Design of landing

Step 2.1: Load calculations

The thickness of the landing is taken equal to the thickness of the flight =140 mm

Landing self-weight = $t_s \times \gamma_c$ = 0.14 × 25=3.5 kN/m^2

w_{ul}= 1.4 w_{DL} + 1.6 w_{LL}=1.4 (self weight +covering material)+ 1.6 w_{LL}

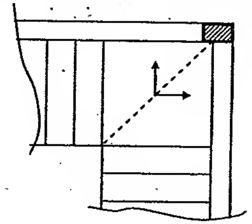
w_{ul}=1.4 × (3.5+0.8) + 1.6 × 3 = 10.82 kN/m^2

Taking a strip of one meter

w_{ul}= 10.82 kN/m′

The landing is a slab that is supported on two sides. The exact analysis of such slabs involves a lengthy calculation procedure. However, since this slab is supported on two beams, the amount transferred in each direction can be approximated by dividing the load between them. Thus the load transferred to each direction equals

w_{ul}= 10.82 /2 =5.41 kN/m′



Step 2.2 Design of sections

$$M_u = \frac{w_{ul} \times L_{eff}^2}{2} = \frac{5.41 \times 1.725^2}{2} = 8.05\,kN.m$$

Assuming 20 mm concrete cover, the effective depth equals

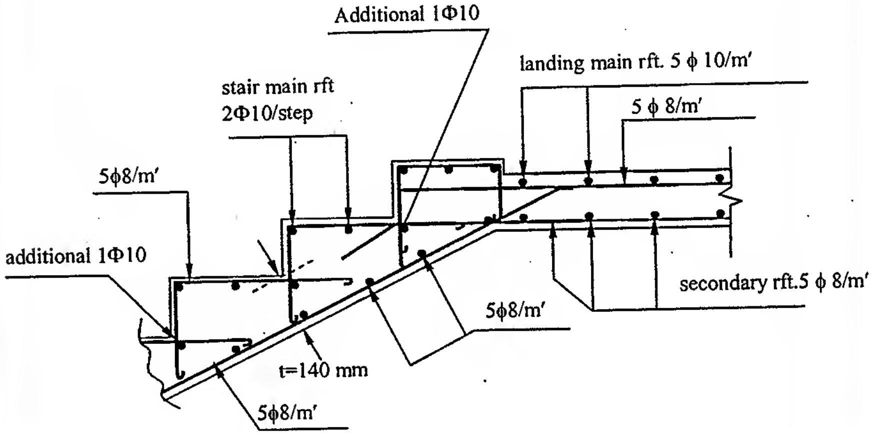
d = t - cover = 140 – 20 = 120 mm

$$R = \frac{M_u}{f_{cu}\ b\ d^2} = \frac{8.05 \times 10^6}{35 \times 1000 \times 120^2} = 0.0159$$

Using the R-ω curve→→ ω=0.019

$$A_s = \omega \frac{f_{cu}}{f_y} b\ d = 0.019 \times \frac{35}{400} \times 1000 \times 120 = 196\ mm^2$$

Choose 5 Φ 10/m′ (393 mm^2) >$A_{s,min}$



Stair reinforcement details

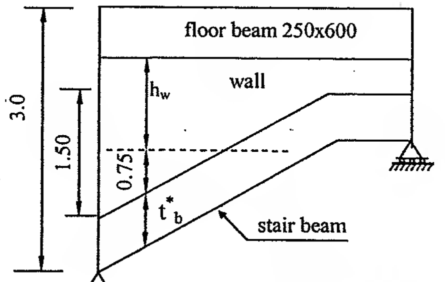
Step 3: Design the supporting beam

Step 3.1: Calculation of loads

Assume the beam cross section is 250 x 800 mm

To obtain the weight in the horizontal projection, calculate the vertical thickness t*

$$t_b^* = \frac{t}{\cos(\theta)} = \frac{800}{\cos(26.56)} = 894.4\ mm$$



Calculation of wall load

Self weight = $1.4 \times \gamma_c \times b \times t^*_b$

Self weight=$1.4 \times 25 \times 0.25 \times 0.8944 = 7.83$ kN/m′ (Horizontal projection)

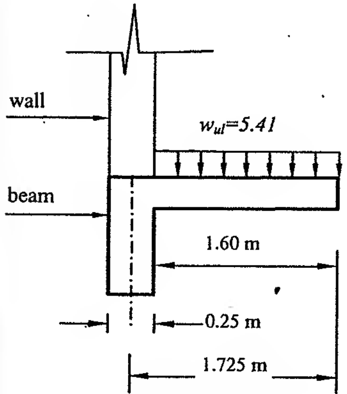
The height of the wall is variable along the beam, however for simplicity an average height will be used throughout the beam. This average height is conservatively calculated as follows:

h_w= 3-floor beam-t^*_b-1.5/2=3.0- 0.6 - 0.89 - 0.75 = 0.755 m

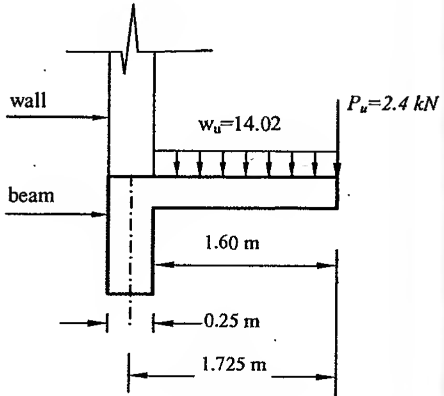
wall load = $1.4 \times \gamma_w \times b \times h_w$=1.4 × 1.2 × 0.25 x 0.755=3.17 kN/m′

w_{ub1}= self weight + w_u × flight width + wall load + edge live load

$$w_{ub1} = 7.83 + 14.02 \times 1.6 + 3.17 + 2.4 = 35.84\ kN/m'$$



Loads on landing



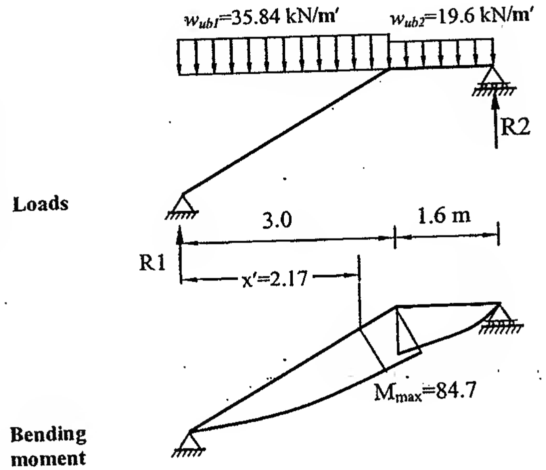
Loads on flight

The loads transmitted to the beam at the level of the landing w_{ub2} equals to

w_{ub2}= self weight + w_{ul} × landing width + wall load

$$w_{ub2} = 7.83 + 5.41 \times 1.6 + 3.17 = 19.6\ kN/m'$$

Note: The reader should observe that the average wall load previously calculated is conservatively used for the whole span.



The reaction R1 equals

$$R1 = \frac{35.84 \times 3 \times (1.5 + 1.6) + 19.6 \times 1.6 \times 0.8}{4.6} = 77.9\ kN$$

$$R2 = \frac{35.84 \times 3 \times 1.5 + 19.6 \times 1.6 \times (3 + 0.8)}{4.6} = 60.96\ kN$$

Step 3.2: Design for flexure

The maximum moment occurs at point of zero shear (x′)

$$x' = \frac{R1}{w_{ub1}} = \frac{77.9}{35.8} = 2.17\ m$$

$$M_{max} = 77.9 \times 2.17 - 35.84 \times \frac{2.17^2}{2} = 84.7\ kN.m$$

Note: Since the beam is inclined with respect to the reaction, R1 will produce shear and normal force on the beam.

This section has positive bending (84.6 kN.m), but we shall neglect the contribution of the stairs and design the beam as rectangular section

$$R = \frac{M_u}{f_{cu} b d^2} = \frac{84.7 \times 10^6}{35 \times 250 \times 750^2} = 0.017$$

Using R-ω curve ⟶ ω=0.02

$$A_s = \omega \frac{f_{cu}}{f_y} b \times d = 0.02 \frac{35}{400} \times 250 \times 750 = 330 \ mm^2$$

$$A_{s\,\min} = smaller \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b \ d = \dfrac{0.225\sqrt{35}}{400} \times 250 \times 750 = 623 \ mm^2 \\ 1.3 A_s = 1.3 \times 330 = 430 \, mm^2 \end{cases}$$

$A_s < A_{s,min}$, use $A_s = A_{s,min} = 430$ mm^2

use (3 Φ 16, 603 mm^2)

Step 3.3: Design the beam for shear and torsion

Step 3.3.1: Shear stresses

The critical section for shear and torsion is at d/2 from the face of the support. Assuming that the column width is 600 mm, the vertical force at the critical section equals

$$Q = 77.9 - 35.8 \times \left(\frac{0.6}{2} + \frac{0.75}{2} \right) = 53.7 \ kN$$

This vertical force produces shear and normal force because the beam is inclined. The normal force is normally small and neglected (<0.04 f_{cu} b t)and the shear force equals

$$Q_u = Q \times \cos(\theta) = 53.7 \times \cos(26.6) = 48.1 \ kN$$

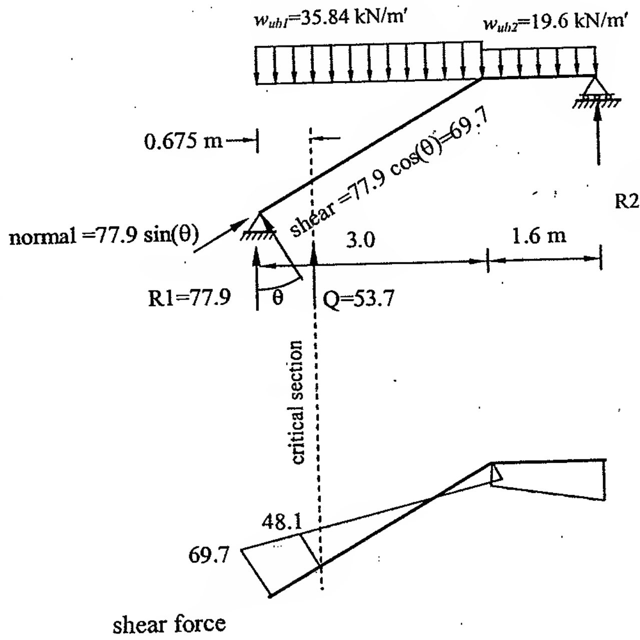
$$q_u = \frac{Q_u}{b \times d} = \frac{48.1 \times 1000}{250 \times 750} = 0.256 \ N/mm^2$$

Step 3.3.2: Shear reinforcement

The concrete shear strength q_{cu} equals

$$q_{cu} = 0.24\sqrt{\frac{35}{1.5}} = 1.16 \ N/mm^2$$

Since the applied shear stress (0.256) is less than q_{cu} (1.16), thus shear reinforcement is not needed.



Loading for Shear and shear force diagram

Step 3.3.3: Torsion stresses

Assume the distance from the concrete cover to the stirrup center line is 40 mm

x_1 = 250 –2 x 40 = 170 mm

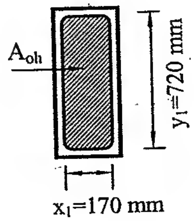
y_1 = 800 –2 x 40 = 720 mm

$$p_h = 2 \times (x_1 + y_1) = 2 \times (170 + 720) = 1780 \ mm$$

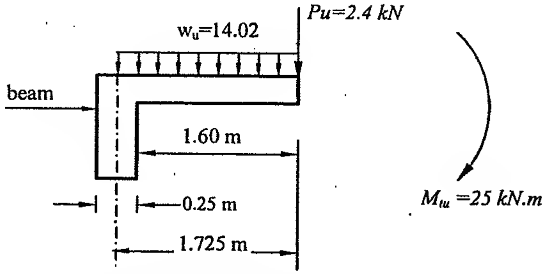
$$A_{oh} = x_1 . y_1 = 170 \times 720 = 122400 \ mm^2$$

$$A_o = 0.85 A_{oh} = 0.85 \times 122400 = 104040 \ mm^2$$

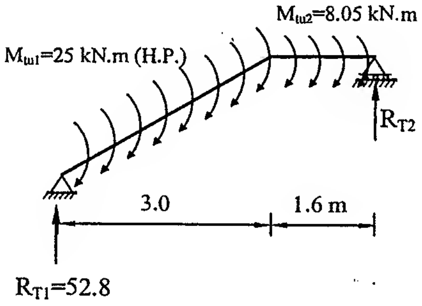
$$t_e = \frac{A_{oh}}{p_h} = \frac{122400}{1780} = 68.76 \ mm$$



The torsional moment (M_{tu}) applied to the beam equals the bending moment developed in the stairs (=25 kN.m) and equals 8.05 kN.m at the landing level.



The torsional moment is distributed along the length of the beam as shown in figure. All these moments are horizontal projection and in the vertical plane.



The torsional reaction R_{T1} equals

$$R_{T1} = \frac{25\times3\times(1.5+1.6)+8.05\times1.6\times0.8}{4.6} = 52.8\ kN$$

The critical section for torsion is at d/2 from the face of the support, thus the vertical torsional moment at the critical section equals to:

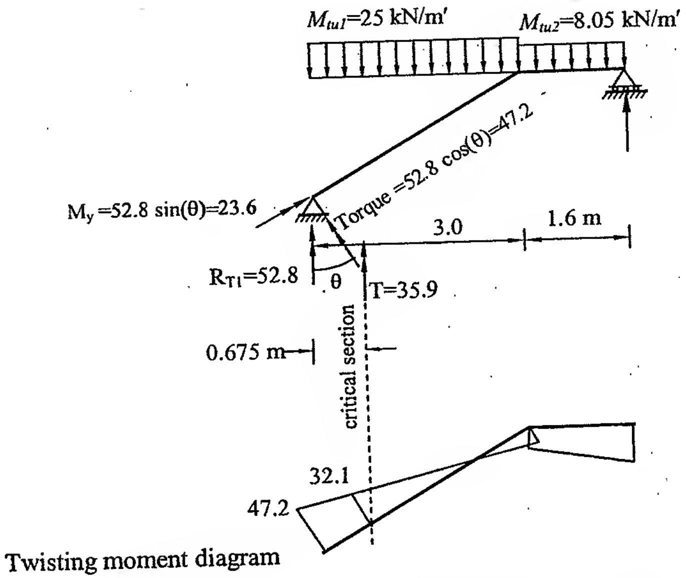
$$T\ = 52.8 - 25\times\left(\frac{0.6}{2}+\frac{0.75}{2}\right) = 35.9\ \ kN.m$$

This vertical torsional moment produces torque and out-of-plane bending moment (M_y) because the beam is inclined. The out-of-plane bending is usually small and can be neglected and the torque equals to:

$$M_{tu} = T\ \times\cos(\theta) = 35.9\times\cos(26.6) = 32.1\ kN$$

The shear stress due to the torque M_{tu} is given by:

$$q_{tu} = \frac{M_{tu}}{2\times A_o\times t_e} = \frac{32.1\times10^6}{2\times104040\times68.76} = 2.24\ N/mm^2$$



Loading for torsion and twisting moment diagram

Step 3.3.4: Check adequacy of the concrete dimensions

$$q_{\max} = 0.70\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.70\times\sqrt{\frac{35}{1.5}} = 3.38 < 4.0\ \ N/mm^2$$

$q_{,\max}$= 3.38 N/mm^2

$$\sqrt{q_u^2 + q_{tu}^2} \le q_{\max}$$

$$\sqrt{0.256^2 + 2.245^2} = 2.26 < 3.38......o.k$$

Thus, the concrete dimensions of the section are acceptable for shear and torsion.

Step 3.3.5: Torsional reinforcement

$$q_{tu,\min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.06\sqrt{\frac{35}{1.5}} = 0.29\ N/mm^2$$

Since $q_{tu} > q_{tumin}$, torsional reinforcement is required. According to the ECP-203, the torsional concrete strength is neglected, and the torsional stresses must be carried by reinforcement

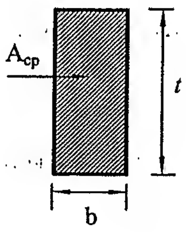
Assuming spacing of 100 mm, the area of one branch A_{str} equals

$$A_{str} = \frac{M_{tu} \times s}{2 \times A_o \times f_{yst}/\gamma_s} = \frac{32.1 \times 10^6 \times 100}{2 \times 104040 \times 240/1.15} = 73.97\ mm^2$$

$$A_{sl} = \frac{A_{str} \times p_h}{s}\left(\frac{f_{yst}}{f_y}\right) = \frac{73.97 \times 1780}{100}\left(\frac{240}{400}\right) = 790\ mm^2$$

Calculate the minimum area for longitudinal reinforcement $A_{sl,min}$

$$A_{sl,\min} = \frac{0.4\sqrt{\frac{f_{cu}}{\gamma_c}}A_{cp}}{f_y/\gamma_c} - \frac{A_{str} \times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)$$



There is a condition on this equation that $\frac{A_{str}}{s} \geq \frac{b}{6 \times f_{yst}}$

$$\frac{73.97}{100} \geq \frac{250}{6 \times 240} \ldots o.k$$

$$A_{sl,\min} = \frac{0.4\sqrt{\frac{35}{1.5}} \times 250 \times 800}{400/1.15} - \frac{73.97 \times 1780}{100}\left(\frac{240}{400}\right) = 320\ mm^2$$

Since $A_{sl} > A_{sl,min}$ ...o.k

Choose 8ϕ12 (904 mm^2)

Step 3.3.6 Reinforcement for combined shear and torsion

Since it was previously computed that no shear reinforcement is needed (A_{st}=0), the area of one branch for combined shear and torsion equals

$A_{str} + A_{st}/2 = 73.97 + 0 = 73.97$ mm^2

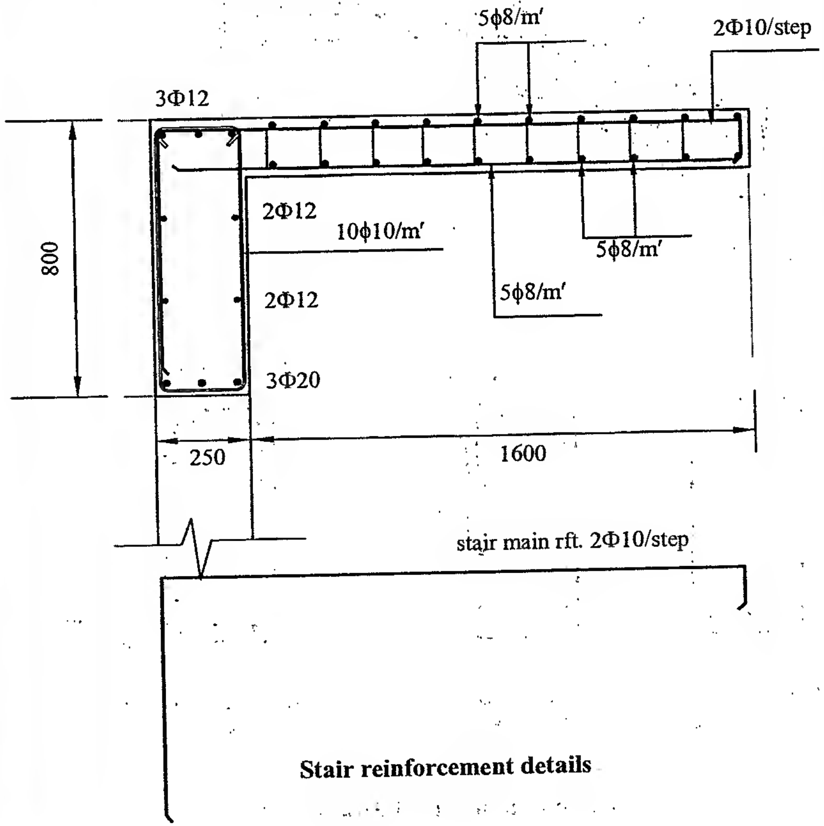
Choose ϕ 10 mm (78.5 mm^2)

$$A_{st,\min} = \frac{0.4}{f_y} b \times s = \frac{0.4}{240} 250 \times 100 = 41.67\ mm^2$$

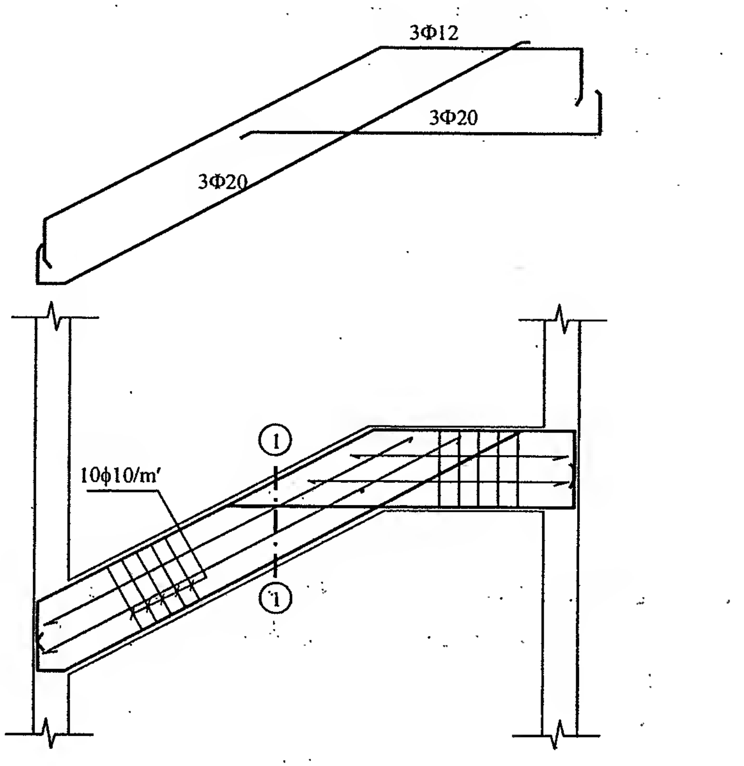
$$Total\ area = 2A_{str} + A_{st} \geq A_{st,\min} \cdots\cdots\cdots\cdots$$

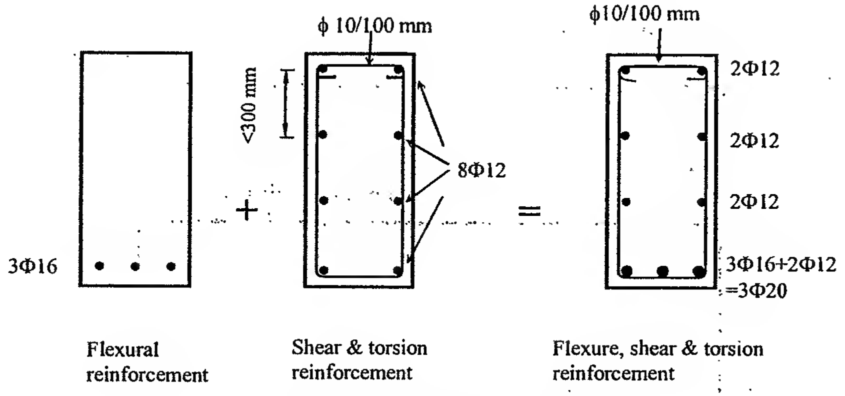
$$= 2 \times 73.97 + 0 = 147.94\ mm^2 > 41.67\ mm^2 \ \ldots\ldots o.k.$$

Final design use ϕ 10 @ 100 mm



Stair reinforcement details

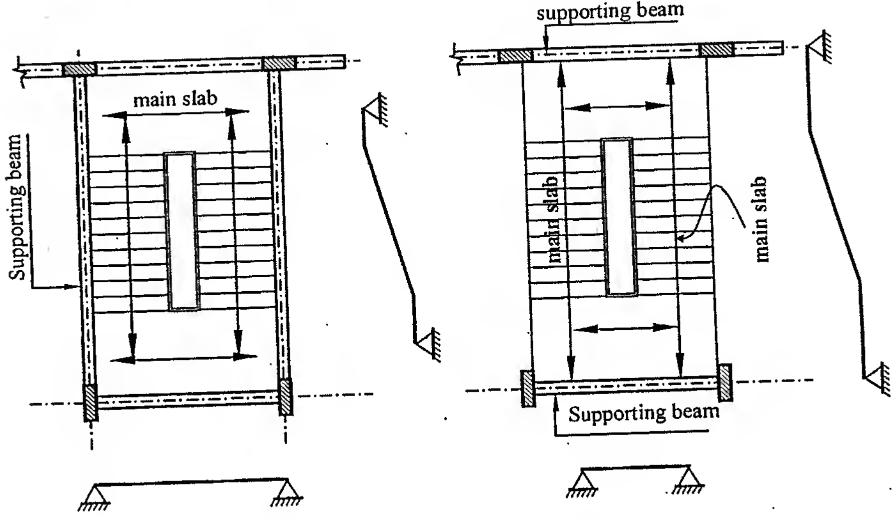




Beam reinforcement details

5.3 Slab Type

In this type, the main supporting element is the slab itself. The flight could be supported on the landing, which is in turn supported on the supporting beams. From the structural point of view, it is better that the main supporting element is spanning in the short direction. However, this depends on the surrounding beams. If the beams exist around the perimeter of the stair well or at least along the long sides, solution A in Fig. 5.8 is more economical. If the supporting beams are only at the short side, solution B is the only valid structural system.



Solution A

Solution B

Fig. 5.8 Proposed structural systems for a slab-type staircase

The applied live loads are based on the plan area (horizontal projection), while the dead load is based on the sloped length. To transform the dead load into horizontal projection the average vertical thickness is used t_{avg} (Eq. 5.11) as shown in Fig. 5.9. However, the depth used in design (*d*) is perpendicular to the slope (Eq. 5.9) as shown in Fig. 5.9. Slab thickness t_s is taken as (1/25 to 1/30 from the slab span).

$$t_s = \frac{span}{25-30} \quad \text{.......} \quad (5.8)$$

$$d = t_s - \text{cover} \quad \text{.......} \quad (5.9)$$

$$t^* = \frac{t}{\cos\theta} \quad \text{.......} \quad (5.10)$$

$$t_{avg} = t^* + \frac{riser(r)}{2} \quad \text{.......} \quad (5.11)$$

$$\text{Stair total self weight} = t_{avg} \times \gamma_c \quad \text{.......} \quad (5.12)$$

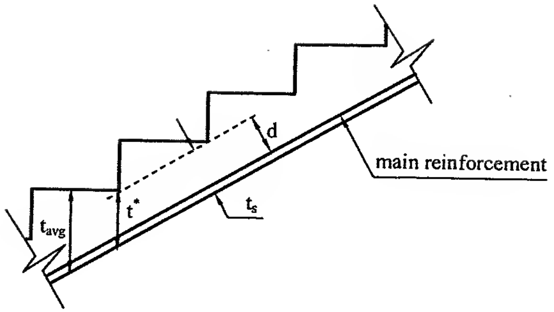


Fig. 5.9 Effective depth and reinforcement in a slab type staircase

The uniform loads w_u is the summation of the (stair +slab) weight (using t_{avg}), covering material (0.8→1 kN/m^2), and live loads. The effective span L_{eff} is taken as

$$w_u = 1.4\, w_{DL} + 1.6\, w_{LL} \quad (5.13)$$

$$w_u = 1.4\, [t_{avg} \times \gamma_c + \text{covering material}(0.8 \rightarrow 1)] + 1.6\, w_{LL}(3 \rightarrow 4) \quad (5.14)$$

If the slab thickness exceeds 160 mm, a top reinforcement mesh must be supplied to control shrinkage and temperature. However, this reinforcement is only required at the landing level as the stair reinforcement acts as shrinkage reinforcement as shown in Fig. 5.10.

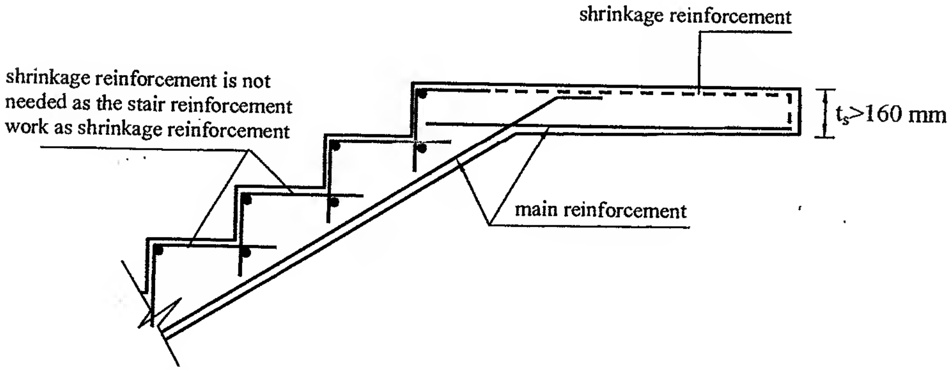


Fig. 5.10 Shrinkage reinforcement for slab greater than 160 mm

Since the landing and the stairs are not straight, internal forces are generated in these sloped elements. The two tensile forces T_1 and T_2 generated at the kink, producing a third outward force F as shown in Fig. 5.11. This force tends to cause splitting cracks if the produced stresses exceed concrete tensile strength. Thus, tension reinforcement should be extended from each side so that no outward force is generated.

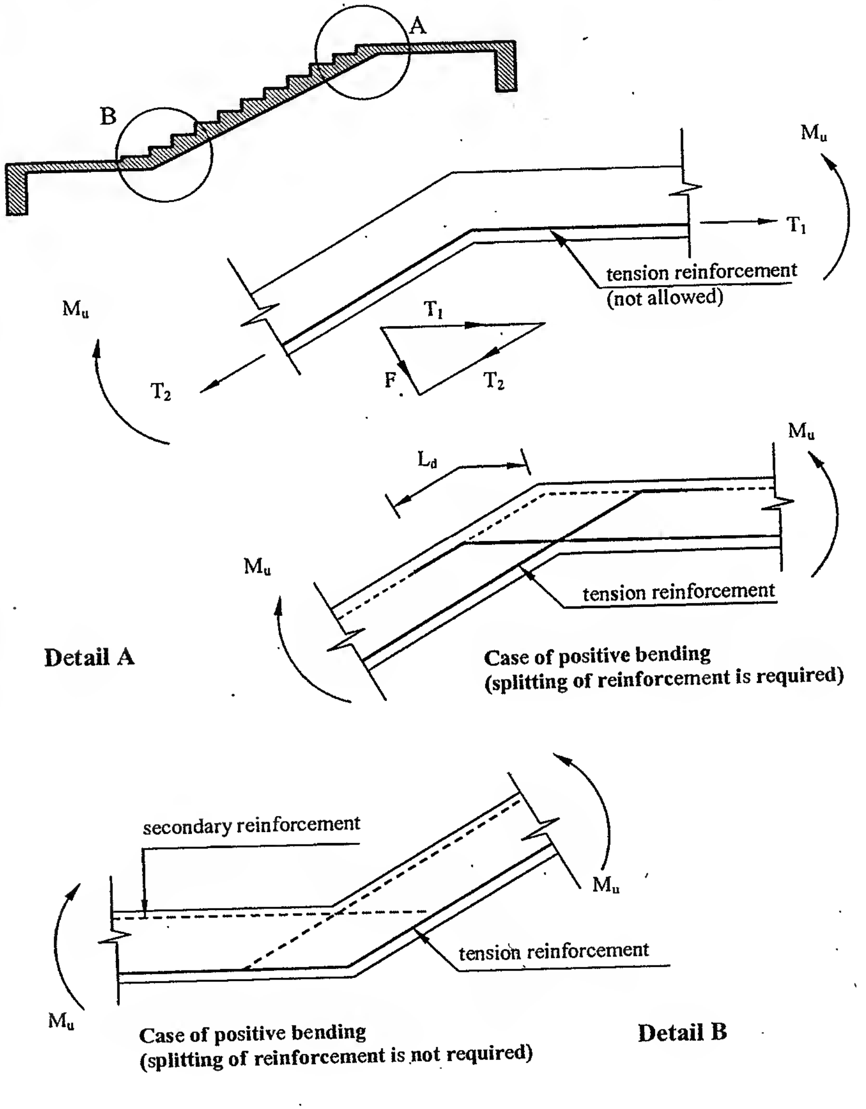


Fig. 5.11 Internal forces developed in the landing and the sloped slab

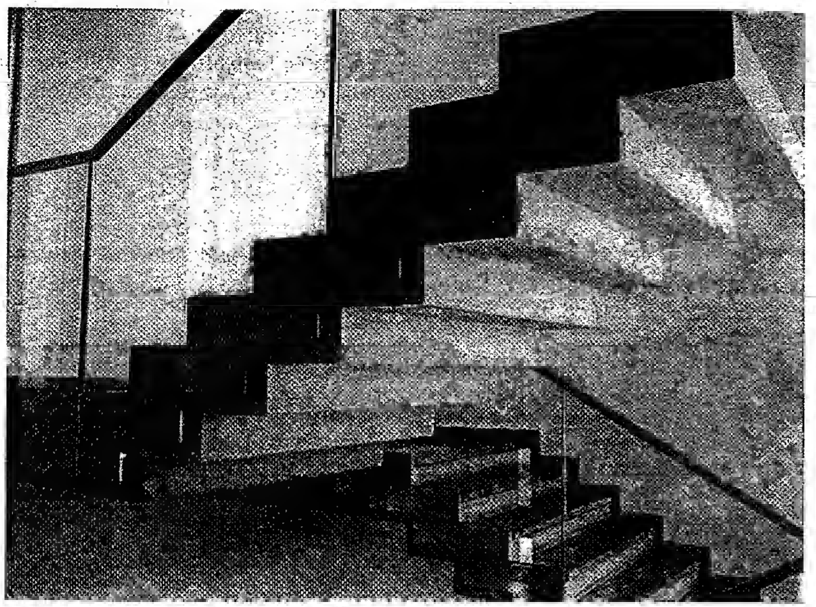


Photo 5.4 Slab type staircase

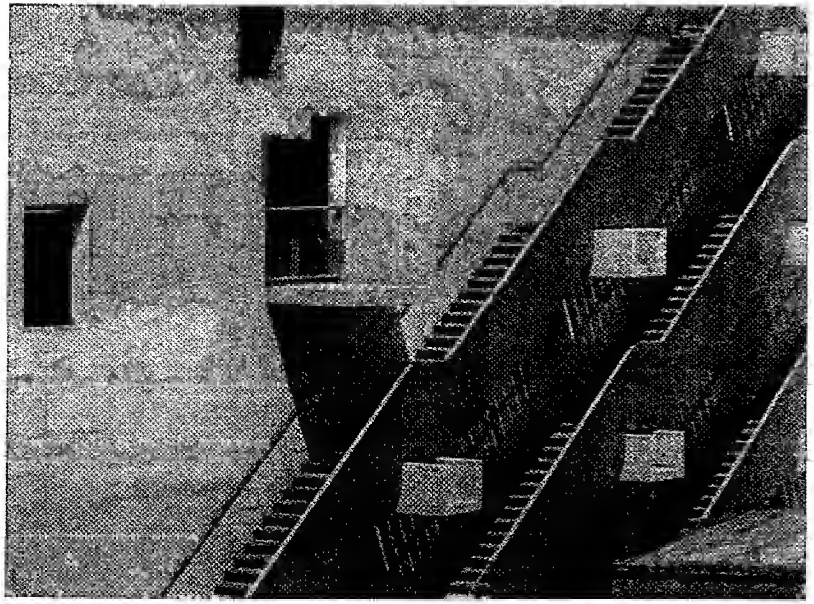
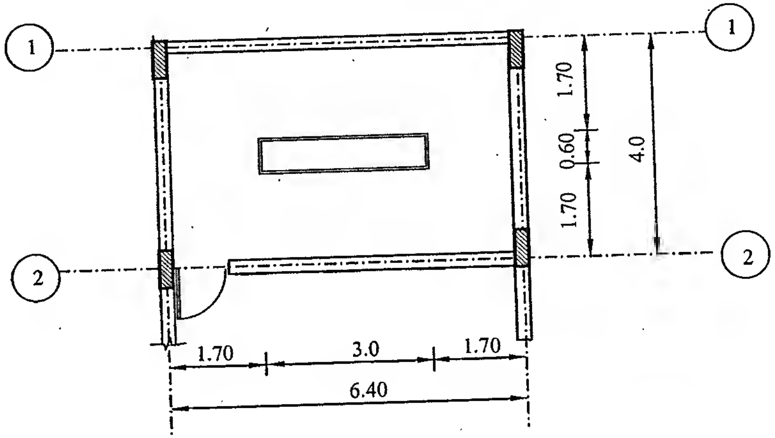


Photo 5.5 Cantilever staircase

Example 5.2

Design the staircase shown in figure below as slab-type. The live load is 4 kN/m^2, the characteristic strength of concrete is f_{cu} =30 N/mm^2 and steel yield stress is f_y=280 N/mm^2. The weight of the covering material is 0.75 kN/m^2 for the stairs and 1.8 kN/m^2 for landing. Floor height=3.3 m and γ_w is 12 kN/m^3



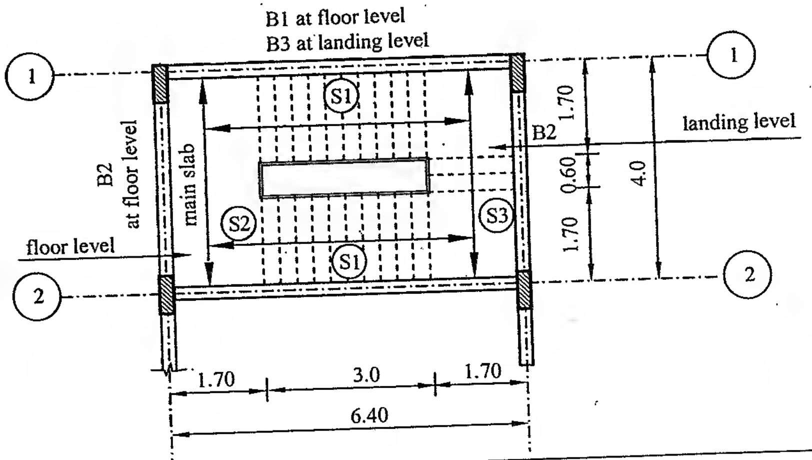
Solution

Step1: Staircase layout and loads

Assume the riser height is 150 mm

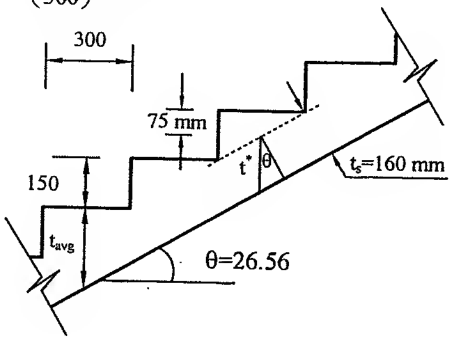
$$\text{Number of goings} = \frac{3.3\times1000}{150} = 22\ going,$$

Using three flights with tread width of 300 mm as shown in figure



The slope of the stair equals

$$\theta = \tan^{-1}\left(\frac{riser}{going}\right) = \tan^{-1}\left(\frac{150}{300}\right) = 26.56^{o}$$



Assume the slab thickness $t_s = \frac{span}{30} = \frac{4700}{30} = 156.7 \approx 160\ mm$

$$t^{*} = \frac{t_s}{\cos(\theta)} = \frac{160}{\cos(26.56)} = 178.9\ mm$$

$$t_{avg} = t^{*} + \frac{riser}{2} = 178.9 + \frac{150}{2} = 253.9\ mm$$

Stair self weight $= t_{avg} \times \gamma_c = 0.2539 \times 25 = 6.347\ kN/m^2$

Step 2: Design of Slab S1

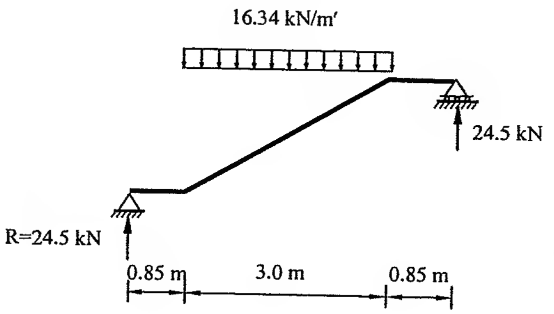
Step 2.1: Calculation of loads

$w_u = 1.4\ w_{DL} + 1.6\ w_{LL} = 1.4$ (self weight +covering material)+ $1.6\ w_{LL}$

$w_u = 1.4 \times (6.347+0.75) + 1.6 \times 4 = 16.34\ kN/m^2$

Taking one meter then $w_u = 16.34\ kN/m'$

Slab S1 is supported at the centerline of slabs S2 and S3



The reaction at each end (R) equals

$$R = \frac{w_u \times 3}{2} = \frac{16.34 \times 3}{2} = 24.5\ kN$$

Step 2.2: Design for flexure

The maximum bending moment is at the middle and equals

$$M_u = R \times (0.85+1.5) - w_u \times \frac{1.5^2}{2} = 24.5 \times 2.35 - 16.34 \times \frac{1.5^2}{2} = 39.2\ kN.m$$

Assuming concrete cover of 30 mm for the secondary direction and 20mm for the main direction

d = 160 – 30 = 130 mm (secondary direction slab S2)

d = 160 – 20 = 140 mm (main direction slab S1)

$$R = \frac{M_u}{f_{cu}\ b\ d^2} = \frac{39.2 \times 10^6}{30 \times 1000 \times 130^2} = 0.0773 \longrightarrow \omega = 0.098$$

$$A_s = \omega \frac{f_{cu}}{f_y} b\ d = 0.098 \times \frac{30}{280} \times 1000 \times 130 = 1365\ mm^2$$

$$A_{s,min} = \frac{0.25}{100} b \times d = \frac{0.25}{100} 1000 \times 130 = 325\ mm^2 < A_s$$

Choose 6 ϕ 18/m' (1527 mm²)

Step 2.3 Design for shear

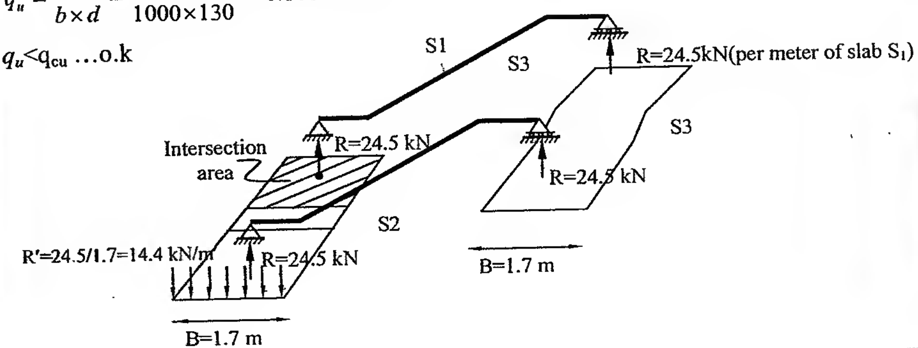
According to the ECP 203, concrete shear strength of slabs equals to:

$$q_{cu} = 0.16\sqrt{\frac{f_{cu}}{1.5}} = 0.16\sqrt{\frac{30}{1.5}} = 0.715\ N/mm^2$$

$Q_u = R = 24.5$ kN

$$q_u = \frac{Q_u}{b \times d} = \frac{24.5 \times 1000}{1000 \times 130} = 0.188\ N/mm^2$$

$q_u < q_{cu}$ ...o.k



Step 3: Design of Slab S2

Step 3.1: Calculation of loads

Slab S2 is supported on the beams located on axis 1,2 at the floor level. The reaction of the slab S1 is applied at the centerline of the slab S2. Since the width of S2 is 1.7 m, the reaction R will be distributed along this width. Thus the load per meter (R′) equals

$$R' = \frac{R_{S1(per\ meter)}}{B} = \frac{24.5}{1.7} = 14.4\ kN/m'$$

Or $$R' = \frac{R_{S1(total)}}{\text{Intersection area between } S_1 \text{ and } S_2} = \frac{24.5\times1.7}{1.7\times1.7} = 14.4\ kN/m'$$

Landing self weight $= t \times \gamma_c = 0.16 \times 25 = 4.0\ kN/m^2$

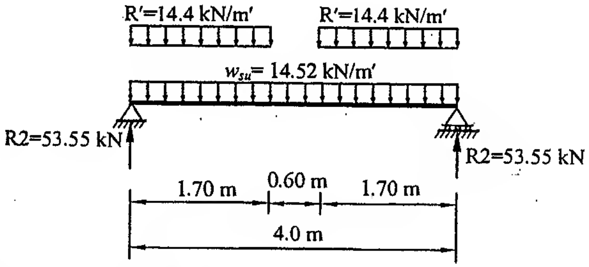
$w_u = 1.4\ w_{DL} + 1.6\ w_{LL} = 1.4$ (self weight +covering material)+ $1.6\ w_{LL}$

The weight of the covering material for the landing is given as $1.8\ kN/m^2$

$w_u = 1.4\times(4.0+1.80) + 1.6\times4 = 14.52\ kN/m^2$

Taking one meter width of the slab (b=1000 mm), then

$w_{su} = w_u \times 1 = 14.52 \times 1.0 = 14.52\ kN/m'$



Step 3.2 Design for flexure

$$R2 = \frac{w_{su}\times4}{2} + R\times1.7 = \frac{14.52\times4}{2} + 14.4\times1.7 = 53.54\ kN$$

The maximum moment is at mid span

$$M_u = R2\times2 - \frac{w_{su}\times2^2}{2} - R'\times1.7\times(\frac{1.7}{2}+0.3)$$

$$M_u = 53.55\times2 - \frac{14.52\times2^2}{2} - 14.4\times1.7\times(\frac{1.7}{2}+0.3) = 49.9\ kN.m$$

d = 160 – 20 = 140 mm →(main direction)

$$R = \frac{M_u}{f_{cu}\ b\ d^2} = \frac{49.9\times10^6}{30\times1000\times140^2} = 0.085$$

Using the R-ω curve →→ ω=0.11

$$A_s = \omega\frac{f_{cu}}{f_y}b\ d = 0.11\times\frac{30}{280}\times1000\times140 = 1650\ mm^2$$

$$A_{s,min} = \frac{0.25}{100}b\times d = \frac{0.25}{100}1000\times140 = 350\ mm^2 < A_s$$

Choose 6 ϕ 20/m′ (1885 mm^2)

Step 3.3: Design for shear

According to the ECP 203 clause 4-2-2-1-6-d , concrete shear strength equals

$$q_{cu} = 0.16\sqrt{\frac{f_{cu}}{1.5}} = 0.16\sqrt{\frac{30}{1.5}} = 0.715\ N/mm^2$$

Q_u= R=53.55 kN

$$q_u = \frac{Q_u}{b\times d} = \frac{53.55\times1000}{1000\times140} = 0.38\ N/mm^2 < q_{cu}\ \ldots\text{o.k}$$

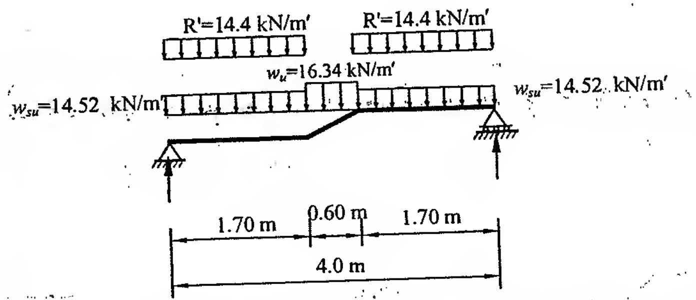
Step 4: Design of slab S3

Slab S3 is supported on the beams, the reaction of the slab S1 is applied at the middle of the slab

Landing self weight $= t_s \times \gamma_c = 0.16 \times 25 = 4.0\ kN/m^2$

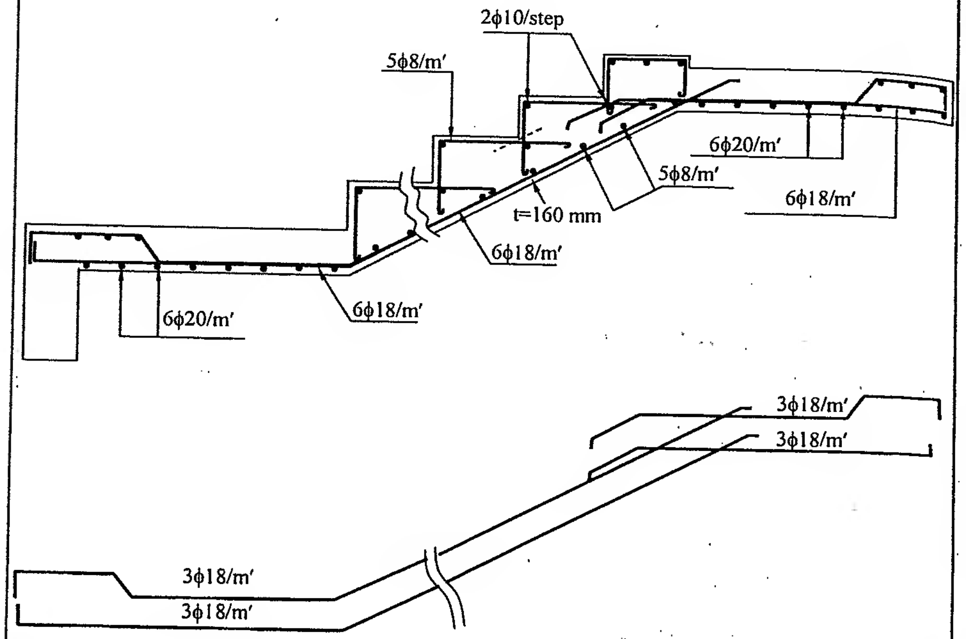
$w_u = 1.4\times(4.0+1.80) + 1.6\times4 = 14.52\ kN/m^2$

The weight of the stairs equals = 16.34 kN/m′ (same as slab S1)

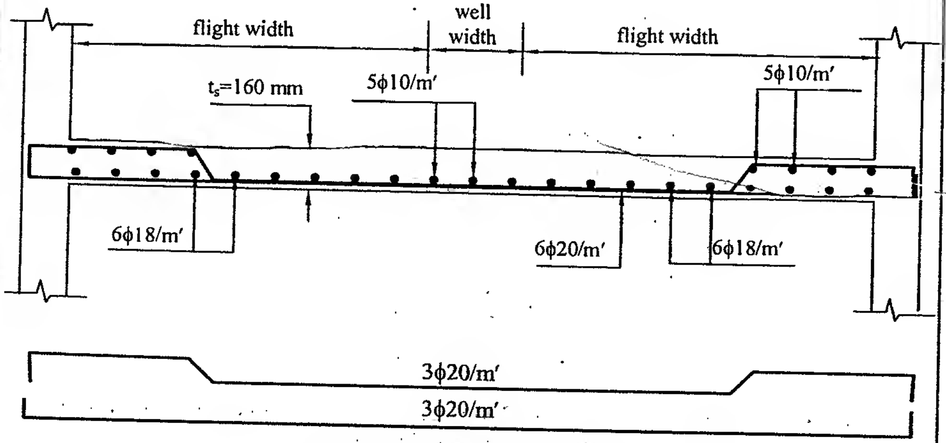


Comparing the loads acting on slab S2 with slab S3 reveals that they are almost the same, thus the same reinforcement used in slab S2 is used in slab S3

Choose 6 ϕ 20/m′ (1885 mm^2).



Reinforcement details for slab S1



Reinforcement details for slab S2

Step 5: Design of supporting beam B1 on axis 1

The supporting beam B1 exists at the floor level and beam B3 exists at the landing level as shown in the figure below. Beam B1 supports the slab S2 while B3 supports the slab S3.

Step 5.1: Calculation of loads

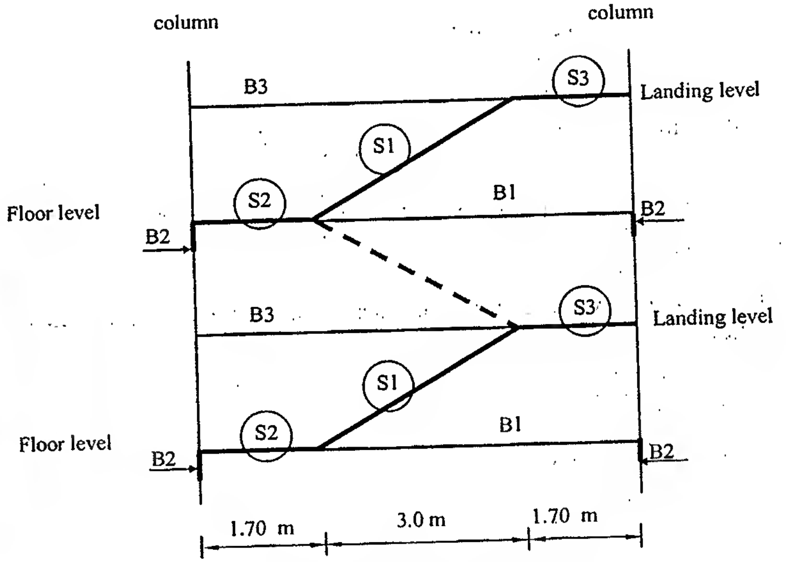
The beam at the landing level B1 carries its own weight, stair load and wall load. Assume beam size is 250x700 mm

Self weight of the beam = 1.4 × γ_c × b × t=1.4 × 25 × 0.25 × 0.7 =6.125 kn/m'

slab load equals to the reaction of the slab S2 (w_{su}) = 53.55 kN/m'

Wall load = 1.4 × γ_w × b × h_w = 1.4 × 12 × 0.25 × (3.3/2-0.7) = 4 kN/m'

w_{ub}=6.125+ 4.0 = 10.125 kN/m'

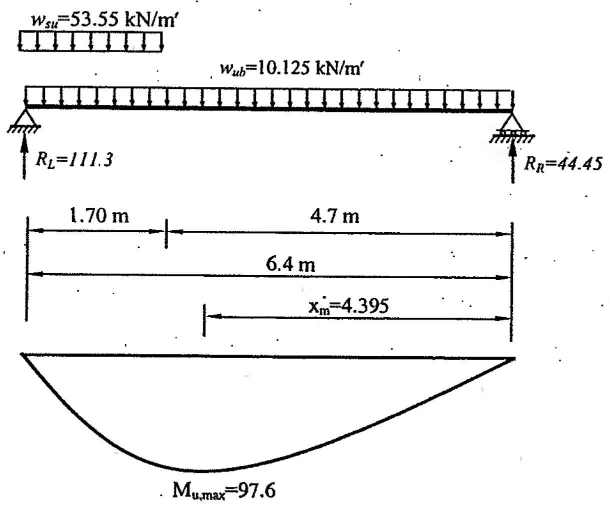


Elevation of beams on axis 1

Step 5.2: Reactions and bending moments

$$R_R = \frac{10.125\times\frac{6.4^2}{2}+53.55\times\frac{1.7^2}{2}}{6.4} = 44.45\ kN$$

$$R_L = 10.125\times6.4+53.55\times1.7-44.45 = 111.3\ kN$$



Point of zero shear from the right support $= x_m = \frac{44.45}{10.125} = 4.395\ m$

$$M_{u,max} = R_R \times x_m - w_{ub} \times \frac{x_m^2}{2} = 44.45 \times 4.395 - 10.125 \times \frac{4.395^2}{2} = 97.6\ kN.m$$

d = 700– 50 = 650 mm

Since some part of the beam is not connected to slab, it shall be designed as rectangular section

$$R = \frac{M_u}{f_{cu}\ b\ d^2} = \frac{97.6 \times 10^6}{30 \times 250 \times 650^2} = 0.031$$

From the R-ω curve ⟶ ω=0.037

$$A_s = \omega \frac{f_{cu}}{f_y} b\ d = 0.037 \times \frac{30}{280} \times 250 \times 650 = 644\ mm^2$$

$$A_{s,min} = \frac{0.225\sqrt{f_{cu}}}{f_y}\ b \times d = \frac{0.225\sqrt{30}}{280}\ 250 \times 650 = 715\ mm^2 < A_s \rightarrow \text{use } A_{smin}$$

Choose 4 ϕ 16 (804 mm^2)

Step 5.3 Design for shear

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{1.5}} = 0.24\sqrt{\frac{30}{1.5}} = 1.06\ N/mm^2$$

$Q_{u,max}$= R_R=111.3 kN

The critical section for shear is at d/2 from the face of the support. The column width (c) equals 0.25 m, thus

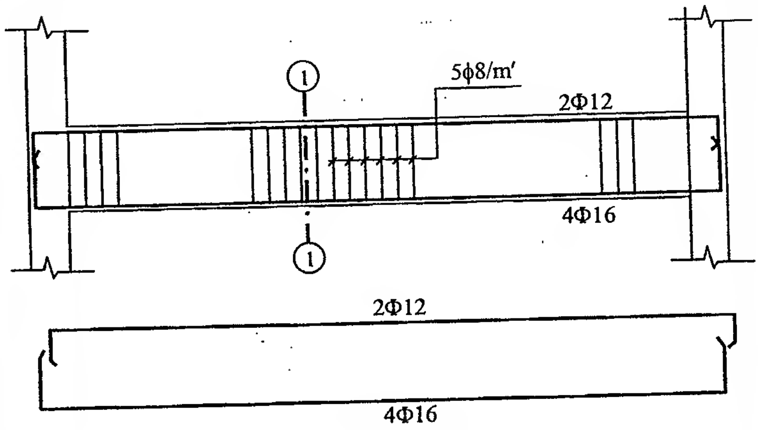
$$Q_u = R_R - (w_{su} + w_{ub}) \times (\frac{c}{2} + \frac{d}{2}) = 111.3 - (53.56 + 10.125) \times (\frac{0.25}{2} + \frac{0.65}{2}) = 82.62\ kN$$

$$q_u = \frac{Q_u}{b \times d} = \frac{82.62 \times 1000}{250 \times 650} = 0.508\ N/mm^2 < q_{cu} \ldots \text{o.k}$$

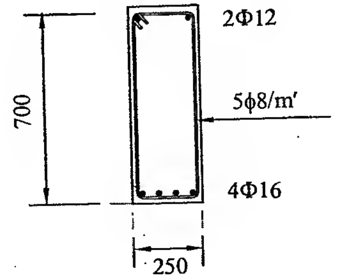
provide minimum shear reinforcement

$$A_{st,min} = \frac{0.4}{f_y} \times b \times s = \frac{0.4}{280} \times 250 \times 200 = 71.43\ mm^2\ for\ two\ branches$$

Choose 5 ϕ 8 /m'



Beam elevation

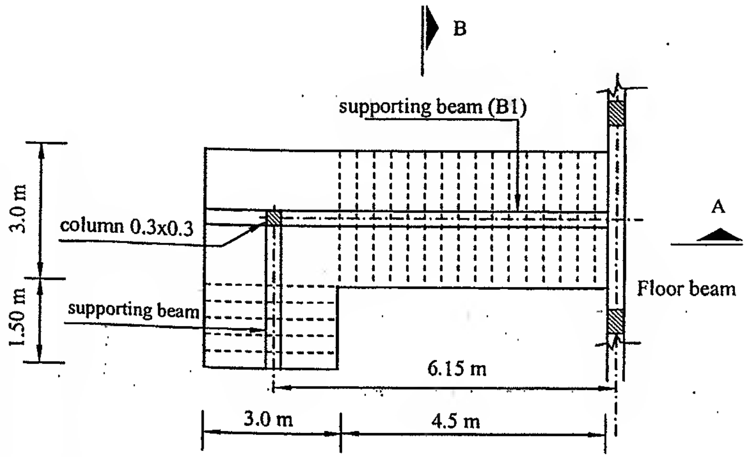


Sec 1-1

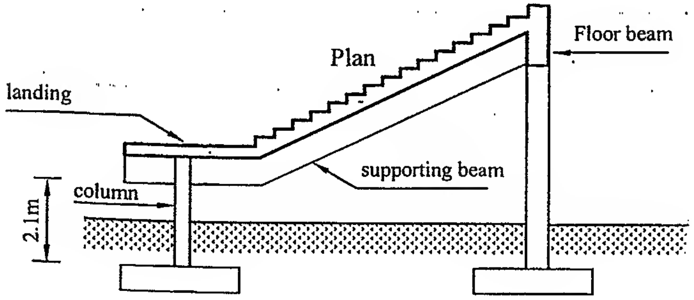
Beam reinforcement details

Example 5.3

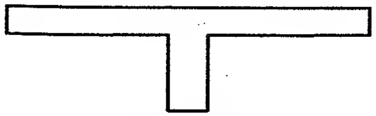
Design the staircase shown in figure knowing that
f_{cu} =25 N/mm^2, f_y=360 N/mm^2 and f_{yst}=280 N/mm^2
The weight of the covering material is 1.0 kN/m^2. The live loads on the stair may be taken as 4 kN/m^2



Plan



section A



section B

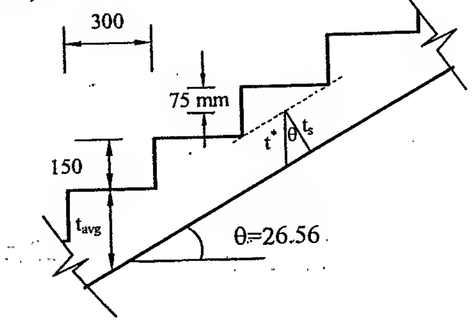
Solution

Step 1: Design of the flight

Step 1.1: Load calculations

Assume the riser height is 150 mm and the tread width is 300 mm, the slope of the stair equals

$$\theta = \tan^{-1}\left(\frac{riser}{tread}\right) = \tan^{-1}\left(\frac{150}{300}\right) = 26.56^o$$



Assume slab thickness t_s equals to 150 mm

$$t^* = \frac{t_s}{\cos(\theta)} = \frac{150}{\cos(26.56)} = 167.7\ mm$$

$$t_{avg} = t^* + \frac{riser}{2} = 167.7 + \frac{150}{2} = 242.7\ mm$$

Stair self weight =$t_{avg} \times \gamma_c$ = 0.2427 × 25=6.068 kN/m^2

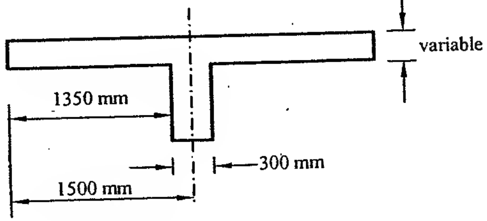
w_u= 1.4 w_{DL} + 1.6 w_{LL}=1.4 (self weight +covering material)+ 1.6 w_{LL}

w_u=1.4x (6.068+1) + 1.6 × 4 = 16.29 kN/m^2

Taking one meter then w_u= 16.29 kN/m'

Step 1.2: Flexure design

Assuming that the supporting beam width is 300 mm, then the effective span for the stairs is given by

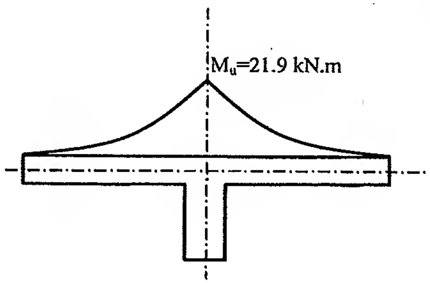


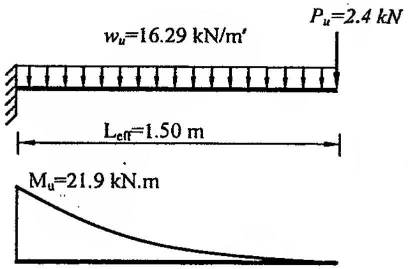
$$L_{eff} = \min \begin{cases} L_{clear} + t_{avg} \\ \text{edge to CL} \end{cases} = \min \begin{cases} (1.35 + 0.24 = 1.59 \ m \\ 1.5 \quad m \end{cases}$$

L_{eff}= 1.5 m

Assuming an edge live load of 1.5 kN/m′. P_u=1.6 x 1.5 = 2.4 kN/m′

The slab is assumed fixed in the beam and the bending moment in the slab equals





The structural system for the sla is cantilever slab form

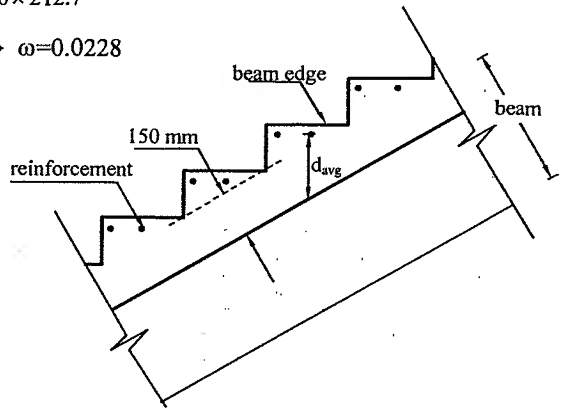
$$M_u = \frac{w_u \times L_{eff}^2}{2} + P_u \times L_{eff} = \frac{16.29 \times 1.50^2}{2} + 2.4 \times 1.50 = 21.9 \ kN.m$$

Thus, assuming 30 mm concrete cover, the effective depth is vertical distance in the slab thickness which is connected to the beam as shown in figure below

$$d_{avg} = t_{avg} - \text{cover} = 242.7 - 30 = 212.7 \ mm$$

$$R = \frac{M_u}{f_{cu} \ b \ d_{avg}{}^2} = \frac{21.9 \times 10^6}{25 \times 1000 \times 212.7^2} = 0.019$$

From the R-ω curve →→ ω=0.0228



$$A_s = \omega \frac{f_{cu}}{f_y} b \ d = 0.0228 \times \frac{25}{360} \times 1000 \times 212.7 = 337 \ mm^2$$

$$A_{s,\min} = \frac{0.6}{f_y} b \ d = \frac{0.6}{360} \times 1000 \times 212.7 = 355 \ mm^2 > \text{As} \longrightarrow \text{use } A_{s,\min}$$

$$A_s / step = 0.3 \times A_s = 0.3 \times 355 = 101 \ mm^2$$

Choose 2 Φ 10/step (157 mm^2)

Step 2: Design of the landing

The thickness of the landing is the same as the stairs =150 mm

Landing self weight = t x γ_c = 0.15 x 25=3.75 kN/m^2

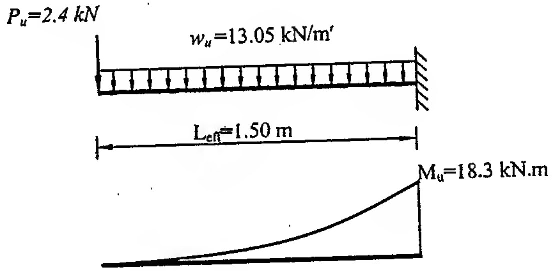
w_{ul}= 1.4 w_{DL} + 1.6 w_{LL}=1.4 (self weight +covering material)+ 1.6 w_{LL}

w_{ul}=1.4x (3.75+1.0) + 1.6 x 4 = 13.05 kN/m^2

Taking a strip of one meter

w_{ul}= 13.05 kN/m′

Assuming an edge live load of 1.5 kN/m′. P_u=1.6 x 1.5 = 2.4 kN/m′



$$M_u = \frac{w_{ul} \times L_{eff}^2}{2} = \frac{13.05 \times 1.5^2}{2} + 2.4 \times 1.5 = 18.3 kN.m$$

- Assuming 30 mm concrete cover, the effective depth equals

d = t - cover = 150 – 30 = 120 mm

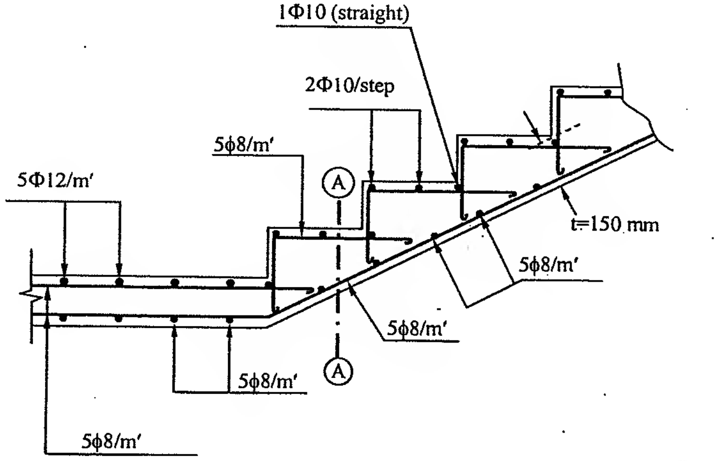
$$R = \frac{M_u}{f_{cu} \ b \ d^2} = \frac{18.3 \times 10^6}{25 \times 1000 \times 120^2} = 0.051$$

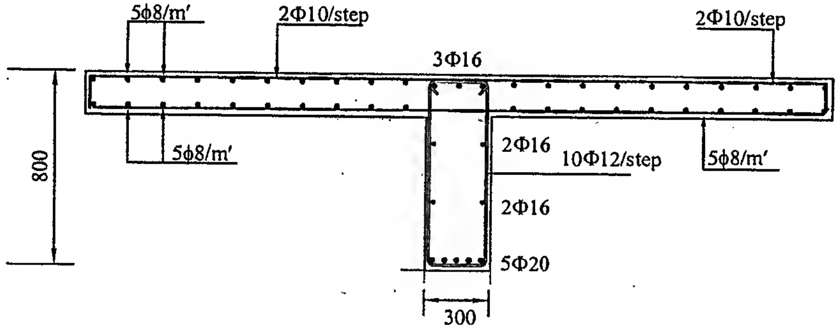
From the R-ω curve

ω=0.062

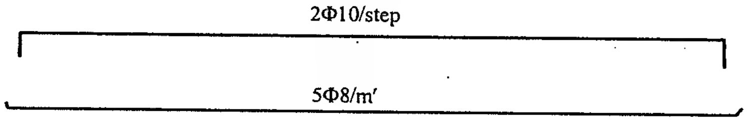
$$A_s = \omega \frac{f_{cu}}{f_y} b \ d = 0.062 \times \frac{25}{360} \times 1000 \times 120 = 518 \ mm^2 > A_{s,\min}$$

Choose 5 Φ 12/m′ (565 mm^2)





Section A-A



Stair main reinforcement

Step 3: Design the supporting beam (B1)

Step 3.1: Design for flexure

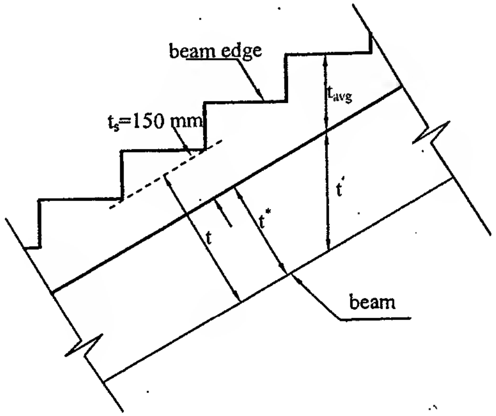
Step 3.1.1: Calculation of loads

Assume the beam cross section is 300x800 mm
The horizontal projection weight can be obtained from

$$t' = \frac{t^*}{\cos(\theta)} = \frac{t - t_s}{\cos(\theta)} = \frac{800-150}{\cos(26.6)} = 726.7\ mm$$

Self weight = 1.4 × γ_c × b × t'

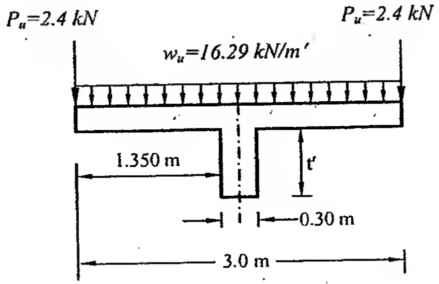
Self weight=1.4 × 25 × 0.30 × 0.7267 = 7.63 kN/m' (H.P.)



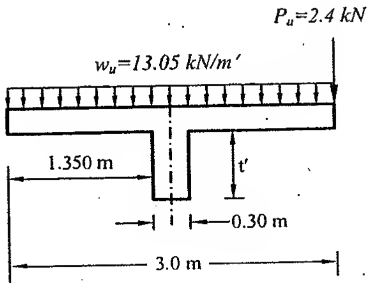
The beam load (w_{ub1}) equals *(in the stairs part)*

w_{ub1}= ow. + w_u x flight width + 2 x hand rail live load

$$w_{ub1} = 7.63 + 16.29 \times 3 + 2 \times 2.4 = 61.3\ kN/m'$$



Stairs loads



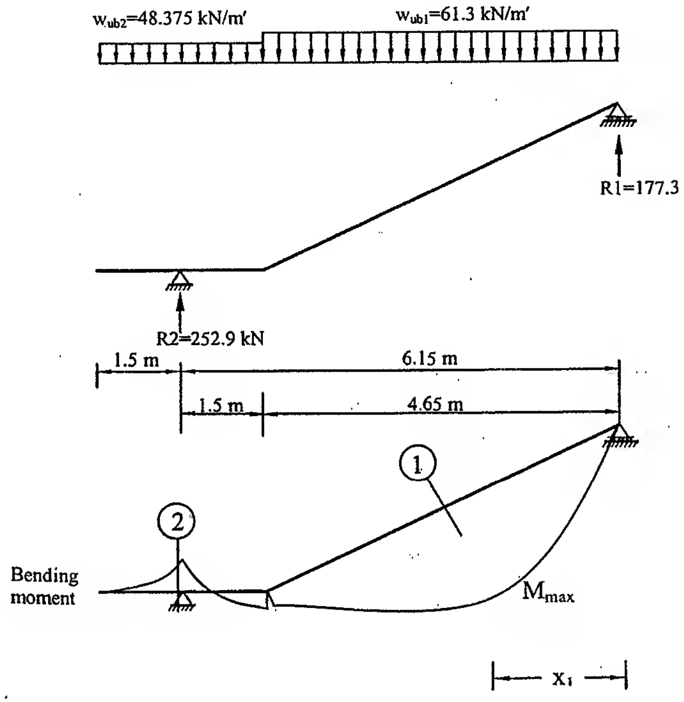
Landing loads

The beam load (w_{ub2}) equals *(in the landing part)*

w_{ub2}= ow. + w_u x flight width + hand rail live load

o.w = 1.4 x γ_c x b x (t-t_s)=1.4 x 25 x 0.30 x (0.8-0.15) = 6.825 kN/m'

$w_{ub2} = 6.825 + 13.05\times3+2.4 = 48.375\ kN/m'$



Step 3.1.2: Calculation of bending moments

$$R_1 = \frac{w_{ub1}\times4.65\times(4.65/2+1.5)+w_{ub2}\times3\times zero}{6.15} = \frac{61.3\times4.65\times(4.65/2+1.5)}{6.15} = 177.3\ \ kN$$

$$R_2 = w_{ub2}\times3+w_{ub1}\times4.65-R1 = 48.375\times3+61.3\times4.65-177.3 = 252.9\ kN$$

Point of zero shear $x_1 = \frac{R_1}{w_{u1}} = \frac{177.3}{61.3} = 2.89\ \ m$

$$M_{\max} = R_1\times x_1 - w_{u1}\times\frac{x_1^2}{2} = 177.3\times2.89-61.3\times\frac{2.89^2}{2} = 256\ kN.m$$

Step 3.1.3: Design of the critical sections

Section 1

This section has positive bending (256 kN.m), but we shall neglect the contribution of the stairs and design the beam as rectangular section

Using R-w curve

d = t - 50 mm= 800- 50= 750 mm

$$R = \frac{M_u}{f_{cu}bd^2} = \frac{256\times10^6}{25\times300\times750^2} = 0.061$$

From the curve it can be determined that ω=0.075

$$A_s = \omega\frac{f_{cu}}{f_y}b\times d = 0.075\,\frac{25}{360}\times300\times750 = 1179\,mm^2 > A_{s,min}$$

$$A_{s\,min} = smaller\ \ of\ \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y}b\ d = \dfrac{0.225\sqrt{25}}{360}\times300\times750 = 703\ mm^2 & < A_s\quad o.k \\ 1.3A_s = 1.3\times1179 = 1532\ mm^2 & \end{cases}$$

use (5 Φ 18, 1272 mm^2)

Section 2

$$M_u = \frac{48.375\times1.5^2}{2} = 54.4\ kN.m$$

Using R-w curve

$$R = \frac{M_u}{f_{cu}bd^2} = \frac{54.4\times10^6}{25\times300\times750^2} = 0.0129$$

From the curve it can be determined that ω=0.015

$$A_s = \omega\frac{f_{cu}}{f_y}b\times d = 0.015\,\frac{25}{360}\times300\times750 = 235\,mm^2 < A_{s,min}$$

$$A_{s\,min} = smaller\ \ of\ \begin{cases} \dfrac{0.225\sqrt{25}}{360}\times300\times750 = 703\ mm^2 & \\ 1.3\times235 = 305\,mm^2 & > A_s\quad o.k \end{cases}$$

Use A_s=$A_{s,min}$=305 mm^2

Choose (2 Φ 20)

Step 3.2: Design the beam for Shear

It should be mentioned that the case of loading that causes the maximum bending moments in the beams results in no torsional moment. However, it produce the maximum shear

Step 3.2.1: Check the adequacy of the concrete dimensions

The critical section for shear and torsion is at d/2 from the support.

$$Q_u = R_2 - w_{u1} \times \left(\frac{c}{2} + \frac{d}{2}\right)$$

$$Q_u = 252.9 - 48.375 \times \left(\frac{0.3}{2} + \frac{0.75}{2}\right) = 227.5\ kN$$

$$q_u = \frac{Q_u}{b \times d} = \frac{227.5 \times 1000}{300 \times 750} = 1.01\ N/mm^2$$

$$q_{u,\max} = 0.70\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.70\sqrt{\frac{25}{1.5}} = 2.8\ N/mm^2$$

since $q_u \langle q_{u,\max}$, the concrete dimensions are adequate.

Step 3.2.2: Design of the transverse reinforcement

$$q_{cu} = 0.24\sqrt{\frac{25}{1.5}} = 0.98\ N/mm^2$$

Since the applied shear is greater than q_{cu} shear reinforcement is needed

$$q_s = q_u - \frac{q_{cu}}{2} = 1.01 - \frac{0.98}{2} = 0.52\ N/mm^2$$

Assume spacing of 100 mm

$$A_{st} = \frac{q_{su} \times b \times s}{f_{yst}/1.15} = \frac{0.52 \times 300 \times 100}{280/1.15} = 64\ mm^2$$

use 10 ϕ 10 /m

Step 3.3: Check the case of loading that produces shear and torsion

Step 3.3.1: Calculate shear and torsion stresses

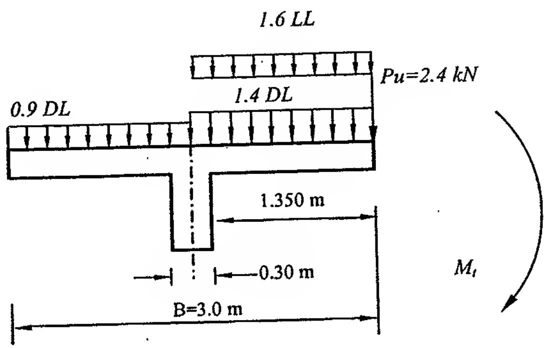
The unsymmetrical loading of the flight produces combined shear and torsion as shown in figure below. For simplicity, the loading of the landing is taken the same as the flight (conservative).

A. Torsion Stresses

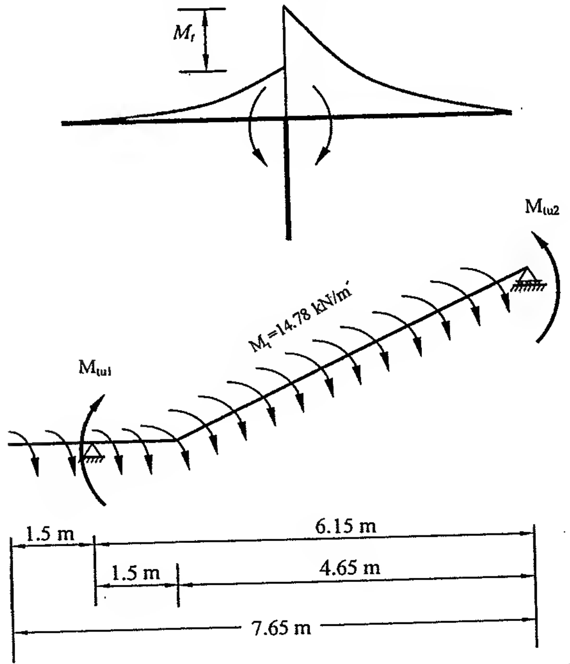
$$M_t = 1.6LL \times \frac{1.5^2}{2} + P_u \times 1.5 + (1.4\ DL - 0.9\ DL) \times \frac{1.5^2}{2}$$

LL = 4 kN/m^2 and DL=(6.068+1)=7.068 kN/m^2

$$M_t = 1.6 \times 4 \times \frac{1.5^2}{2} + 2.4 \times 1.5 + (1.4 \times 7.07 - 0.9 \times 7.07) \times \frac{1.5^2}{2} = 14.78\ t.m/m'$$



Load case that produce maximum torsion on beam



$$M_{tu1}=\frac{M_t\times\frac{7.65^2}{2}}{6.15}=\frac{14.78\times\frac{7.65^2}{2}}{6.15}=70.3\ kN.m$$

The critical section is at d/2 from the face of the column (300x300 mm)

$$M_{tu}=M_{tu1}-M_t\left(\frac{c}{2}+\frac{d}{2}\right)=70.3-14.78\times\left(\frac{0.30}{2}+\frac{0.75}{2}\right)=62.5\ kN.m$$

Assume the distance from the concrete cover to the stirrup center line is 35 mm

x_1 = 300 –2 x 35= 230 mm

y_1 = 800 –2 x 35 = 730 mm

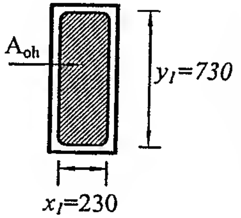
$$p_h=2\times(x_1+y_1)=2\times(230+730)=1920\ mm$$

$$A_{oh}=x_1.y_1=230\times730=167900\ mm^2$$

$$A_o=0.85A_{oh}=0.85\times167900=142715\ mm^2$$

$$t_e=\frac{A_{oh}}{p_h}=\frac{167900}{1920}=87.4\ mm$$

$$q_{tu}=\frac{M_{tu}}{2\times A_o\times t_e}=\frac{62.5\times10^6}{2\times142715\times87.4}=2.5\ N/mm^2$$



B. Shear Stresses

The critical section for shear is at d/2

$$w_{u2}=o.w_{ultimate}+1.4\times DL\times(B/2)+1.6\times LL\times(B/2)+P_u+0.9\times DL\times(B/2)$$

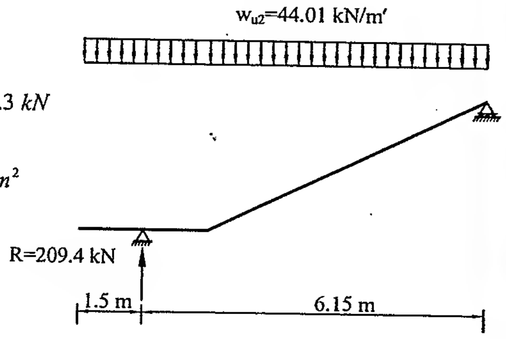
$$w_{u2}=7.63+1.4\times7.07\times1.5+1.6\times4\times1.5+2.4+0.9\times7.07\times1.5=44.01\ kN/m'$$

$$R=\frac{w_{u2}\times(6.15+1.5)^2/2}{6.15}=\frac{44.01\times(6.15+1.5)^2/2}{6.15}=209.4\ kN$$

$$Q_u=R-w_{u2}\times\left(\frac{c}{2}+\frac{d}{2}\right)$$

$$Q_u=209.4-44.01\times\left(\frac{0.3}{2}+\frac{0.75}{2}\right)=186.3\ kN$$

$$q_u=\frac{Q_u}{b\times d}=\frac{186.3\times1000}{300\times750}=0.828\ N/mm^2$$



Step 3.3.2: Check the adequacy of the concrete dimensions

$$q_{max}=0.70\sqrt{\frac{f_{cu}}{\gamma_c}}=0.70\times\sqrt{\frac{25}{1.5}}=2.86<4.0\ N/mm^2$$

q_{max}= 2.86 N/mm^2

$$\sqrt{q_u^2+q_{tu}^2}\le q_{max}$$

$$\sqrt{0.828^2+2.5^2}=2.63<2.86......o.k$$

Thus, section dimension is acceptable for shear and torsion

Note: Since the section is very close to the maximum values, it is advisable to increase the beam cross-section. However, from the architectural point of view, smaller depth is preferable.

Step 3.3.3: Reinforcement for shear

The concrete shear strength q_{cu} equals

$$q_{cu}=0.24\sqrt{\frac{25}{1.5}}=0.98\ N/mm^2$$

Since the applied shear is less than q_{cu} shear reinforcement is not needed

Step 3.3.4: Reinforcement for torsion

$$q_{tu,min}=0.06\sqrt{\frac{f_{cu}}{\gamma_c}}=0.06\sqrt{\frac{25}{1.5}}=0.24\ N/mm^2$$

Since q_{tu} (2.5)>q_{tumin} (0.24) then reinforcement is required, and torsional concrete strength is neglected. Assuming spacing of 100mm, the area of one branch A_{str} equals

$$A_{str}=\frac{M_{tu}\times s}{2\times A_o\times f_{yst}/\gamma_s}=\frac{62.5\times10^6\times100}{2\times142714\times280/1.15}=90\ mm^2$$

$$A_{sl}=\frac{A_{str}\times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)=\frac{90\times1920}{100}\left(\frac{280}{360}\right)=1343\ mm^2$$

Calculate the minimum area for longitudinal reinforcement $A_{sl,min}$

$$A_{sl,min}=\frac{0.40\sqrt{\frac{f_{cu}}{\gamma_c}}A_{cp}}{f_y/\gamma_s}-\frac{A_{str}\times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)$$

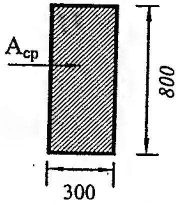
There is a condition on this equation that $\frac{A_{str}}{s} \geq \frac{b}{6 \times f_{yst}}$

$$\frac{90}{100} \geq \frac{300}{6 \times 280} \ldots o.k$$

$$A_{sl,\min} = \frac{0.40\sqrt{\frac{25}{1.5}} \times 300 \times 800}{360/1.15} - \frac{90 \times 1920}{100}\left(\frac{280}{360}\right) = -ve$$

Since $A_{sl} > A_{sl,min}$ ...o.k

Choose 8ϕ16 (1600 mm^2)



Step 3.3.5: Reinforcement for combined shear and torsion

The area for two branches= $2A_{str} + A_{st} \geq A_{st,\min}$, or the area of one branch for combined shear and torsion equals

$A_{str}+A_{st}/2=90+0=90$ mm^2

Choose ϕ 12 mm (113 mm^2) (one branch)

$$A_{st,\min} = \frac{0.4}{f_y} b \times s = \frac{0.4}{280} 300 \times 100 = 42.85\ mm^2 \qquad \text{(two branches)}$$

$A_{st,min}$ (one branch) = 42.85/2=21.4 mm^2 <113 mm^2 .....o.k

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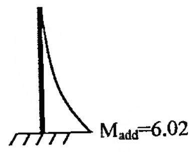
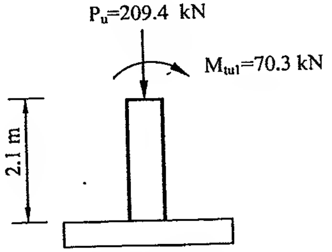
Step 4: Design of column (Refer to Chapter 8)

The column is subjected to compressive force in addition to bending resulted from the torsion load case of the beam.



Additional moment



Original column

The column is considered as case 1 ($t_{beam}>t_{column}$) at the top and case 3 (foundation) at the bottom in the in-plane and out of plane directions

H_{column} =2.1= m, From Table 6-10 in the code for unbraced columns k= 1.6

H= k x H_{column} =1.6 × 2.1 = 3.36 m

$$\lambda = \frac{H_e}{b} = \frac{3.36 \times 1000}{300} = 11.2 > 10 \rightarrow \text{additional moment is developed}$$

$$\delta = \frac{\lambda^2 \times b}{2000} = \frac{11.2^2 \times 0.30}{2000} = 0.0188$$

$$M_{add} = P_u \times \delta = 209.4 \times 0.0188 = 3.94\ kN.m$$

$$M_{tot,(in-plane)} = M_u + M_{add} = 70.3 + 3.94 = 74.24\ kN.m$$

$M_{tot(out\ of\ plan)} = M_u + M_{add} = 0 + 3.94 = 3.94$ kN.m

$$R_b = \frac{P_u}{f_{cu} \times b \times t} = \frac{209.4 \times 1000}{25 \times 300 \times 300} = 0.093$$

The column is subjected to biaxial bending, from code Table 6.12.a with R_b=0.1→β=0.8

$$M'_x = M_x + \beta\left(\frac{a'}{b'}\right) M_y = 74.24 + 0.8 \times \left(\frac{270}{270}\right) \times 3.94 = 77.39 \text{ kN.m}$$

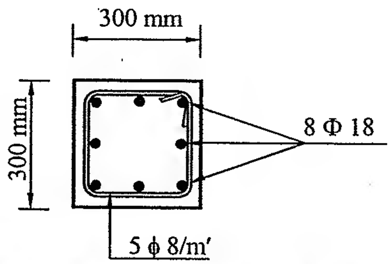
$$\frac{M'_x}{f_{cu} \times b \times t^2} = \frac{77.39 \times 10^6}{25 \times 300 \times 300^2} = 0.115$$

Using interaction diagram with uniform steel, f_y=360 N/mm^2 and ζ=0.8→ →ρ = 8.0

$$\mu = \rho \times f_{cu} \times 10^{-4} = 8.0 \times 25 \times 10^{-4} = 0.02 > \mu_{min}$$

$A_{s,total} = \mu \times b \times t = 0.02 \times 300 \times 300 = 1800\,mm^2$

Choose 8Φ18 (2035 mm²)



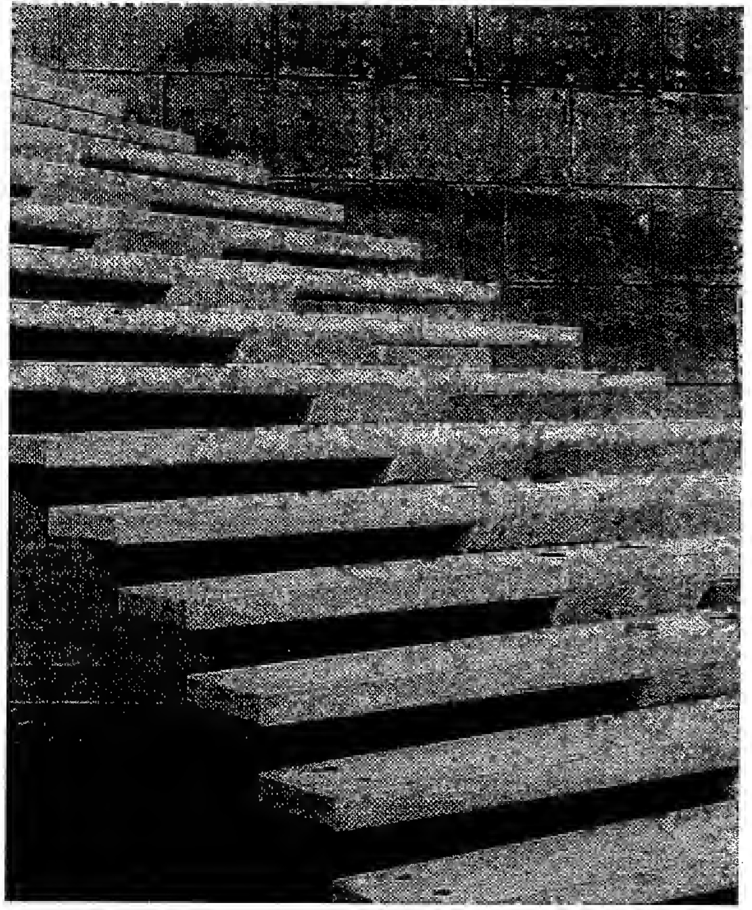
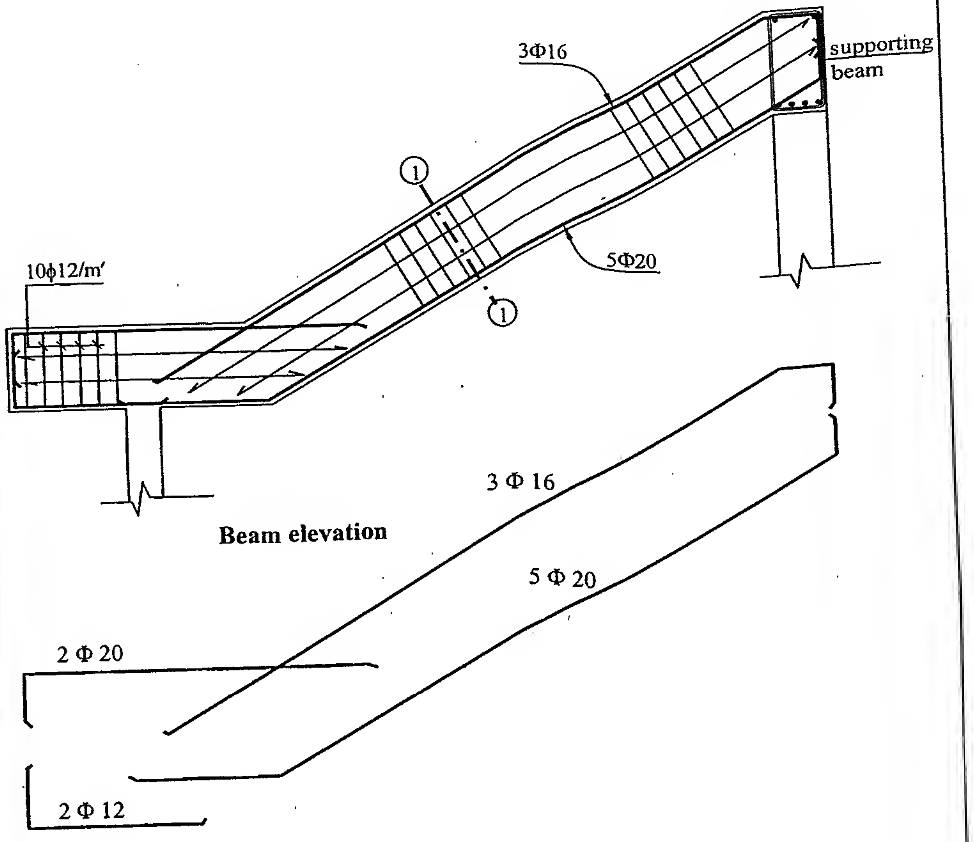
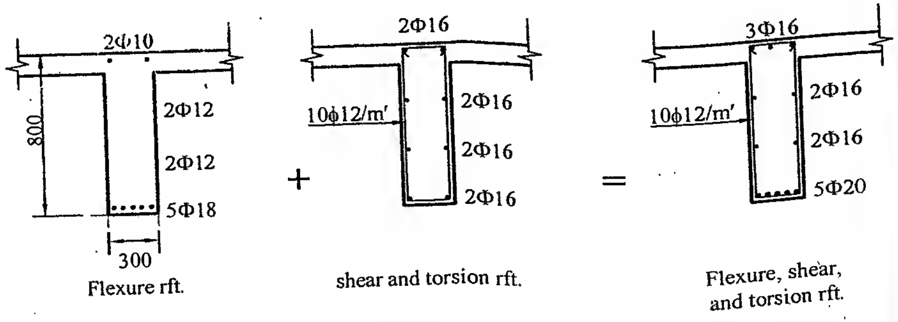


Photo 5.6 Cantilever stairs from a middle beam



Beam elevation



Flexure rft.

shear and torsion rft.

Flexure, shear, and torsion rft.

Sec 1-1

Beam reinforcement details

6

SHORT COLUMNS SUBJECTED TO CONCENTRIC COMPRESSION

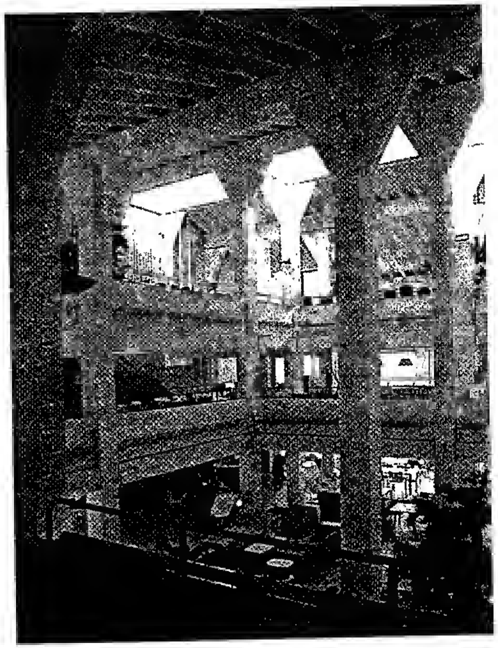


Photo 6.1 Reinforced concrete columns in a shopping center

6.1 Introduction

Columns are the most important structural element in buildings. A great number of structural failures are attributed to column failure. A plain concrete column can carry compression forces, however its ultimate capacity is greatly enhanced by adding vertical bars. For normal ratios of reinforcement, the increase in strength due to the addition of vertical reinforcement can range from 15-40 percent of the total carrying axial capacity. Lateral reinforcement or ties are added to provide support to the longitudinal bars and decrease the tendency of the bars to buckle out. They also prevent the concrete from expanding laterally due to Poisson's effect and accordingly increase the concrete ultimate strain. Reinforced concrete columns are classified as tied or spiral depending on the lateral confinement type.

In actual practice there are no perfect axially loaded columns. Some percentage of eccentricity will occur due to the reduction of the size of the column from one floor to another or the misalignment of the column. Hence, a minimum longitudinal reinforcement ratio has to be provided to account for any stresses resulting from the eccentricity.

Reinforced concrete columns are usually classified as short or long depending on the length to width ratio and end restraint conditions (refer to Chapter 7). The discussion in this chapter is limited to short columns either subjected to axial loads.

6.2 Axially Loaded Tied Columns

6.2.1 Behavior and Strength

The failure of the tied column is usually initiated by the spalling (*falling*) of the concrete cover followed by the buckling of the longitudinal bars due to the lack of the lateral support provided by the cover. The failure of axially loaded reinforced concrete columns is brittle with little or no warning. Up to approximately 80% of the total load, no sign of cracking appears. Suddenly vertical cracks start to appear with concrete cover failure leading to the collapse of the column.

Since failure of columns is often sudden with a high potential for loss of life, columns are designed with a much higher safety factor than beams. Because perfect straight columns subjected to pure axial loading are subjected to the brittle failure mode, the Egyptian code increases the strength reduction factors for concrete and steel to1.75 and 1.34, respectively.

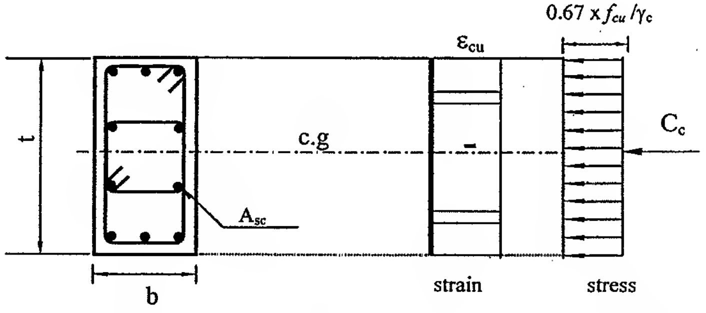


Fig. 6.1 Strain and stress distributions for columns under axial loads

When a column is subjected to axial loads, longitudinal strain develops in both concrete and steel. Because of the perfect bond between steel and concrete, the strains in the concrete and steel are equal. The total carrying capacity of the column is the summation of concrete and steel contributions. At failure, all the steel reinforcement is assumed to reach yielding. Applying the equilibrium equation for the section shown in Fig. 6.1 gives

$$P_u = \frac{0.67 f_{cu}\, b\, t}{\gamma_c} + \frac{A_{sc} \times f_y}{\gamma_s} \qquad \text{.......... (6.1)}$$

$$P_u = \frac{0.67 f_{cu}\, b\, t}{1.75} + \frac{A_{sc} \times f_y}{1.34} = 0.383 f_{cu}\, A_c + 0.75 A_{sc}\, f_y \qquad \text{.......... (6.2)}$$

Where, A_c is the area of the concrete and A_{sc} is the total area of the steel reinforcement.

The previous behavior is applied for perfect straight columns, which are practically almost do not exist. Even for concentrically loaded columns, most codes impose a minimum eccentricity to be considered in column design to account for dimensional inaccuracies and uncertainties in the line of action of axial loads. The ECP-203 minimum eccentricity is given by Eq. 6.3 as follows

$$e_{min} = bigger\ of \begin{cases} 0.05\, t \\ 20\ mm \end{cases} \qquad \text{.......... (6.3)}$$

The existence of moments leads to a reduction in axial load capacity. Thus the code imposes a further reduction on the column strength by reducing the capacity by about 10% giving the following equation

$$P_u = 0.35 f_{cu}\, A_c + 0.67 f_y\, A_{sc} \qquad \text{.......... (6.4)}$$

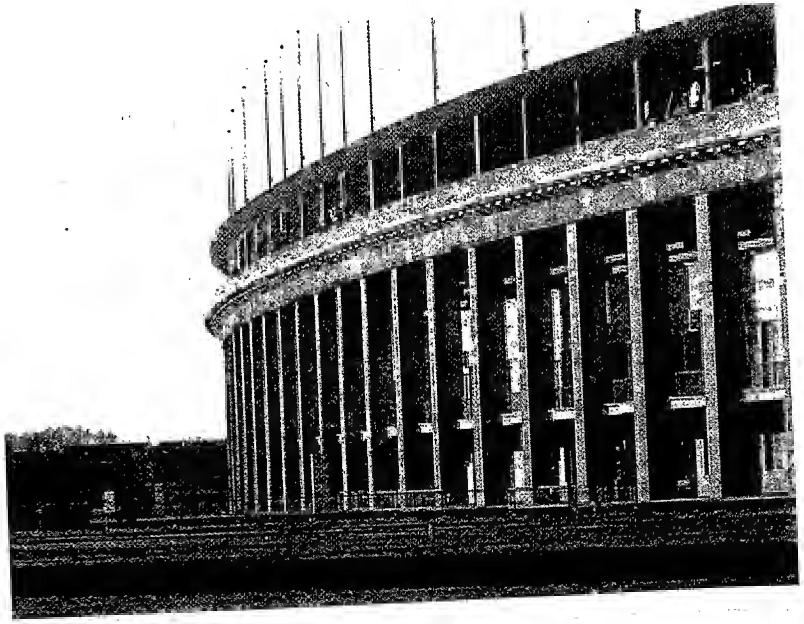


Photo 6.2 Olympic stadium in Berlin

6.2.2 Code Provisions for Tied Columns

- For short tied columns, the minimum vertical reinforcement is 0.8% of the required cross sectional area but not less than 0.6% of the chosen concrete area.
- The maximum reinforcement ratio in columns is
  - 4% for interior columns
  - 5% for exterior columns
  - 6% for corner columns
- The minimum vertical bar diameter is 12mm
- The minimum column dimension is 200 mm for both circular and rectangular columns. However, in practice, column dimensions are usually not less than 250 mm.
- Intermediate bars should be added if the column width is greater than 300 mm as shown in Fig. 6.2.
  - The maximum distance between two bars supported by ties is 250 mm. The maximum distance between unsupported bars and supported bars is 150 mm as shown in Fig. 6.2.
  - The maximum vertical distance for ties is 15×the smallest longitudinal bar diameter but not more than 200mm as shown in Fig. 6.3.
- The minimum stirrup diameter is one quarter of the longitudinal bars but not less than 8 mm
- The minimum stirrups volume is 0.25% of the concrete volume for one meter of the column.

$$V_s = n \times A_{sp} \times primeter \ge \frac{0.25}{100} \times b \times t \times 1000 \quad \text{.........................(6.5a)}$$

$$V_s = n \times A_{sp} \times primeter \ge 2.5 \times b \times t \quad (mm^3) \quad \text{.............................. (6.5b)}$$

where A_{sp} is the area of the stirrups and n is number of stirrups per meter.

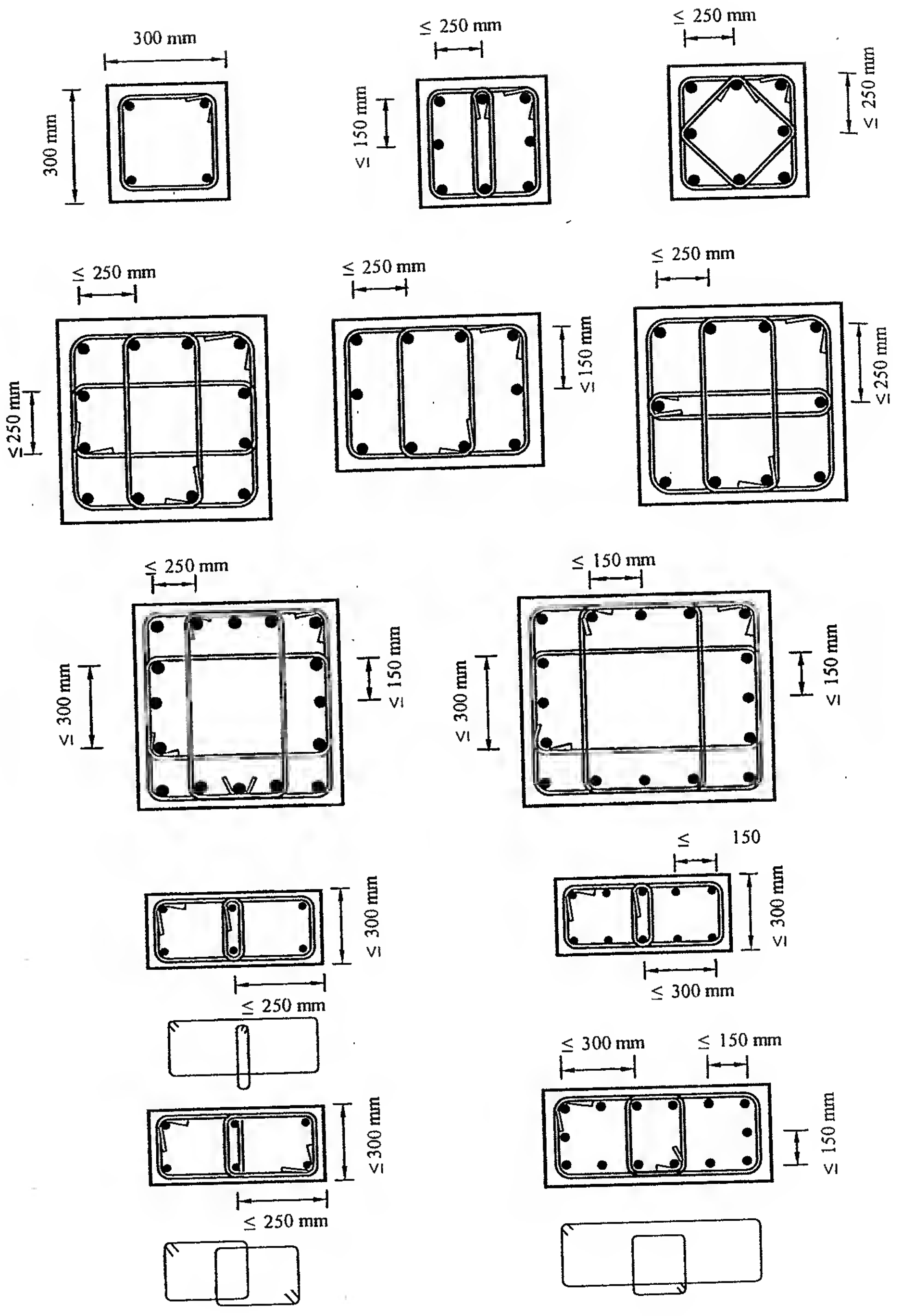


Fig. 6.2 Column Reinforcement Details

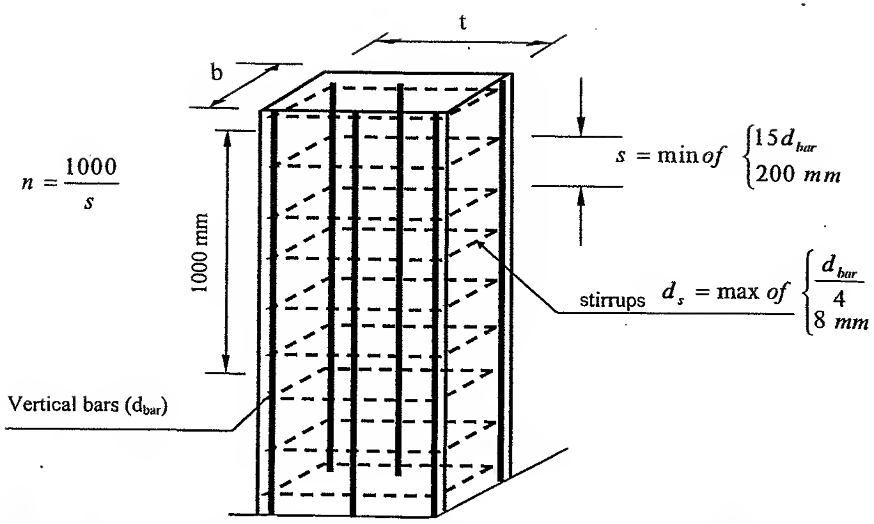


Fig. 6.3 Stirrups spacing requirements

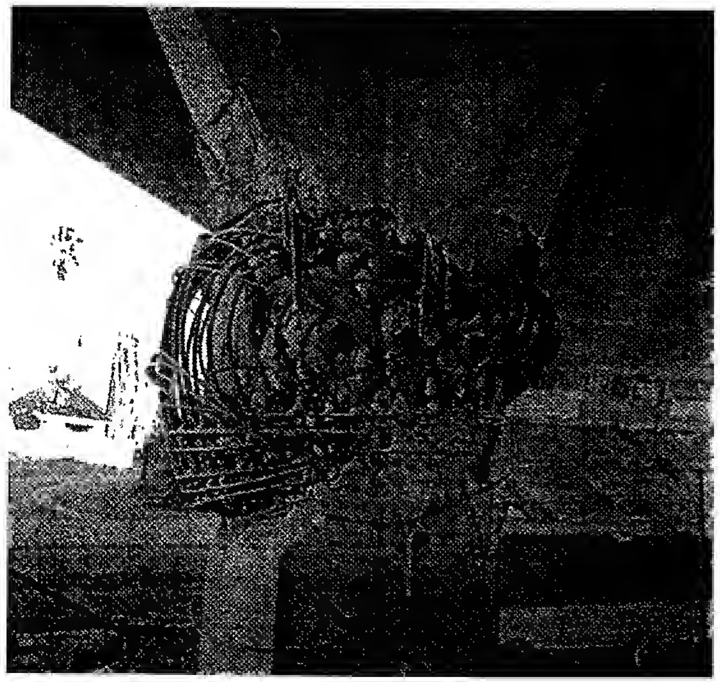
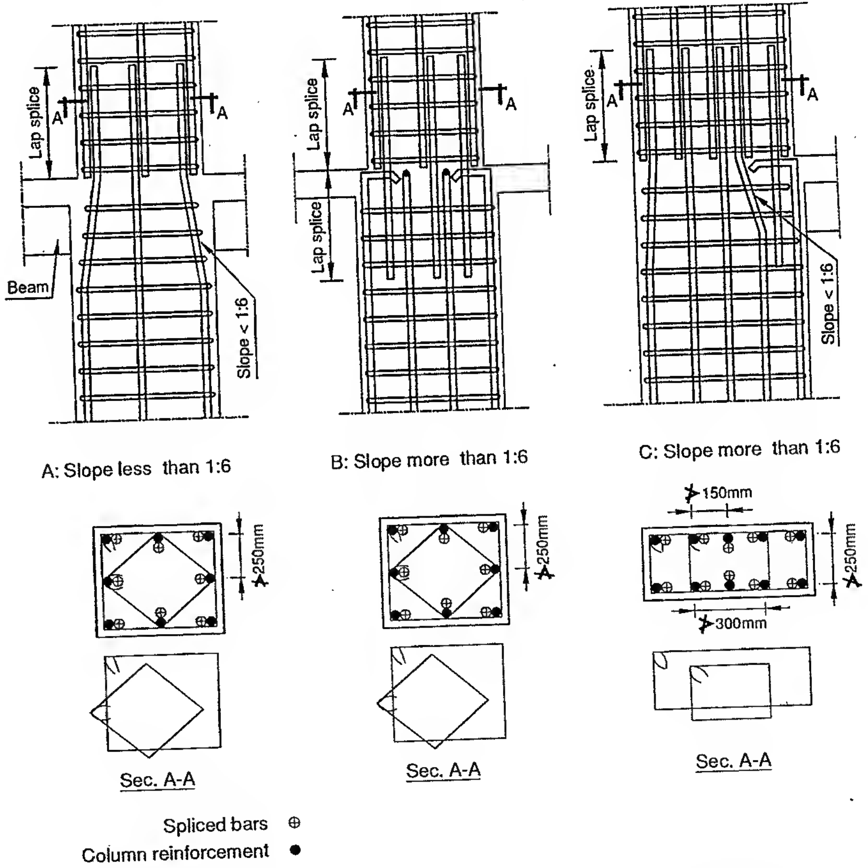


Photo 6.3 Bridge column failure due to lack of lateral reinforcement

6.2.3 Splicing of Vertical Reinforcement

For columns that are not used as a part of a lateral load resisting system, the reinforcement is spliced above each floor. Figure 6.4 shows lap details of the most widely used types in Egypt. The requirement for lap splice depends on the state of stress at ultimate state. If the column is subjected to combined bending and axial loads, tensile stress may occur and tension lap splice should be provided. For concentrically load columns, the ECP requires compression lap splice of 40 Φ for high grade steel and 35 ϕ for mild steel.

Column cross sections might change from one floor to another due to change in the axial applied force. Thus, the longitudinal bars may be discontinued or laterally displaced. The maximum allowable slope for the bars is 1:6 as shown in Fig 6.4.A. If the slope exceeds this limit the detail shown in Fig. 6.4.B-C should be followed. For the columns subjected to high lateral forces such as those used in seismic regions, the splice is made at the mid-height of the column as shown in Fig. 6.4.D.

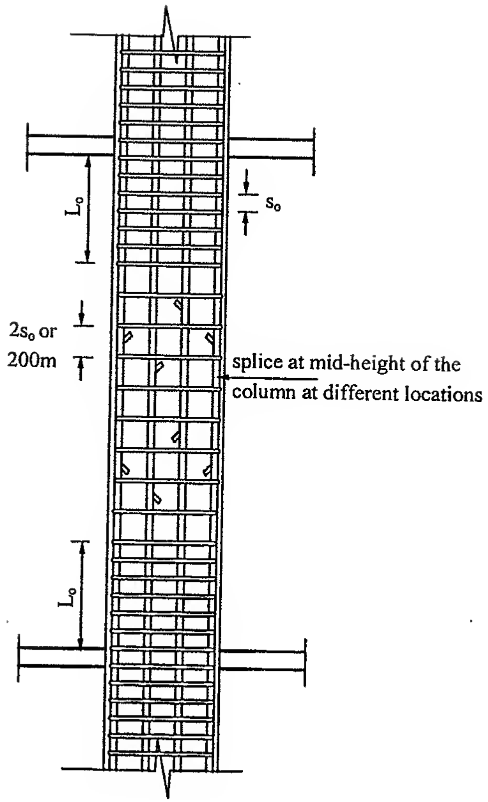


A: Slope less than 1:6

B: Slope more than 1:6

C: Slope more than 1:6

Fig. 6.4 Column lap splice requirements in structures with limited ductility



D: columns subjected to high lateral forces

$$L_o = \text{bigger of} \begin{cases} 500\,mm \\ Clear\ height\ /\ 6 \\ \text{Column bigger dimension (t)} \end{cases}$$

$$s_o = \text{smaller of} \begin{cases} b_{column}\ /\ 2 \\ 8\,\Phi_{\text{longintudnal}} \\ 24\,\phi_{striuups} \\ 150\,\text{mm} \end{cases}$$

Fig. 6.4 Reinforcement of columns in ductile structures subjected to large lateral force

Design steps for short columns (dimensions are not known)

1) Assume $A_{sc}= 0.01\,A$

Calculate $P_u = 1.4\,P_{DL} + 1.6\,P_{LL}$

Solve the column equation to find A_c

$P_u = A_c(0.35\,f_{cu} + 0.0067\,f_y\,)$

2) Calculate reinforcement area A_{sc}

$A_{sc}= 0.01\,A_{c,required} > 0.006\,A_{sc,chosen}$

3) Design the stirrups diameter d_s and spacing s

Check that $d_s \geq 8$ mm $\geq d_{bar}/4$. (biggest bar diameter)

Check that s ≤ 200 mm $\leq 15\,d_{bar}$ (smallest bar diameter)

4) Check that stirrups volume $\geq$ 0.25%

$$n \times A_{sp} \times primeter \geq 2.5 \times b \times t \quad (mm^3)$$

Note: In some cases, and due to architectural requirements, the cross-section of the column could be limited to certain dimensions. Accordingly, the designer has to provide the amount of longitudinal reinforcement that satisfies the strength requirement. If such amount might case a difficulty in casting the column, the designer could specify a higher concrete strength.

Design steps for short columns (dimensions are known)

1) Solve the column equation to find A_{sc}

$P_u = 0.35\,f_{cu}\,A_c + 0.67\,f_y\,A_{sc}$

2) Check that

$A_{sc} > A_{sc,min}$ and $A_{sc} < A_{sc,max}$

3) Design the stirrups diameter d_s and spacing s

Check that $d_s \geq 8$ mm $\geq d_{bar}/4$ (biggest bar diameter)

Check that s ≤ 200 mm $\leq 15\,d_{bar}$ (smallest bar diameter)

4) Check that stirrups volume $\geq$ 0.25%

$$n \times A_{sp} \times primeter \geq 2.5 \times b \times t \quad (mm^3)$$

Example 6.1

Design a tied column that is subjected to the following axial compression loads

P_{DL} = 1057 kN
P_{LL} = 400 kN

The material properties are as follows:

f_{cu} = 35 N/mm^2
f_y =400 N/mm^2

Solution

Step 1: Calculate column dimension

Calculate the ultimate load

$$P_u = 1.4\ P_{DL} + 1.6\ P_{LL} = 1.4\times 1057 + 1.6\times 400 = 2119.8\ kN$$

Assume $A_{sc} = 0.01\ A_c$

$$P_u = A_c(0.35\ f_{cu} + 0.0067\ f_y)$$

$$2119.8\times 1000 = A_c(0.35\times 35 + 0.0067\times 400)$$

Ac = 141982 mm^2

Assume column width b of 250 mm, then column thickness t equals

$$t = \frac{A_c}{b} = \frac{141982}{250} = 568\ mm$$

t= 600 mm

Step 2: Calculate reinforcement area

$$A_{sc} = 0.01\ A_c, required > 0.006\ A_c, chosen$$

$$A_{sc} = 0.01\times 141982 = (1419\ mm^2) > 0.006\ (250\times 600)\quad o.k.$$

Choose 8 Φ 16 (1608 mm^2)

Step 3: Calculate stirrups

Chose stirrup diameter of 8 mm (>16/4) and spacing of 200 mm <(16 x 15)

Choose 5 ϕ 8 /m'

Assume concrete cover of 25 mm from each side, the dimensions of the stirrups equal:

Stirrup A (200 x 550) Stirrup B (200 x 250)

The perimeter of the center line of the stirrups

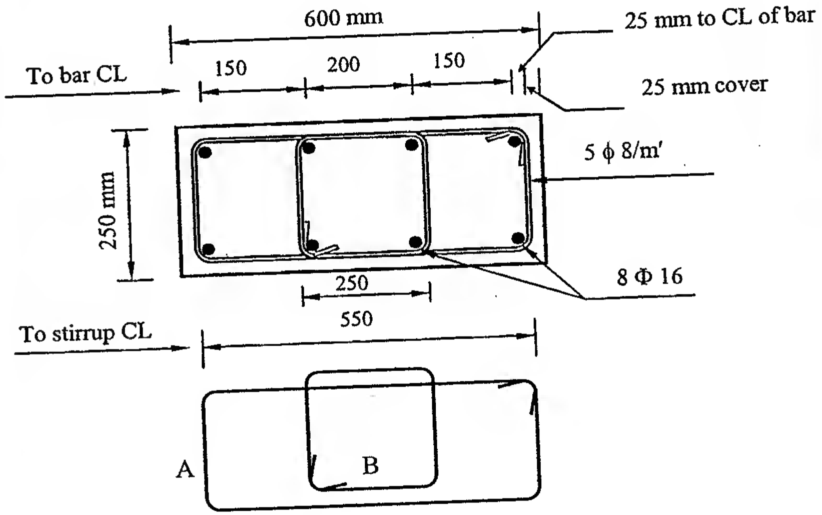
$$p = 2\times(200+550) + 2\times(200+250) = 2400\ mm$$

The volume of the stirrups in 1 meter equals

Noting that we have 5 stirrups per meter and A_{sp} for ϕ 8mm =50 mm^2

$$V_{s,min} = 2.5\times 250\times 600 = 375\times 10^3\ mm^3$$

$$V_s = n\times A_{sp}\times p = 5\times 50\times 2400 = 600\times 10^3 mm^3 > V_{s\,min} \ldots\ldots o.k$$



Example 6.2

Calculate the maximum and the minimum loads that an interior column can carry according the following data:

Column cross section (250 mm x 800 mm)

f_{cu} = 30 N/mm^2

f_y =360 N/mm^2

Solution

Since the column dimensions, material properties are given, the only variable is the reinforcement area.

The maximum area steel for interior column is 4% thus A_{sc} equals

$$A_{sc,\max}=\frac{4}{100}\times 250\times 800=8000\ mm^2 \qquad \rightarrow (18\ \Phi\ 25)$$

$$P_u=0.35\ f_{cu}\ A_c+0.67\ f_y\ A_{sc}$$

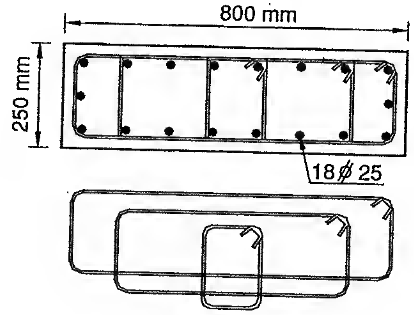
$$P_{u,\max}=\frac{1}{1000}\{0.35\times 30\times 250\times 800+0.67\times 360\times 8000\}=4029.6\ kN$$

The minimum area of steel for a column is 0.8% of the required area

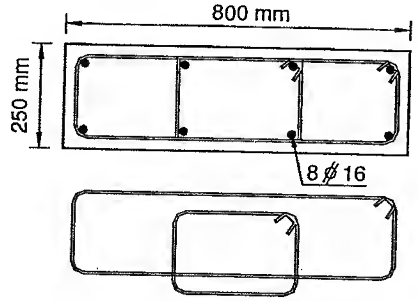
$$A_{sc,\min}=\frac{0.8}{100}\times 250\times 800=1600\,mm^2 \qquad \rightarrow (8\ \Phi\ 16)$$

$$P_u=0.35\ f_{cu}\ A_c+0.67\ f_y\ A_{sc}$$

$$P_{u,\min}=\frac{1}{1000}\{0.35\times 30\times 250\times 800+0.67\times 360\times 1600\}=2485.92\ kN$$



Column with maximum reinforcement



Column with minimum reinforcement

6.3 Axially Loaded Spiral Columns

6.3.1 Behavior and Strength

Tied columns are commonly used in buildings and structures in non-seismic regions. Occasionally, when ductility or higher strength is required, a continuous circular steel reinforcement in the form of a spiral is used instead of individual stirrups. This type of column is called a *spiral column.*

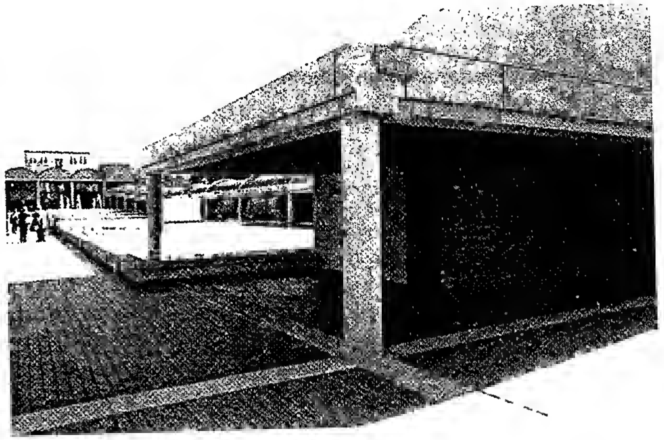


Photo 6.4 Spiral reinforced concrete column

The main advantage of using the spiral reinforcement is the enhancement of the concrete confinement developed by the closer spacing of spiral reinforcement as shown in Fig. 6.5.

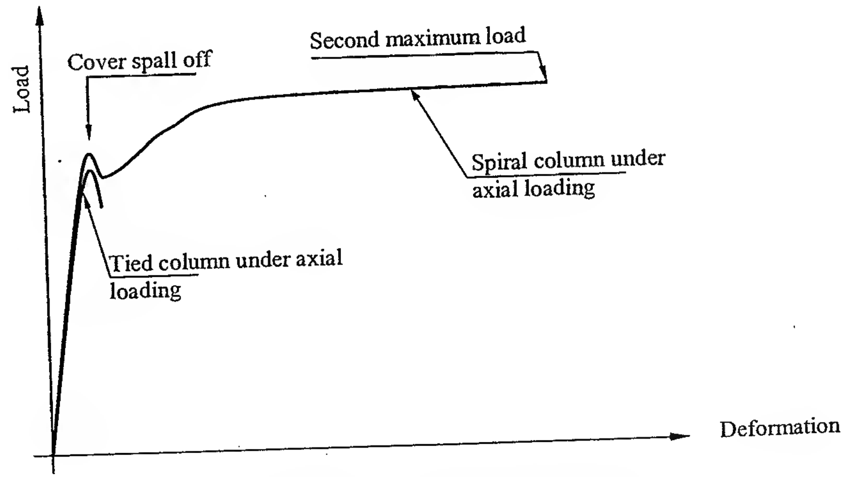


Fig. 6.5 Behavior of tied and spiral columns

The closely spaced stirrups of the spiral and the vertical bars confine the concrete very effectively. As a result, the concrete cover will spall off but the core will continue to carry loads larger than the initial load that caused spalling *(falling)*. This is due to the enhancement of the compressive strength produced by the spiral. The spalling of the cover gives a warning of failure, and shortly after that the column will reach another maximum load but under very large deformations.

Recognizing the difference between the failure modes of tied and spiral columns, the ECP 203 specifies (refer to Eq. 6.6) an additional increase of approximately 14% in the ultimate load capacity than regular tied columns Eq. 6.4.

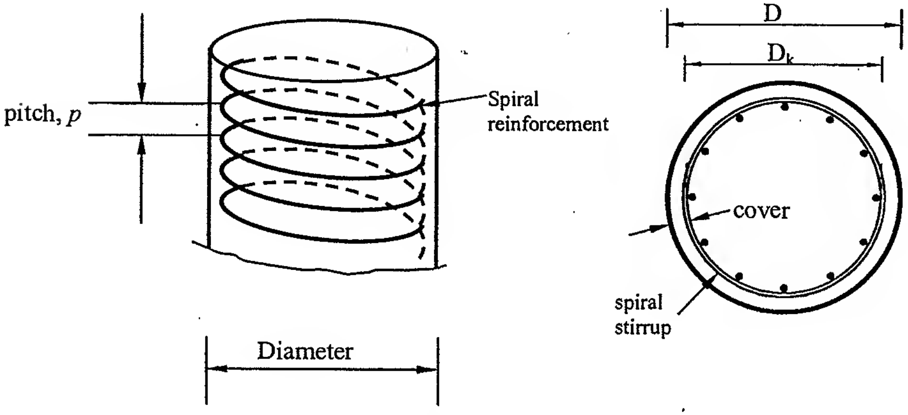


Fig. 6.6 Spiral reinforcement details

The ECP 203 states that the ultimate load a spirally loaded column is the smaller of two values. The first value given by Eq. 6.6 is based on the axial capacity of the concrete gross area A_c. The second value given by Eq. 6.7 considers the confining effect of the spiral on the core strength

$$P_u = 0.4 f_{cu}\, A_c + 0.76 f_y\, A_{sc} \quad \text{......} (6.6)$$

$$P_u = 0.35 f_{cu}\, A_k + 0.67 f_y\, A_{sc} + 1.38 \times f_{yp}\, V_{sp} \quad \text{......} (6.7)$$

where

A_k is the area of concrete core enclosed by spiral stirrups
V_{sp} is the spiral reinforcement ratio
p is the pitch of the spiral stirrups
f_{yp} is the steel yield strength of the stirrups
A_{sc} is the area of the vertical steel

$$A_c = \frac{\pi}{4} D^2 \qquad A_k = \frac{\pi}{4} D_k^2 \qquad D_k = (D - \text{cover})$$

$$V_{sp} = \frac{\pi\, A_{sp}\, D_k}{p} \geq V_{sp,\min} \quad \text{......} (6.8)$$

A_{sp} is the area of the spiral.

P is the pitch of the spiral (30→80mm)

$$\mu_{sp,\min} = 0.36 \left(\frac{f_{cu}}{f_{yp}}\right)\left[\frac{A_c}{A_k} - 1\right] \quad \text{......} (6.9)$$

but

$$\mu_{sp,\min} = \frac{V_{sp,\min}}{A_k} \quad \text{......} (6.10)$$

$$V_{sp,\min} = 0.36 \left(\frac{f_{cu}}{f_{yp}}\right)\left[A_c - A_k\right] \quad \text{......} (6.11)$$

The designer assumes a cross sectional area for the rebar used as spiral *(8mm or bigger diameter)* and computes the required pitch. The pitch used must be within the limitations of the ECP 203 of 30 mm to 80mm. If the required pitch is less than 30 mm, a bigger diameter should be assumed. On the other hand, if the calculated pitch is greater than 80mm, a decrease of the spiral diameter or the use of the same diameter but with spacing of 80 mm should be considered.

The minimum area of the longitudinal steel for spiral columns is more than that for a tied column, and is related to both the gross and core cross sectional areas of concrete, and is given by

$$A_{s,\min} = \text{the maximum of} \begin{cases} 0.01\, A_c & \textit{(gross area)} \\ 0.012\, A_k & \textit{(core area)} \end{cases} \quad \text{......} (6.12)$$

6.3.2 Minimum Spiral Reinforcement

The ability of the confined concrete to carry additional loads is attributed to the lateral pressure developed on the concrete core by the heavy coils of the spiral. Experimental tests proved that concrete axial compressive strength increases to the order of 4.1 times the applied lateral pressure. The spiral column is designed so that the increased capacity of the core due to spiral lateral pressure f_2 equals the loss that may occur if the concrete cover spalls off as shown in Fig. 6.7. Thus, the capacity of the column without applying the strength reduction factor equals,

$$4.1 f_2 \times A_k = 0.67 \times f_{cu} (A_c - A_k) \quad \text{......} (6.13)$$

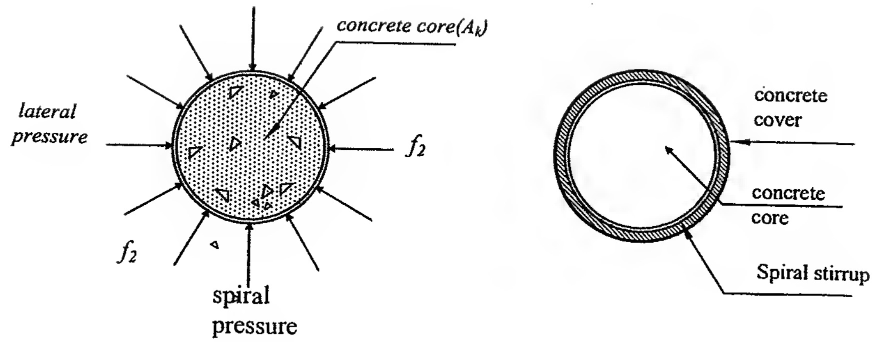


Fig. 6.7 Lateral pressure developed in spiral columns

From the free body diagram shown in Fig. 6.8, and by equating the force in the steel to the pressure on concrete, it can be concluded that

$$f_2 \times p \times D_k = 2\times A_{sp} \times f_{ysp} \quad \text{.......................................} (6.14)$$

$$f_2 = \frac{2\,A_{sp} \times f_{ysp}}{p \times D_k} \quad \text{...} (6.15)$$

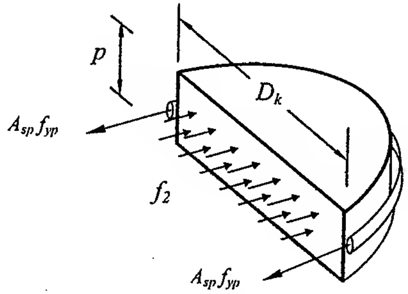


Fig. 6.8 Analysis of forces

Substituting in with expression 6.15 into Eq. 6.13 gives

$$4.1\frac{2\,A_{sp} \times f_{ysp}}{p \times D_k} A_k = 0.67 \times f_{cu} (A_c - A_k) \quad \text{...........................} (6.16)$$

Noting that $A_k = \pi\, D^2_k/4$

$$\frac{A_{sp} \times f_{ysp}}{p \times}(\pi\, D_k) = 0.33 \times f_{cu} (A_c - A_k) \quad \text{..................................} (6.17)$$

but $V_{sp,\min} = \dfrac{\pi\, A_{sp}\, D_k}{p}$

Rounding some numbers, Eq. 6.17 can be put in the following form

$$V_{sp,\min} = 0.36 \times \frac{f_{cu}}{f_{ysp}} (A_c - A_k) \quad \text{.......................................} (6.18)$$

or

$$\mu_{sp,\min} = 0.36 \times \left(\frac{f_{cu}}{f_{ysp}}\right) \times \left(\frac{A_c}{A_k} - 1\right) \quad \text{................................} (6.19)$$

The previous equation is the ECP 203 equation for minimum stirrups.

6.3.3 Code Provisions for Spiral Columns

- The minimum area of steel is 1 % of the gross sectional area but not less than 1.2 from the core area.
- The minimum spiral bar diameter is 8 mm
- The maximum pitch for a spiral column is 80 mm and the minimum is 30 mm.

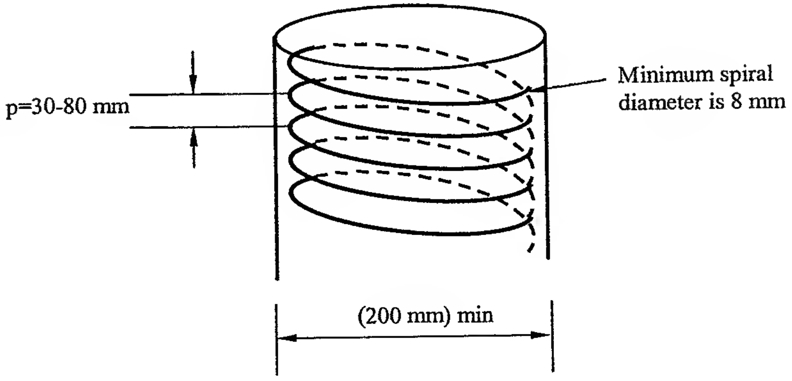


Fig. 6.9 Spiral column code requirements

Design Steps

1. Assume $A_{sc}= 0.01\ A_c$
2. Solve the first equation to find A_c

   Calculate $P_u = 1.4 P_{DL} + 1.6 P_{LL}$

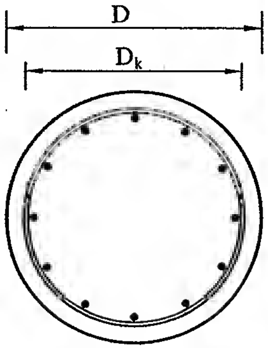
   $P_u = A_c (0.4\ f_{cu} + 0.0076\ f_y)$
3. Solve the second equation to get V_{sp}

   $P_u = 0.35 f_{cu}\ A_k + 0.67 f_y\ A_{sc} + 1.38 \times f_{yp}\ V_{sp}$

   if V_{sp} =-ve use $V_{sp,min}$
4. check $V_{sp,min}$

   $$V_{sp,\min} = 0.36 \left(\frac{f_{cu}}{f_{yp}} \right) [A_c - A_k]$$
5. calculate the pitch (*p*) using

   $$p = \frac{\pi\ A_{sp}\ D_k}{V_{sp}}$$



if $p > 80$ mm use $p = 80$ mm

if $p < 30$ mm increase spiral area (A_{sp}) and recalculate p

Note: In some cases, and due to architectural requirements, the cross-section of the column could be limited to certain dimensions. Accordingly, the designer has to provide the amount of longitudinal reinforcement that satisfies the strength requirement. If such amount might case a difficulty in casting the column, the designer could specify a higher concrete strength.

Example 6.3

Design a spiral column to support an unfactored dead load of 1500 kN and an unfactored live load of 700 kN. The material properties are f_{cu}=25 N/mm^2 , f_y = 240 N/mm^2 , f_{yp}=240 N/mm^2

Solution

Step 1: Determine cross section and A_{sc}

$P_u = 1.4\ P_{DL} + 1.6\ P_{LL} = 1.4 \times 1500 + 1.6 \times 700 = 3220\ kN$

The most economical percentage of steel μ is 1% to1.5%. Assume that μ= 1%, substitute in equation to find the area of the cross section as first trial.

$P_u = 0.4\ f_{cu}\ A_c + 0.76\ f_y\ A_{sc}$

$3220 \times 1000 = 0.4 \times 25 \times A_c + 0.76 \times 240 \times (0.01\ A_c)$

$A_c = 272327\ mm^2$

$A_c = \frac{\pi}{4} D^2$

D = 588.84 mm

The nearest round number is 600 mm. Assume that the concrete cover is 25 mm then the core diameter D$_k$ equals

D$_k$=600-50 =550 mm

The area of the concrete, $A_c = \frac{\pi}{4} D^2 = \frac{\pi}{4} 600^2 = 282743\ mm^2$

The area of the core, $A_k = \frac{\pi}{4} D_k^2 = \frac{\pi}{4} 550^2 = 237583\ mm^2$

$$A_{s,\min} = \text{the maximum of} \begin{cases} 0.01\ A_c & = 0.010 \times 282743 = 2827\ mm^2 \\ 0.012\ A_k & = 0.012 \times 237583 = 2850.\ mm^2 \end{cases}$$

$A_{sc}= 0.01\ A_c = 2827.43\ mm^2 < A_{smin} \rightarrow$ use $A_{sc}=A_{smin}$

A_{sc}=2850 mm^2

Choose 12 ϕ 18 (3053 mm^2)

Step 2: Determine spiral reinforcement

Applying in the second equation to determine the volume of the spiral (V_{sp})

$$P_u = 0.35\, f_{cu}\, A_k + 0.67\, f_y\, A_{sc} + 1.38\times f_{yp}\, V_{sp}$$

Note that A_{sc} chosen will be used

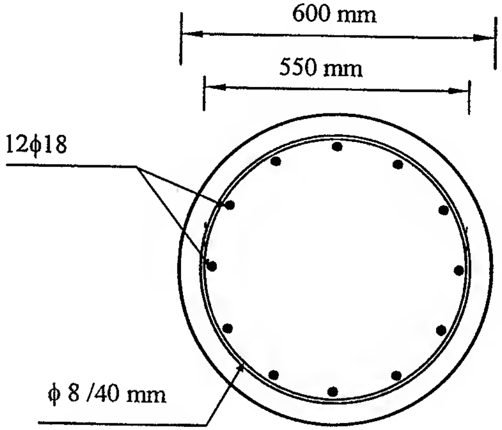
$$3220\times 1000 = 0.35\times 25\times 237583 + 0.67\times 240\times 3053 + 1.38\times 240\times V_{sp}$$

$V_{sp} = 1963\ mm^2$

Check that $V_{sp} > V_{sp,min}$

$$V_{sp,\min} = 0.36\left(\frac{f_{cu}}{f_{yp}}\right)[A_c - A_k] = 0.36\left(\frac{25}{240}\right)[282743 - 237583]$$

$$V_{sp,\min} = 1693\ mm^2 < V_{sp} \text{ ......o.k}$$



Step 3: Design of spiral

Assuming that bar diameter of the spiral is 8 mm, A_{sp}= 50 mm^2.Use the following equation to determine the stirrup pitch p

$$p = \frac{\pi\, A_{sp}\, D_k}{V_{sp}}$$

$$p = \frac{\pi\times 50\times 550}{1962} = 44.2\ mm$$

Round to the smallest pitch p=40 mm

p<80 mm and p>30 mm ....o.k.

Example 6.4

Determine the ultimate load that can be supported by a spiral column having a cross section of 800 mm with minimum area of steel required by the code. The spiral reinforcement is ϕ10 every 50 mm. The material properties are f_{cu}=30 N/mm^2 , f_y = 360 N/mm^2 , f_{yp}=240 N/mm^2

Solution

Step 1: calculate section properties

Calculate the cross sectional area of the column

$$A_c = \frac{\pi}{4} D^2 = \frac{\pi}{4} 800^2 = 502655\ mm^2$$

D_k=800-50 =750 mm

$$A_k = \frac{\pi}{4} D_k^2 = \frac{\pi}{4} 750^2 = 441786\ mm^2$$

The area of the reinforcement equals

$$A_{sc} = bigger\ of \begin{cases} 0.01\, A_c & 0.01\times 502655 = 5026\ mm^2 \\ 0.012\, A_k & 0.012\times 441786 = 5301\ mm^2 \end{cases}$$

A_{sc}=5301 mm^2

Step 2: calculate ultimate load

Substitute in the first equation

$$P_u = 0.4\, f_{cu}\, A_c + 0.76\, f_y\, A_{sc}$$

$$P_u = (0.4\times 30\times 502655 + 0.76\times 360\times 5301)/1000$$

P_u= 7482 kN

Calculate the volume of the spiral, for ϕ10 mm A_{sp}= 78.53 mm^2

$$V_{sp} = \frac{\pi\, A_{sp}\, D_k}{p}$$

$$V_{sp} = \frac{\pi\times 78.53\times 750}{50} = 3701\ mm^2$$

Applying in the second equation to determine the second ultimate load

$$P_u = 0.35\, f_{cu}\, A_k + 0.67\, f_y\, A_{sc} + 1.38\times f_{yp}\, V_{sp}$$

$P_u = 0.35 \times 30 \times 441786 + 0.67 \times 360 \times 5301 + 1.38 \times 240 \times 3701$ $\quad P_u = 7143$ kN

P_u is the smaller of the two ultimate loads, thus

$\boxed{P_u = 7143\ \text{kN}}$

Choose 12 ϕ 25 (5890 mm^2)

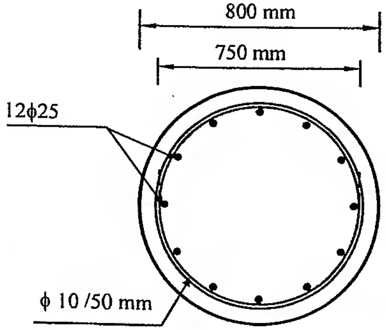


Photo 6.5 Circular reinforced concrete columns supporting a bridge

6.4 Design of Composite Columns

The use of composite columns has become increasingly popular in high-rise buildings construction due to several advantages such are:

1- Significant saving in material and construction time.
2- Smaller cross section and higher strength to weight ratio than conventional reinforced concrete columns.
3- Inherent ductility and toughness that can be useful in resisting lateral loads.
4- Higher load carrying capacity due to the composite action of steel and concrete. Furthermore the confinement of the outer shell in case of in-filled columns, increase the compressive strength of concrete.

Composite columns can include concrete filled into FRP shell (fiber reinforced plastics) or into steel pipe. The ECP-203 defines composite columns as compression members reinforced longitudinally with one of the following (refer to Fig. 6.10):

1- Internal structural steel shapes.
2- External steel pipe.
3- External steel tubing.

6.4.1 Design Guidelines

1. Forces required to be resisted by concrete in the composite member shall be transmitted to concrete through direct bearing on concrete. Bearing strength should be checked in accordance to clause (4-2-4-1) or clause (5-6) of ECP-203. All forces not directly transmitted to the concrete should be transmitted to the steel section through connections attached to the steel sections.
2. Interaction diagrams for composite columns subjected to eccentric compression force can be developed in a manner similar to that followed for regular reinforced concrete columns.
3. All axial load strength not assigned to concrete of a composite member shall be developed by direct connection to the structural steel shape, pipe, or tube. This achieved through welding of shear connectors (small steel pieces) to the steel shape or pipe before casting the concrete.
4. The maximum yield strength of the structural steel core shall not exceed 350 N/mm^2.
5. Spiral reinforcement pitch and diameter should confirm to that mentioned in non-composite reinforced concrete columns.
   - $p = 30 - 80\ mm$
   - $\phi \geq 8 mm$

6. The ratio of the longitudinal reinforcement (μ) shall not be less than 1% and not more than 6% of the net area of the cross section as follows:

$$\mu = \frac{A_{sc}}{A_g - A_t} \geq 1\%$$

$$< 6\%$$

$$A_t = A_{sc} + A_{ss}$$

Where A_g is the gross cross sectional area, A_{sc} is the area of the reinforcement and A_{ss} is the structural steel area.

7. The moment of inertia of the longitudinal reinforcement may be added to the structural steel moment of inertia as follows

$$I_t = I_{sc} + I_{ss}$$

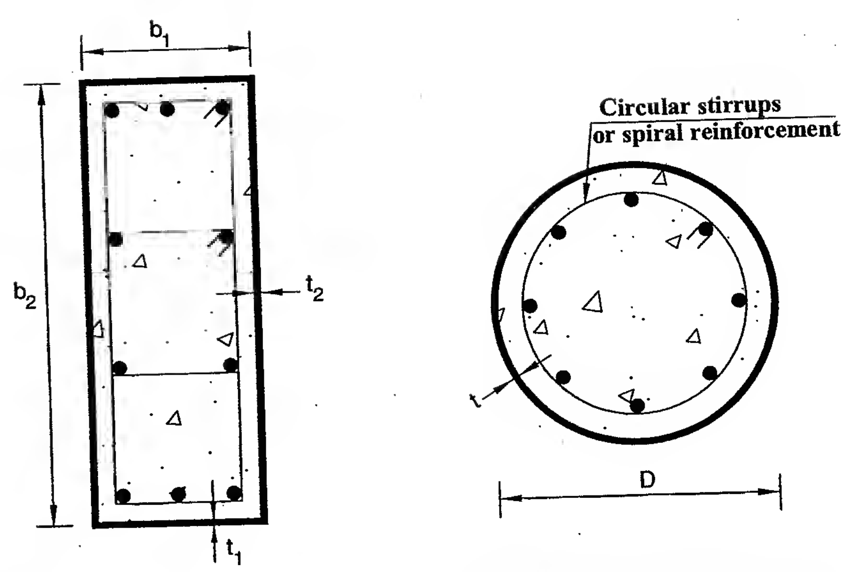
I_{sc} =moment of inertia of the longitudinal steel.
I_{ss} = moment of inertia for the structural steel about the centroid.

8. The slenderness ratio of the composite column may be evaluated using the radius of gyration given by the following equation:

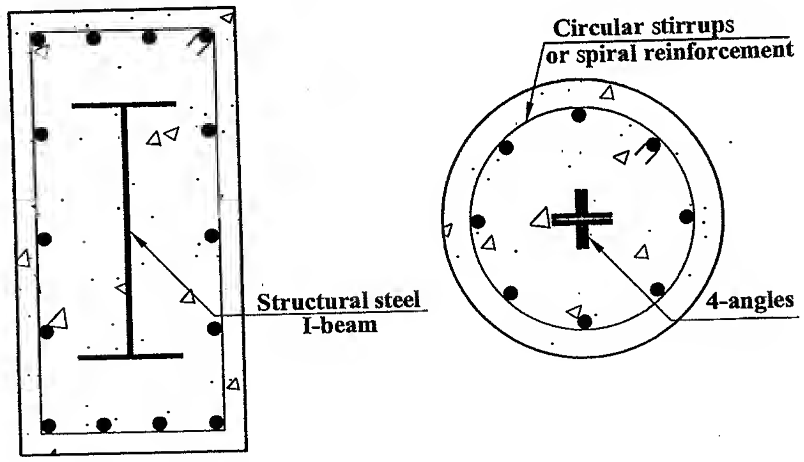
$$r = 0.80\sqrt{\frac{(E_c\, I_g/5) + E_s I_t}{(E_c\, A_g/5) + E_s A_t}} \qquad \text{.. (6.20)}$$

A_g = gross area of the cross section
A_t = total area of steel (A_{sc}+A_{ss})
E_c = Young's modulus of concrete $\rightarrow E_c = 4400\sqrt{f_{cu}}$
E_s = Young's modulus of the structural steel (=200000 N/mm^2)
I_g = moment of inertia of the concrete section about the centroid neglecting the effect of the reinforcement
I_t . = moment of inertia of the structural steel and reinforcement (I_{sc}+I_{ss})

In reinforced concrete columns subject to sustained loads, creep transfers some of the load from the concrete to the steel, increasing the steel stresses. In the case of lightly reinforced columns, the load transfer between concrete and steel may cause the compression steel to yield prematurely (too early), resulting in a loss of the effective EI due to creep in both steel and concrete. However, For heavily reinforced columns or for composite columns in which the pipe or structural shape makes up a large percentage of the cross section, the load transfer due to creep is not significant. Accordingly, in Eq. (6.20) only the term *EI* of the concrete is reduced for sustained load effects.



a)External tube + steel reinforcement **b)External pipe + steel reinforcement**



c)Internal I-beam + steel reinforcement **d)Internal angles + steel reinforcement**

Fig. 6.10 Examples of composite columns.

6.4.2 Types of Composite Columns

6.4.2.1 Structural steel confining concrete core

Structural steel confining concrete core should have a wall thickness large enough to reach yield stress before buckling outward. The ECP-203 requires that for a composite member with concrete encased by structural steel tube or pipe, the thickness of the steel wall shall be not less than:

For rectangular column with width b, the minimum thickness for each face as shown in Fig. 6.10 is given by

$$t_{\min} \geq b\sqrt{\frac{f_y}{3E_s}} \quad \text{...} (6.21)$$

For circular columns with diameter D

$$t_{\min} \geq D\sqrt{\frac{f_y}{8E_s}} \quad \text{...} (6.22)$$

Where E_s is the modulus of elasticity of external steel casing.

- **Capacity of columns with ordinary stirrups**

  The capacity of the encased concrete columns subjected to axial load with minimum eccentricity ($e < e_{min}$) equals to:

$$P_u = 0.35 f_{cu}\ A_c + 0.67 f_{yss}\ A_{ss} + 0.67 f_{ysc}\ A_{sc} \quad \text{..........................} (6.23)$$

Where:

f_{yss} = Steel yield strength of the outer steel tube or pipe.

f_{ysc} = Steel yield strength of internal steel vertical reinforcement.

A_{ss} = Cross sectional area of the outer steel tube or pipe.

A_{sc} = Cross sectional area of the internal vertical reinforcement.

- **Capacity of circular columns with spiral reinforcement**

  The capacity of circular composite columns that are laterally confined is given by:

$$P_u = 0.40 f_{cu}\ A_c + 0.67 f_{yss}\ A_{ss} + 0.76 f_{ysc}\ A_{sc} \quad \text{..................} (6.24)$$

Where:

f_{yss} = Steel yield strength of the internal structural steel shape.

f_{ysc} = Steel yield strength of internal steel reinforcement.

A_{ss} = Cross sectional area of the internal structural steel shape.

A_{sc} = Cross sectional area of the internal reinforcement.

In such a case, the amount of the spiral reinforcement should satisfy the minimum requirements given by the following equations:

$$V_{sp} = \frac{\pi A_{sp} D_k}{p} \geq V_{sp,\min} \quad \text{.................................} (6.25a)$$

$$V_{sp,\min} = 0.36\left(\frac{f_{cu}}{f_{yp}}\right)[A_c - A_k] \quad \text{..............................} (6.25b)$$

$$A_c = \frac{\pi}{4}D^2 \qquad A_k = \frac{\pi}{4}D_k^2 \qquad D_k = (D - cover)$$

where

A_c	is the gross cross sectional area of the column.
A_k	is the area of concrete core enclosed by spiral stirrups.
A_{sp}	is the cross sectional area of the spiral stirrups.
V_{sp}	is the spiral reinforcement volume.
p	is the pitch of the spiral stirrups.
f_{yp}	is the steel yield strength of the stirrups.

6.4.2.2 Concrete surrounding Structural Steel Core

To maintain the concrete around the structural steel core in composite columns, it is reasonable to require more lateral ties than needed for ordinary reinforced concrete columns. The yield strength of the structural steel core should be limited to prevent separation of the concrete. It has been assumed that axially compressed concrete will not separate at strains less than 0.0018. According to the ECP-203 yield strength of 350 N/mm^2 represents an upper limit of the useful maximum steel stress.

The axial capacity of the composite column with concrete surounding the structural steel core may be calculated according to the type of ties as follows:

- **Ordinary stirrups surrounding structural steel core**

In case of using ordinary stirrups, the following requirements should be satisfied:

1. The diameter of stirrups shall not be less than 8 mm and shall completely surround the structural steel core.
2. Stirrups shall have a diameter not less than 0.02 times the greatest side dimension of the composite member but not more than 16 mm.
3. Vertical spacing of stirrups shall not exceed (16Φ) longitudinal bar diameters.
4. A longitudinal bar shall be located at every corner of a rectangular cross section, with other longitudinal bars spaced not more than one-half the least side dimension of the composite member or 150 mm.

The ultimate carrying capacity of the column is calculated by

$$P_u' = 0.35 f_{cu} A_c + 0.67 f_{yss} A_{ss} + 0.67 f_{ysc} A_{sc} \quad \text{.........................} \; (6.26)$$

Where:

f_{yss} = Steel yield strength of the internal structural steel.
f_{ysc} = Steel yield strength of the longitudinal steel reinforcement.
A_{ss} = Cross sectional area of the internal structural steel.
A_{sc} = Cross sectional area of the longitudinal reinforcement.

- **Spiral reinforcement surrounding structural steel core**

Concrete that is laterally confined by spiral reinforcement has increased load-carrying strength.

In case of using spiral reinforcement, the capacity is given by:

$$P_u = 0.35 f_{cu} A_k + 0.67 f_{yss} A_{ss} + 0.67 f_{ysc} A_{sc} + 1.38 f_{yp} V_{sp} \quad \text{.............} \; (6.27)$$

Where:

f_{yss} = Steel yield strength of the internal structural steel shape.
f_{ysc} = Steel yield strength of internal steel reinforcement.
f_{yp} = Steel yield strength of the spiral stirrups.
A_{ss} = Cross sectional area of the internal structural steel shape.
A_{sc} = Cross sectional area of the internal reinforcement.
V_{sp} = Spiral reinforcement volume.

The amount of the spiral reinforcement should satisfy the minimum requirement given by the following equation:

$$V_{sp} = \frac{\pi A_{sp} D_k}{p} \geq V_{sp,\min} \quad \text{....................................} \; (6.28)$$

Where $V_{sp,min}$ is determined from Eq. 6.25.

The designer assumes a cross sectional area for the rebar used as spiral *(8mm or bigger diameter)* and computes the required pitch. The pitch used must be within the limitations of the ECP-203 (30 mm→80mm). If the required pitch is less than 30 mm, a bigger diameter should be assumed. On the other hand, if the calculated pitch is greater than 80mm, a decrease of the spiral diameter or the use of the same diameter but with spacing of 80 mm should be considered.

Example 6.5

Design a rectangular composite column with an internal IPE No. 300 if the column is subjected to the following axial compression loads

P_{DL} = 1450 kN
P_{LL} = 760 kN

The material properties are as follows

f_{cu} = 35 N/mm^2
f_{ysc} =360 N/mm^2
f_{yss} =300 N/mm^2

Solution

Step 1: Calculate column dimension

Calculate the ultimate load

$$P_u = 1.4\,P_{DL} + 1.6\,P_{LL} = 1.4\times1450 + 1.6\times760 = 3246\ kN$$

The cross sectional area of IPE 300=5380 mm^2

The axial capacity of a composite column is given by

$$P_u = 0.35 f_{cu}\,A_c + 0.67 f_{ysc}\,A_{sc} + 0.67\,f_{yss}\,A_{ss}$$

Assume $A_{sc}= 0.01\,A_c$, thus

$$P_u = A_c(0.35 f_{cu} + 0.0067 f_{ysc}) + 0.67\,f_{yss}\,A_{ss}$$

$$3246\times1000 = A_c\,(0.35\times35 + 0.0067\times360\,) + 0.67\times300\times5380$$

Ac = 147635 mm^2

Assume column width b of 250 mm, then column thickness t equals

$$t = \frac{A_c}{b} = \frac{147635}{250} = 590.5\ mm$$

t= 600 mm

Step 2: Calculate longitudinal reinforcement area

$$A_{sc} = 0.01\,A_c$$

$$A_{sc} = 0.01\times147635 = 1476\ mm^2$$

Choose 16 Φ 12 (1810 mm^2)

The maximum horizontal spacing between the vertical bars equals

$$\le\frac{b}{2} = \frac{250}{2} = 125\ mm \le 150mm\ ok.$$

- The ratio of the longitudinal reinforcement μ should not be less than 1% and not more than 6% from the net cross section

$$\mu = \frac{A_{sc}}{A_g - A_t} \ge 1\%$$
$$< 6\%$$

$$A_t = A_{sc\,chosen} + A_{ss} = 1810 + 5380 = 7190\ mm^2$$

$$\mu = \frac{1810}{250\times600 - 7190} = 1.27\% \ge 1\%.....ok.$$
$$< 6\%\ ....ok$$

Step 3: Calculate stirrups

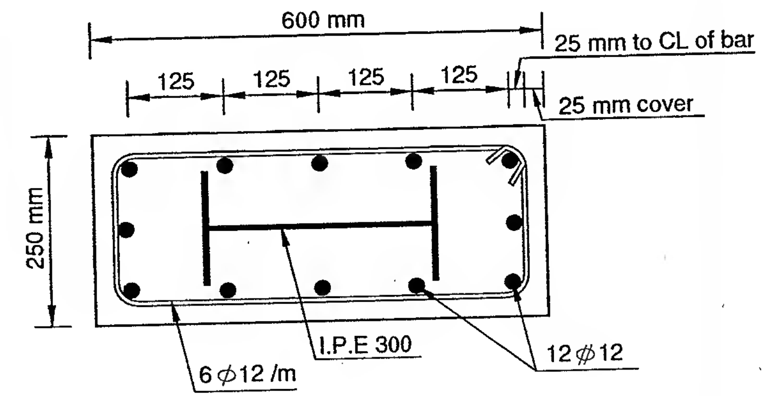
In this type of composite columns, more lateral ties is required than for ordinary reinforced concrete columns.

$$\text{The minimum stirrup diameter}\begin{cases} \ge 8mm \\ \ge t/50 = 600/50 = 12\,mm \\ \le 16\,mm \end{cases}$$

The vertical spacing between the ties should be less than 16 times the vertical bars

Chose stirrup diameter of 12 mm and spacing of 167 mm <(16 x 12=192 mm)

Choose 6 ϕ 12 /m'

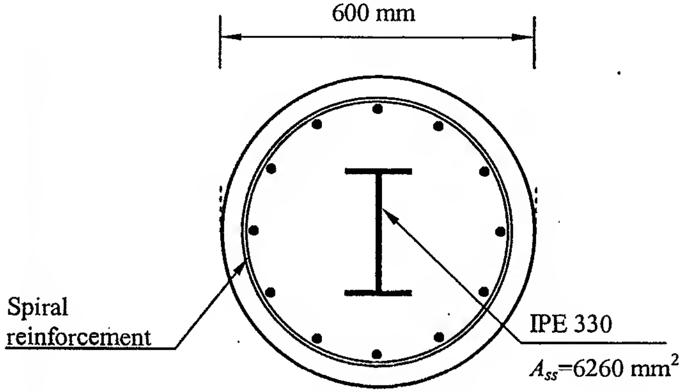


Example 6.6

Design a spiral composite column (D=600 mm) with internal IPE 330 to support an unfactored dead load of 1940 kN and an unfactored live load of 1620 kN.

The material properties are as follows

f_{cu} = 25 N/mm^2
f_{ysc} =360 N/mm^2
f_{yp} =240 N/mm^2
f_{yss} =400 N/mm^2



Solution

Step 1: Determine cross section and A_{sc}

$P_u = 1.4\,P_{DL} + 1.6\,P_{LL} = 1.4\times1940 + 1.6\times1620 = 5308\ kN$

The area of the concrete, $A_c = \frac{\pi}{4}D^2 = \frac{\pi}{4}600^2 = 282743\ mm^2$

D$_k$=600-50 =550 mm

The area of the core, $A_k = \frac{\pi}{4}D_k^2 = \frac{\pi}{4}550^2 = 237583\ mm^2$

Assume that μ= 1% $\longrightarrow$ A_{sc}= 0.01 A_c = 2827.43 mm^2

The chosen A$_{sc}$ is less than 1.2% of A_k (2850 mm^2), thus take A_{sc}=2850 mm^2

Choose 12 ϕ 18 (3054 mm^2)

- The ratio of the longitudinal reinforcement μ should not be less than 1% and not more than 6% from the net cross section

$$\mu = \frac{A_{sc}}{A_c - A_t} \geq 1\%$$

$$A_t = A_{sc,chosen} + A_{ss} = 3054 + 6260 = 9314\ mm^2$$

$$\mu = \frac{3054}{282743 - 9314} = 1.12\% \geq 1\% .....ok.$$

$$< 6\% \;....ok$$

Step 2: Determine spiral reinforcement

Applying in the column equation to determine the volume of the spiral (V_{sp})

$$P_u = 0.35\,f_{cu}\,A_k + 0.67\,f_{yss}\,A_{ss} + 0.67 f_{ysc}\,A_{sc} + 1.38\,f_{yp}V_{sp}$$

Note that f_{yss} =400 N/mm^2, however, according to the ECP-203, the maximum usable structural steel yield strength is 350 N/mm^2 $\therefore$ f_{yss} =350 N/mm^2

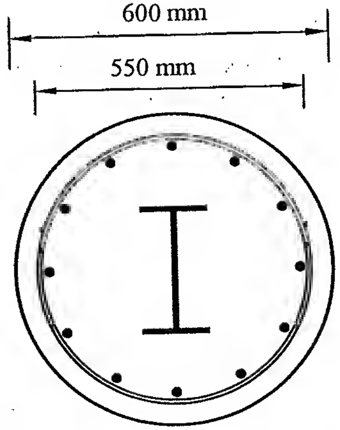
$$5308\times1000 = 0.35\times25\times237583 + 0.67\times350\times6260 + 0.67\times360\times3053 + 1.38\times240\times V_{sp}$$

V_{sp} = 3094 mm^2

Check that $V_{sp} > V_{sp,min}$

$$V_{sp,\min} = 0.36\left(\frac{f_{cu}}{f_{yp}}\right)[A_c - A_k] = 0.36\left(\frac{25}{240}\right)[282743 - 237583]$$

$$V_{sp,\min} = 1694\ mm^2 < V_{sp} ......o.k$$



Step 3: Design of spiral

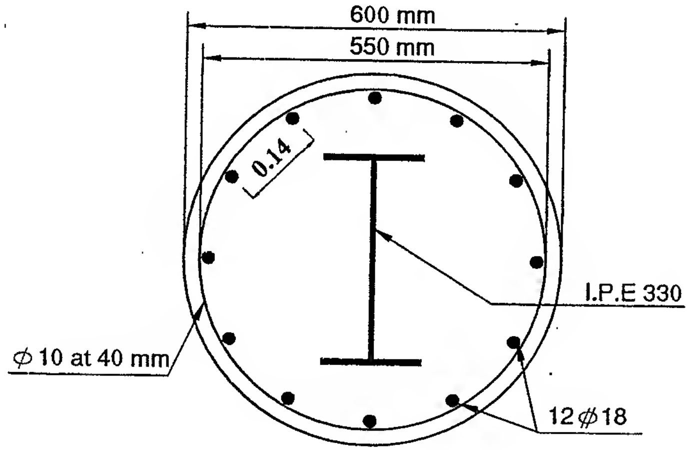
Assuming that bar diameter of the spiral is 10 mm, A_{sp}= 78.5 mm^2. Use the following equation to determine the stirrup pitch p

$$p = \frac{\pi\, A_{sp}\, D_k}{V_{sp}}$$

$$p = \frac{\pi \times 78.5 \times 550}{3094} = 43.8\ mm$$

Round to the smallest pitch p=40 mm

$p<80$ mm and $p>30$ mm ....o.k.



Example 6.7

Design a spiral composite column with an external pipe to support an unfactored dead load of 2400 kN and an unfactored live load of 1500 kN.

The material properties are as follows

f_{cu} = 25 N/mm^2
f_{ysc} =280 N/mm^2
f_{yp} =240 N/mm^2
f_{yss} =350 N/mm^2

Solution

Step 1: Determine cross section and A_{sc}

$$P_u = 1.4\, P_{DL} + 1.6\, P_{LL} = 1.4 \times 2400 + 1.6 \times 1500 = 5760\ kN$$

According to the ECP-203, the minimum thickness of the pipe equals

$$t_{\min} \geq D \sqrt{\frac{f_y}{8E_s}}$$

Thus minimum area of the pipe equals

$$A_{ss,\min} = \pi D\, t_{\min} = \pi D^2 \sqrt{\frac{f_{yss}}{8E_s}} = 4\sqrt{\frac{f_{yss}}{8E_s}}\, A_c$$

$$A_{ss} = 4\sqrt{\frac{350}{8 \times 200000}}\, A_c = 0.059\, A_c \qquad \rightarrow A_c = \frac{\pi}{4} D^2$$

Assume that the reinforcement ratio μ for the vertical bars= 1%

$$P_u = 0.40 f_{cu}\, A_c + 0.67 f_{yss}\, A_{ss} + 0.76 f_{ysc}\, A_{sc}$$

$$5760 \times 1000 = 0.4 \times 25 \times A_c + 0.67 \times 350 \times 0.059 \times A_c + 0.76 \times 280 \times (0.01\, A_c)$$

$$A_c = 221850\ mm^2 \qquad A_c = \frac{\pi}{4} D^2$$

D = 531 mm →→D=550 mm

Assuming that concrete cover is 25 mm then the core diameter D$_k$ equals

D$_k$=550-50 =500 mm

The area of the concrete, $A_c = \frac{\pi}{4} D^2 = \frac{\pi}{4} 550^2 = 237583\ mm^2$

The area of the core, $A_k = \frac{\pi}{4} D_k^2 = \frac{\pi}{4} 500^2 = 196350\ mm^2$

A_{sc}= 0.01 A_c = $2376\ mm^2$ >1.2% of A_k (2356 mm^2)....ok

A_{sc}=2376 mm^2 →→→ Choose 12 ϕ 16 (2413 mm^2)

Step 3: Determine the Pipe thickness

$$t_{\min} = D\sqrt{\frac{f_y}{8E_s}} = 550\sqrt{\frac{350}{8\times 200000}} = 8.13\,mm \qquad \longrightarrow\!\!\longrightarrow \text{Take } t = 9 \text{ mm}$$

$$A_{ss} = \pi D\,t = \pi\times 550\times 9 = 15551\,mm^2$$

- The ratio of the longitudinal reinforcement (μ) should not be less than 1% and not more than 6% from the net cross section

$$\mu = \frac{A_{sc}}{A_g - A_t} \geq 1\%$$

$$A_t = A_{sc} + A_{ss} = 2413 + 15551 = 17964\ mm^2$$

$$\mu = \frac{2413}{237583 - 17964} = 1.1\% \geq 1\% \ldots\ldots ok.$$

$$< 6\% \ldots\ldots ok$$

Step 3: Determine spiral reinforcement

The used spiral should be greater than the minimum spiral reinforcement specified by the code

$$V_{sp,\min} = 0.36\left(\frac{f_{cu}}{f_{yp}}\right)[A_c - A_k] = 0.36\left(\frac{25}{240}\right)[237583 - 196350]$$

$$V_{sp,\min} = 1546.3\ mm^2$$

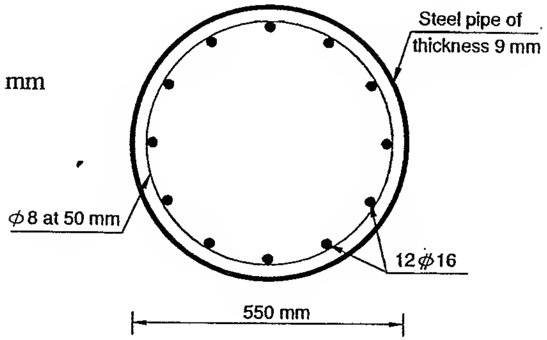
Assuming that bar diameter of the spiral is 8 mm, A_{sp}= 50 mm^2. Use the following equation to determine the stirrup pitch p

$$p = \frac{\pi\ A_{sp}\ D_k}{V_{sp}}$$

$$p = \frac{\pi\times 50\times 500}{1546.3} = 50.8\ mm$$

Round to the smallest pitch p=50 mm

p<80 mm and p>30 mm ….o.k.



6.5 Calculation of Axial Loads on Columns

The axial load on a column is the sum of the total loads that comes from all structural elements such as beams, slabs, and walls. There are two approaches for calculating the axial loads on columns namely, the area method and the reaction method.

6.5.1 Area Method

The first method is the area method in which the column is assumed to carry loads acting on an area bounded by the centerline of the previous bay to the centerline of the next bay in both directions as shown in Fig. 6.11. This method is very effective in calculating loads on columns with a symmetrical layout or on columns with flat slab floors.

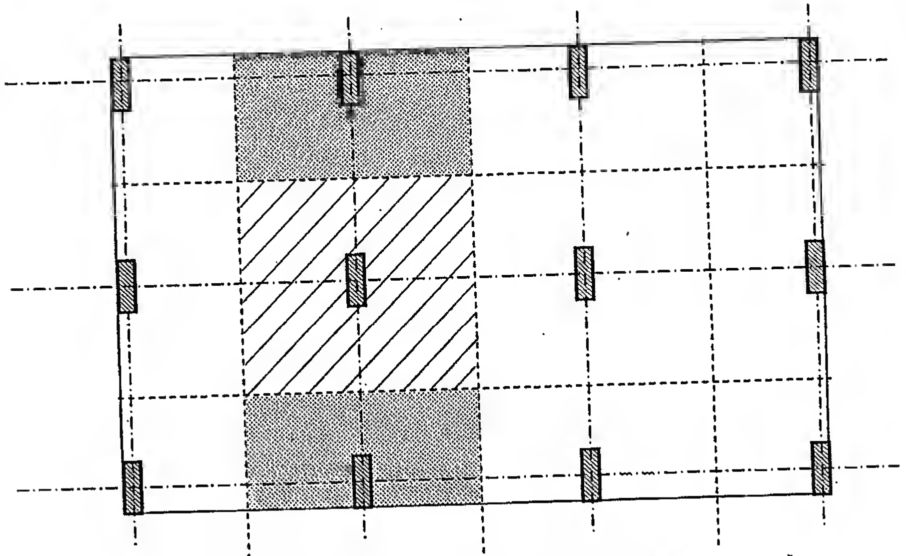


Fig. 6.11 Calculations of column load using area method

This method can be used for floors containing projected beams. The reaction is calculated by ignoring the effect of continuity and treating every beam as a simple span. This method should be limited to plans with nearly equal spans. The effect of continuity at point B can be implemented by multiplying the reaction by 1.1 as shown in Fig 6.12.

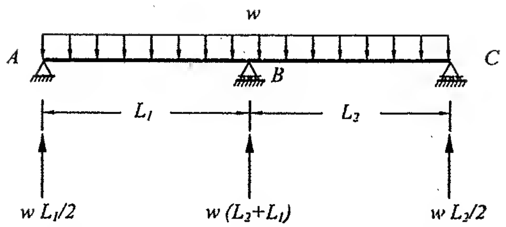


Fig. 6.12 Effect of continuity on the calculation of column load

However, this method may lead to serious errors in buildings where large differences exist in the adjacent spans either for flat slabs or slab beams floors. This is attributed to neglecting the effect of continuity especially for unequal spans. For example, the reaction on column B (shown in Fig. 6.13) when calculated by the area method gives 80 kN whereas the reaction obtained from the structural analysis equals 126 KN (about 57% error)

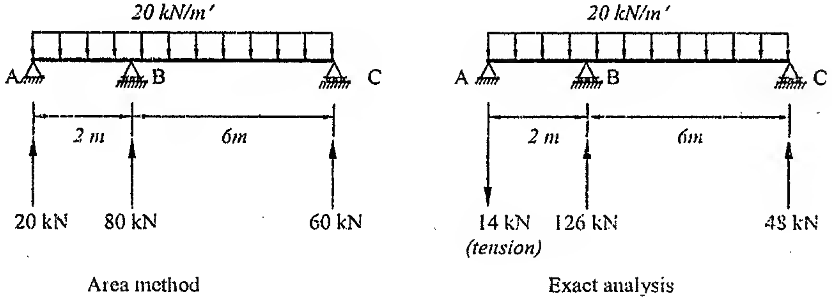


Fig. 6.13 Comparison between area method and exact analysis (where the use of area method is not recommended)

6.5.2 Reaction Method

This method depends on the exact structural analysis of the structure either using a computer analysis program or classical structural analysis. The reactions are calculated from the shear loads. When using the computer, the whole structure is modeled and the reactions are obtained from the final solution. For hand calculations, the structure is divided into individual beams and the reaction of each beam should be added to give the final column load.

The self-weight of the column should be added to these reactions or can be estimated as 5-10% of the column load.

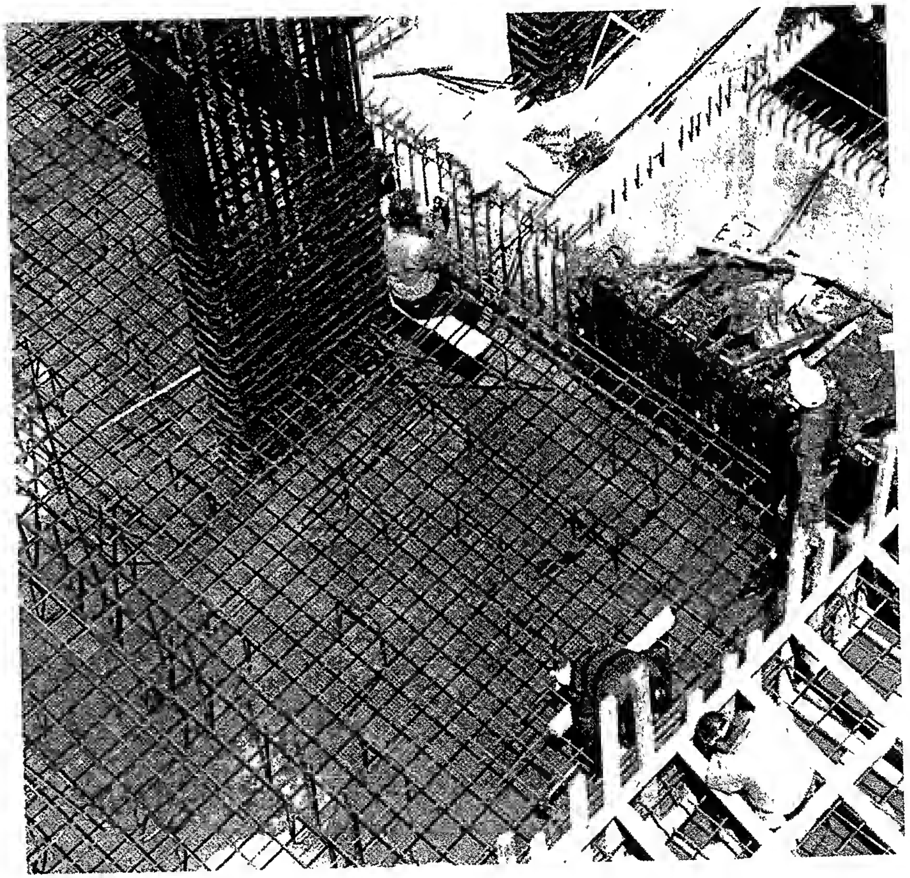


Photo 6.6 Column reinforcement placement in a high-rise building

7

DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC FORCES

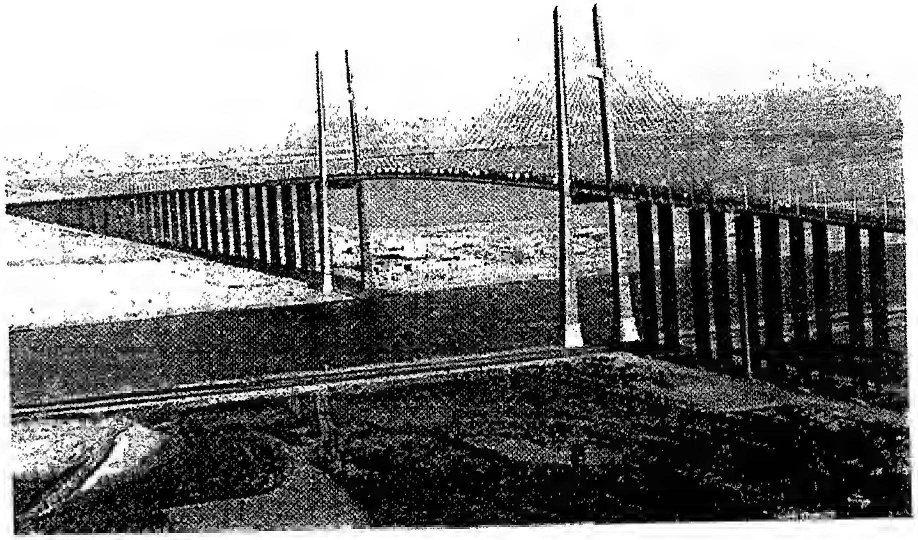
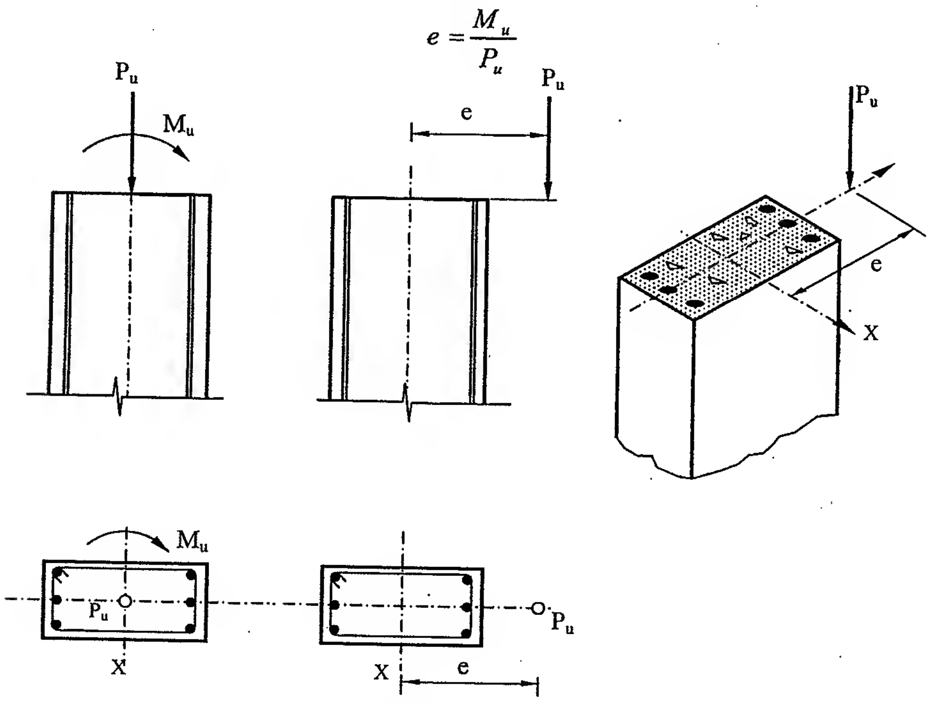


Photo 7.1 The bridge over Suez canal

7.1 Introduction

This chapter deals with the analysis and design of cross sections subjected to axial loads and bending moments. Concrete sections may be subjected to eccentric compression or eccentric tension. The eccentricity of the load could be in one direction *"uniaxial"* or in two directions *"biaxial"*.

The behavior of the sections under combined axial compression loads and bending moments depends on the magnitude of the moment M_u and the axial force P_u. If M_u is relatively small compared to P_u, the eccentricity e will be small and the section will be subjected to a small eccentricity. In this case, most of the section will be in compression and column behavior will dominate. On the contrary, if M_u is large the eccentricity e will be large enough so that the normal force will be outside the cross section and the section will be subjected to a big eccentricity. In this case, the near side of the section will be subjected to compression and the far side will be subjected to tension, and beam behavior will dominate. The combination of an axial load P_u and bending moment M_u is equivalent to a load P_u applied at eccentricity $e = M_u / P_u$ as shown in Fig. 7.1.



(a) Axial load and moment (b) Eccentric Load (c) Eccentric Load

Fig. 7.1 Representation of axial load and bending moment

7.2 Interaction Diagrams

7.2.1 Definition

The interaction diagram or *"the failure envelope"* of a reinforced concrete cross-section contains the different combinations of M_u and P_u that result in the failure of the cross section as shown in Fig. 7.2. Thus, an interaction diagram is a graphical representation of all possible combinations of axial loads and bending moments that cause failure for a given cross-section. In order to develop the interaction diagram, one has to know the concrete dimensions of the section, the longitudinal reinforcement, f_{cu} and f_y.

Developing the interaction diagram of a reinforced concrete cross-section can be achieved through the application of:

1. Compatibility of strains.
2. Equilibrium of forces and moments.

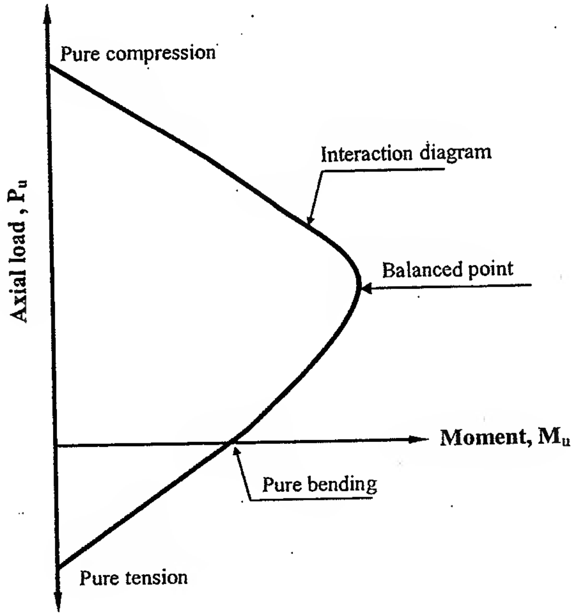


Fig. 7.2 Interaction diagram of a reinforced concrete cross section

7.2.2 Modes of Failure

Fig. 7.3 presents a series of strain distributions and the resulting points on the interaction diagram. The state of stress developed in the concrete and steel controls the type of failure. These modes are explained as follows

7.2.2.1 Compression failure mode

Point A corresponds to the case of pure axial loading and point A′ represents the case of axial loading with code minimum eccentricity (i.e. e/t=0.05). Point B represents the case of compression failure mode where the concrete reaches its ultimate strain of 0.003 and the tension steel does not yield. This failure mode is brittle, as the column fails as soon as the concrete compressive strain reaches 0.003 without large deformations or sufficient warning. Accordingly, the Egyptian code increases the strength reduction factors for concrete and steel for such modes of failure.

This mode of failure belongs to columns with a relatively small bending moment.

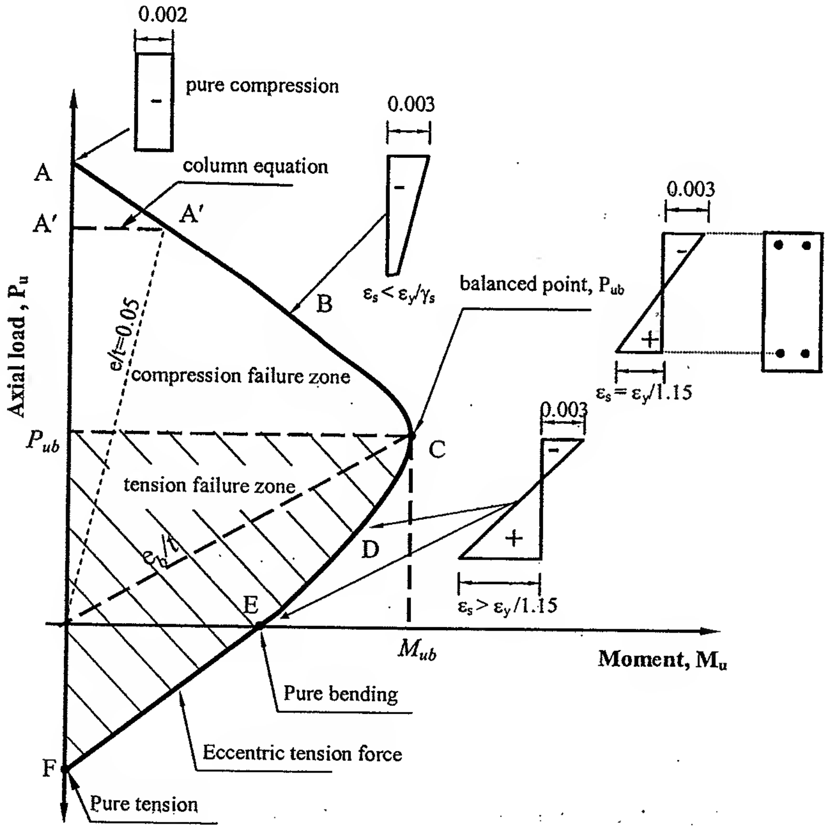


Fig. 7.3 Modes of failure for a section subjected to eccentric forces

7.2.2.2 Balanced Failure Mode

At this point the concrete reaches its ultimate strain of 0.003 at the same time that the tension steel reaches (f_y/1.15) (point C). The maximum bending moment capacity for the section occurs at this point (M_{ub}). Loads larger than the balanced load P_{ub} cause compression failure and loads smaller than the balanced load P_{ub} cause tension failure. The position of the neutral axis c_b can be obtained from the following equation:

$$c_b = \frac{690}{690 + f_y (N/mm^2)} \qquad \text{.......... (7.1)}$$

7.2.2.3 Tension failure mode

This is represented by point D on the interaction diagram. The strain in the steel is larger than the yield strain and the steel will reach the yielding stress. This is a ductile mode of failure since the section will crack and develop large deformations before failure, thus giving sufficient signs of warning before the complete collapse. This behavior is similar to that of beams rather than columns. If the axial force is equal to zero, the section will reach the case of pure bending represented by point E. Both the axial loads and moment capacities increase in this zone up to the balanced point.

Point F corresponds to the section axial tension capacity. The part E-F represents the section capacity under the combined axial tension and bending. The modes of failure and steel stress can be established from Eq. 7.2 and Table 7.1 as follows:

$$\text{if } P_u > P_{ub} \begin{cases} \text{Compression Failure} \\ f_s < f_y/\gamma_s \text{ (steel did not yield)} \\ c > c_b \\ e < e_b \\ M_u < M_{ub} \\ \gamma_s \geq 1.15 \text{ and } \gamma_c \geq 1.50 \end{cases} \qquad \text{..........(7.2a)}$$

$$\text{if } P_u < P_{ub} \begin{cases} \textit{Tension Failure} \\ f_s = f_y/\gamma_s \text{ (steel yields)} \\ c < c_b \\ e > e_b \\ M_u < M_{ub} \\ \gamma_s = 1.15 \text{ and } \gamma_c = 1.50 \end{cases} \qquad \text{..........(7.2b)}$$

Table 7.1 Steel stresses according to the type of failure

Type of failure	Tension steel stress	Compression steel stress
Tension failure $P_u<P_{ub}$	yields, $f_s=f_y/1.15$	yields, $f_s'=f_y/1.15$, or does not yield $f_s'=600\frac{c-d'}{c}$
Compression failure $P_u>P_{ub}$	does not yield, $f_s=600\frac{d-c}{c}$	yields, $f_s'=f_y/\gamma_s$, or does not yield, $f_s'=600\frac{c-d'}{c}$

7.2.3 Development of the Interaction Diagram

In this section the relationships needed to calculate different points on the interaction diagram are derived using the principles of strain compatibility and equilibrium of forces. The procedure followed to obtain a point on the interaction diagram is as follows:

- Concrete strain at the outermost stressed fiber of the section is assumed to be equal to 0.003.
- Because it is difficult to solve the equilibrium equations for a specific axial load and bending moment, the neutral axis distance c is usually assumed and the developed forces and moments in the concrete and steel are evaluated as shown in Fig. 7.4a. Incrementing the neutral axis distance for enough points gives the interaction diagram as shown in Fig. 7.3.
- According to the ECP-203, the strength reduction factors change depending on the applied eccentricity (*mode of failure*) and are given by

$$\gamma_c=1.5\times\left(\frac{7}{6}-\frac{e/t}{3}\right)\geq 1.5 \quad \text{.......................................} (7.3)$$

$$\gamma_s=1.15\times\left(\frac{7}{6}-\frac{e/t}{3}\right)\geq 1.15 \quad \text{..................................} (7.4)$$

Note: if $e/t>0.50$ then $\gamma_s=1.15$ and $\gamma_c=1.50$

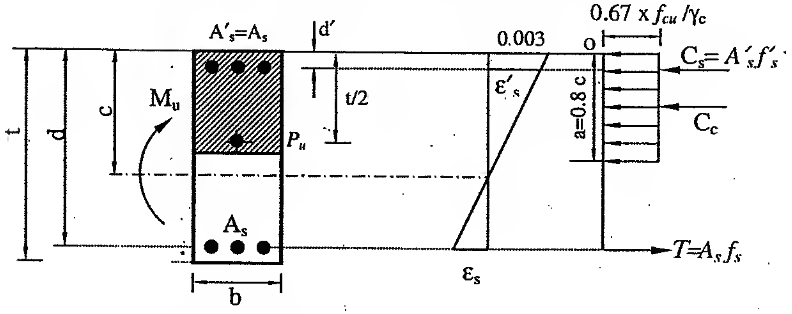


Fig. 7.4a Strain and stress distribution eccentrically loaded sections

- Steel strains in both compression and tension steel are calculated using similarity of triangles. The developed stress using the computed strain is checked to ensure the yielding of the steel. Noting that Young's steel modulus equals 200,000 N/mm^2, the stress in the tension steel is obtained using the following ***compatibility equation***

$$f_s=E_s\times 0.003\times\frac{d-c}{c}=600\frac{d-c}{c}\leq\frac{f_y}{\gamma_s} \quad \text{(tension if positive) ....} (7.5)$$

$$\boxed{f_s=600\frac{d-c}{c}\leq\frac{f_y}{\gamma_s}}$$

Similarly, the stress in the compression steel equals

$$\boxed{f_s'=600\frac{c-d'}{c}\leq\frac{f_y}{\gamma_s}} \quad \text{(compression if positive) ....} (7.6)$$

The values of strength reduction factors and steel stresses are illustrated in Fig.7.4b. It is clear from this figure that $f_s=f_y/1.15$ if the point is in the ductile failure mode (below the balanced point) and Eq. 7.5 should if the point in the brittle failure zone (above the balanced point).

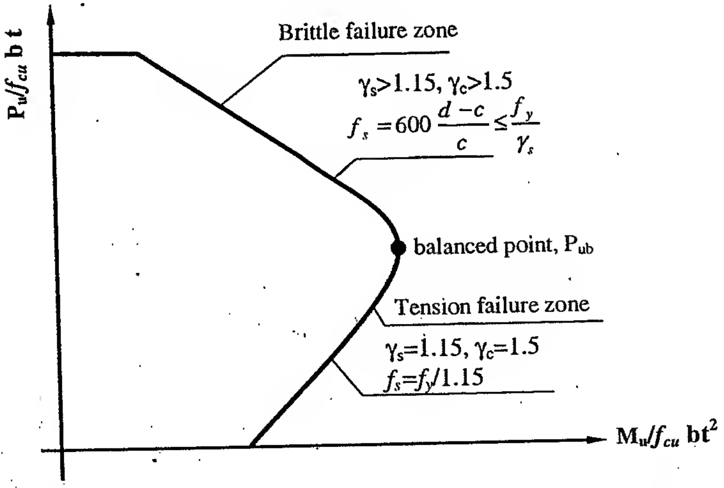


Fig. 7.4b Calculation of tension steel stress(f_s) in developing interaction diagram

The equivalent stress block distance (a) is assumed to be equal to 0.8c as permitted by the code, and the corresponding concrete force is evaluated. The total axial capacity is given by the following equilibrium equation

$$P_u = C_c + C_s - T \quad \text{...} (7.7a)$$

$$P_u = \frac{0.67 f_{cu}\, b\, (0.8c)}{\gamma_c} + A_s' \times f_s' - A_s \times f_s \quad \text{..........................} (7.7b)$$

- For symmetrical sections, the bending moment capacity of the section is obtained by taking the moment of the individual forces about the c.g as follows

$$M_u = \frac{0.67 f_{cu}\, b\, (0.8c)}{\gamma_c} \times \left(\frac{t}{2} - \frac{a}{2}\right) + A_s' \times f_s' \left(\frac{t}{2} - d'\right) + A_s \times f_s \left(\frac{t}{2} - d'\right) \quad (7.8)$$

- It should be clear to the reader that the corresponding bending moment can be computed at any point in the cross-section similar to the case of pure bending, but with one major difference in that the axial load is located at the c.g along with the bending moment. Thus, if the bending is calculated at any point rather than the c.g (or plastic centroid in case of unsymmetrical sections), the normal force should be included. For example, the bending moment at point "o" in Fig. 7.4 equals

$$M_u = P_u \times \frac{t}{2} - \frac{0.67 f_{cu}\, b\, (0.8c)}{\gamma_c} \times \frac{a}{2} - A_s' \times f_s' \times d' + A_s \times f_s \times d \quad \text{......} (7.9)$$

An example of an interaction diagram obtained using the aforementioned procedure is shown in Fig. 7.8.

7.2.4 Plastic Centroid

Most of reinforced concrete columns are symmetrically reinforced. However, in the cases where the eccentricity is large, it is more economical to place most of the reinforcement on the tension side. In reinforced concrete sections with unsymmetrical reinforcement the load must pass through a point known as the plastic centroid. The plastic centroid is defined as the point of application of the resultant force(p_{up})when the column is compressed uniformly to the failure strain. Eccentricity must be measured with respect to the plastic centroid as shown in Fig. 7.5. The strength reduction factors γ_c and γ_s are taken as 1.75 and 1.34 receptively

For symmetrically reinforced sections, the plastic centroid coincides with the center of gravity. The procedure for calculating the plastic centroid is illustrated in example 7.2

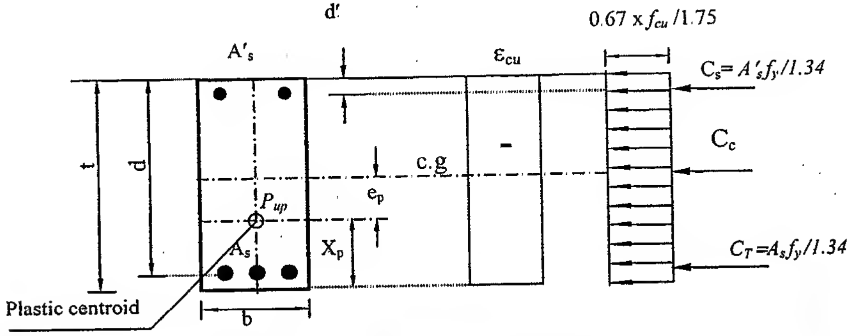


Fig. 7.5 Plastic centroid for sections with unsymmetrical reinforcement

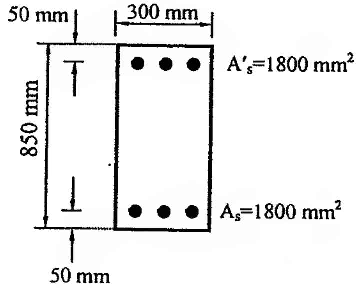
Photo 7.2 Unconventional support for a Multifamily Building -Warsaw

Example 7.1

Calculate the balanced load and balanced moment for the section shown in figure knowing that the material properties are

$f_{cu}=35\ N/mm^2$

$f_y=400\ N/mm^2$



Solution

Step 1: Calculate C_b

The position of the neutral axis position at balanced failure is given by

$$c_b=\frac{690}{690+f_y}d=\frac{690}{690+400}800=506.42\ mm$$

$$a_b=0.8\times c=0.8\times 506.42=405.14\ mm$$

Assume that γ_c=1.5,γ_s=1.15 (e/t>0.5) *(will be verified later)*

$$f_s'=600\frac{c_b-d'}{c_b}=600\frac{506.42-50}{506.42}=540.76\ N/mm^2>>\frac{400}{1.15}\text{......steel yields}$$

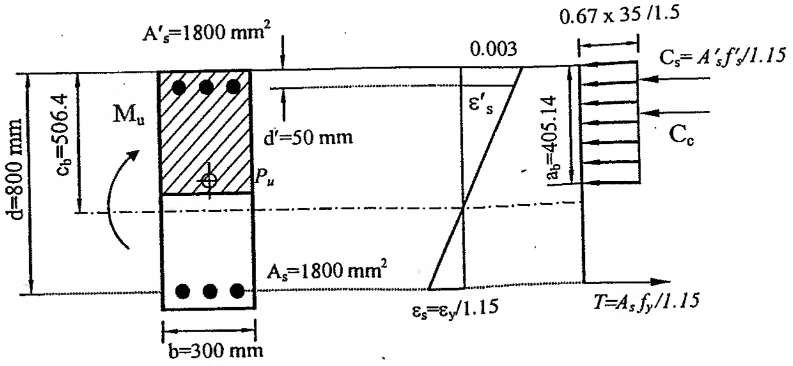
$$f_s'=\frac{400}{1.15}$$

Step 2: Calculate the forces

$$C_c=\frac{0.67f_{cu}\times b\times a_b}{1.5}=\frac{0.67\times 35\times 300\times 405.14}{1.5}\frac{1}{1000}=1900.1kN$$

$$C_s=A_s'\times\frac{f_y}{1.15}=1800\times\frac{400}{1.15}\times\frac{1}{1000}=626.1\ kN$$

$$T=A_s\times\frac{f_y}{1.15}=1800\times\frac{400}{1.15}\times\frac{1}{1000}=626.1\ kN$$



Step 3: Calculate the bending moment

Computing moment at plastic centroid (c.g. in this case)

$$M_{ub}=C_c(\frac{t}{2}-\frac{a_b}{2})+C_s(\frac{t}{2}-d')+T(\frac{t}{2}-d')$$

$$M_{ub}=\left[1900.1\left(\frac{850}{2}-\frac{405.14}{2}\right)+626.1\left(\frac{850}{2}-50\right)+626.1\left(\frac{850}{2}-50\right)\right]\frac{1}{1000}=892.21\ kN.m$$

$$\frac{e_b}{t}=\frac{892.21}{1900.1\times 0.85}=0.552>0.50\text{...(our assumption that }\gamma_c\text{=1.5,}\gamma_s\text{=1.15 is correct)}$$

Note: The bending moment can be taken at any other point as long as the ultimate load P_u is considered, thus

$$M_{ub}=\left[1900.1\times\frac{850}{2}-1900.1\times\frac{405.14}{2}-626.1\times 50+626.1\times 800\right]\frac{1}{1000}=892.21\ kN.m$$

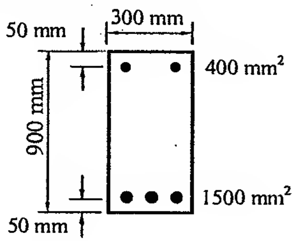
(same as before)

Example 7. 2

Locate the plastic centroid for the section shown in figure, knowing that

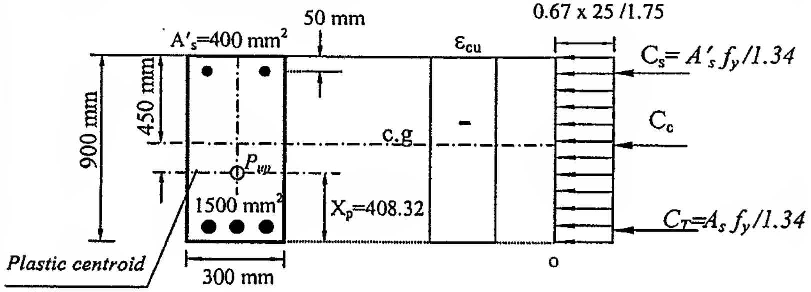
f_{cu} =25 N/mm^2

f_y =400 N/mm^2



Solution

To find the plastic centroid, the entire section is subjected to compression force, thus γ_c=1.75 and γ_s=1.34. Assume 50 mm concrete cover



$$C_s = A_s' \times \frac{f_y}{\gamma_s} = 400 \times \frac{400}{1.34} \times \frac{1}{1000} = 119.4\ kN$$

$$C_c = \frac{0.67 \times f_{cu} \times b \times t}{1.75} = \frac{0.67 \times 25 \times 300 \times 900}{1.75} \times \frac{1}{1000} = 2584.286\ kN$$

$$C_T = A_s \times \frac{f_y}{\gamma_s} = 1500 \times \frac{400}{1.34} \times \frac{1}{1000} = 447.76\ kN$$

The resultant of the three forces P_{up} equals

$$P_{up} = C_s + C_c + C_T = 119.4 + 2584.286 + 447.76 = 3151.45\ kN$$

Computing the moment of all forces at point o (bottom fibers)

$$P_{up} \times X_p = C_s \times 850 + C_c \times 450 + C_T \times 50$$

$$X_p = \frac{119.4 \times 850 + 2584.286 \times 450 + 447.76 \times 50}{3151.45} = 408.32\ mm$$

The distance of the plastic centroid to the c.g (e_p) equals

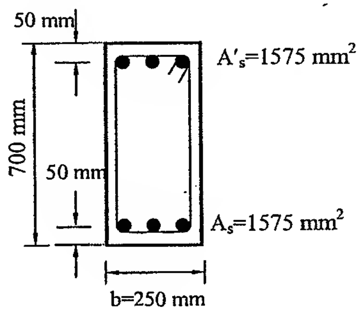
$$e_p = \frac{t}{2} - X_p = \frac{900}{2} - 408.32 = 41.68\ mm$$

Example 7.3

Construct the interaction diagram for the section shown in figure knowing that the material properties are

f_{cu} =30 N/mm^2

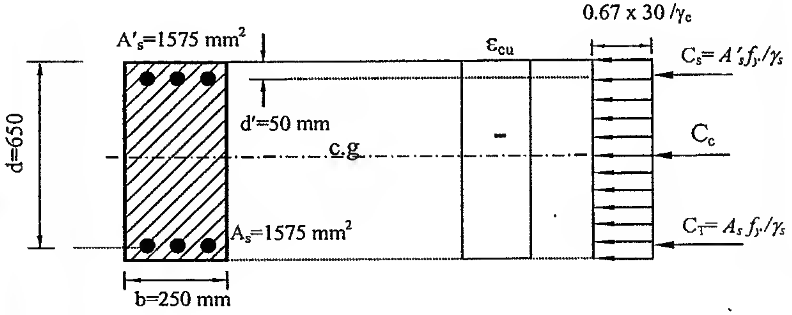
f_y = 400 N/mm^2



Solution

Point 1 (Pure axial compression)

The entire section is under axial compression and the neutral axis is considered at infinity. The strain distribution is uniform at the ultimate strain and all the steel has yielded in compression. Summing all the forces gives the total section capacity



$$P_u = \frac{0.67\, f_{cu}\, b\, t}{\gamma_c} + A_s' \times \frac{f_y}{\gamma_s} + A_s \times \frac{f_y}{\gamma_s}$$

Since the section is in pure compression e=0 thus γ_c=1.75 and γ_s=1.34

$$P_u = \frac{0.67 \times 30 \times 250 \times 700}{1.75} + 1575 \times \frac{400}{1.34} + 1575 \times \frac{400}{1.34} = 2950\ kN$$

Since the column has symmetrical reinforcement $M_u=0$

However, the ECP-203 does not permit the use of this capacity and assumes that any column will be subjected to a minimum eccentricity of (e_{min}=0.05 t). Thus, use the tied column equation

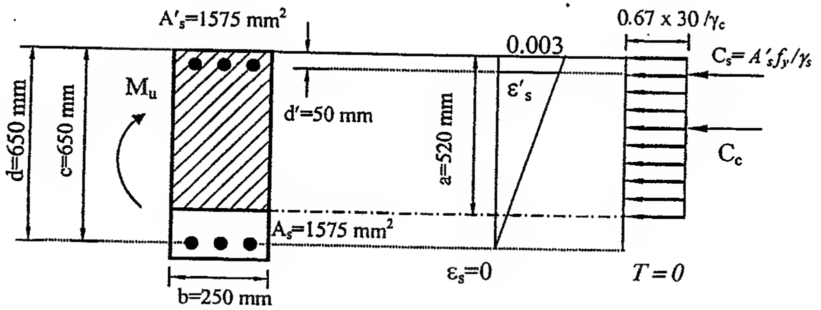
$$P_u = 0.35\ f_{cu} A_c + 0.67\ A_{sc}\ f_y$$

$$P_u = 0.35 \times 30 \times 250 \times 700 + 0.67\ (1575 + 1575)\ 400 = 2681\ kN$$

$$M_u = P_u . e_{min} = 2681 \times (0.05 \times 700) = 93835\ \text{Kn.mm} = 93.84\ \text{kN.m}$$

Point 2 (compression failure)

The compression failure occurs when the depth of the neutral axis is greater than its depth at the balanced position. In this situation the tension steel stress is below the yield stress and for the purpose of simplicity, the neutral axis position is chosen at the tension steel location (*c=d*). Thus, the developed force in the tension steel is equal to zero.



c=650 mm and a= 0.8 × c= 520 mm

To estimate the strength reduction factors γ_s and γ_c, assume that e/t=0.2 (will be verified later), thus

$$\gamma_c = 1.5 \times \left(\frac{7}{6} - \frac{0.20}{3}\right) = 1.65 \qquad \gamma_s = 1.15 \times \left(\frac{7}{6} - \frac{0.20}{3}\right) = 1.265$$

Since $c=d$ thus, $f_s=0$ and $T=0$

$$f'_s = 600 \frac{c - d'}{c} \le \frac{f_y}{\gamma_s}$$

$$f'_s = 600 \frac{650 - 50}{650} = 553.85 > \frac{400}{1.265}$$

$$\therefore f'_s = \frac{f_y}{\gamma_s} = \frac{400}{1.265} = 316.2\ N/mm^2$$

$$C_s = A'_s \times f'_s = 1575 \times 316.2 \times \frac{1}{1000} = 498\ kN$$

$$C_c = \frac{0.67 \times f_{cu} \times b \times a}{\gamma_c} = \frac{0.67 \times 30 \times 250 \times 520}{1.65} \times \frac{1}{1000} = 1583.6\ \ kN$$

$P_u = C_c + C_s - T = 1583.6 + 498 - 0 = 2081.6$ kN

Computing moment at plastic centroid (the c.g. in this case)

$$M_u = C_c\left(\frac{t}{2} - \frac{a}{2}\right) + C_s\left(\frac{t}{2} - d'\right) = \left[1583.6 \times (350 - 260) + 498 \times (350 - 50)\right]\frac{1}{1000} = 291.92\ kN.m$$

Check strength reduction factors

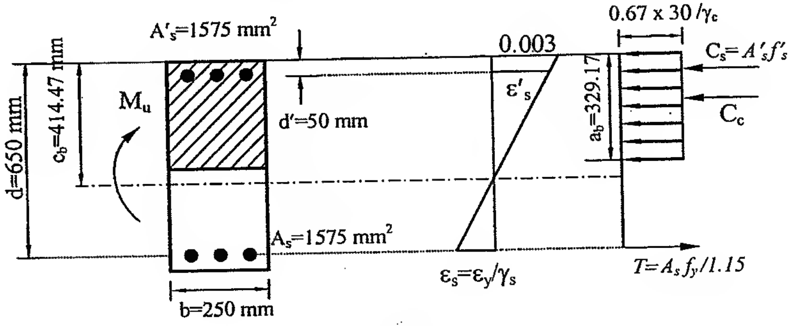
$$e = \frac{M_u}{P_u} = \frac{291.92}{2081.6} = 0.14$$

$$\gamma_c = 1.5 \times \left(\frac{7}{6} - \frac{0.14}{3}\right) = 1.68 \approx 1.65 \ldots o.k$$

$$\gamma_s = 1.15 \times \left(\frac{7}{6} - \frac{0.14}{3}\right) = 1.288 \approx 1.265 \ldots o.k$$

Point 3 (balanced point)

By definition at the balanced point the strain in the tension steel equals $\varepsilon_y/1.15$. Thus, the stress in the tension steel equals $f_y/1.15$.



$$c_b = \frac{690}{690+f_y} d = \frac{690}{690+400} \times 650 = 411.47\ mm$$

a= 0.8 c= 329.17 mm

Assume that e/t>0.50, thus γ_c=1.5 and γ_s=1.15 (will be verified later)

$$T = A_s \times \frac{f_y}{1.15} = 1575 \times \frac{400}{1.15} = 547.83\ kN$$

$$f_s' = 600 \frac{c-d'}{c} \le \frac{f_y}{\gamma_s}$$

$$f_s' = 600 \frac{411.47-50}{411.47} = 527 > \frac{f_y}{\gamma_s}$$

$$\therefore f_s' = \frac{f_y}{\gamma_s} = \frac{400}{1.15} = 347.83\ N/mm^2$$

$$C_s = A_s' \times f_s' = 1575 \times 347.83 = 547.83\ kN$$

$$C_c = \frac{0.67 \times f_{cu} \times b \times a}{\gamma_c} = \frac{0.67 \times 30 \times 250 \times 329.17}{1.50} = 1102.72\ \ kN$$

$P_u = C_c + C_s - T$ = 1102.72+547.83-547.83=1102.72 kN

Computing moment at plastic centroid (*i.e. c.g due to symmetry*)

$$M_u = C_c(\frac{t}{2} - \frac{a}{2}) + C_s(\frac{t}{2} - d') + T(\frac{t}{2} - d')$$

$$M_u = 1102.72 \times (350 - \frac{329.17}{2}) + 547.83 \times (350-50) + 547.83 \times (350-50) = 533.16\ kN.m$$

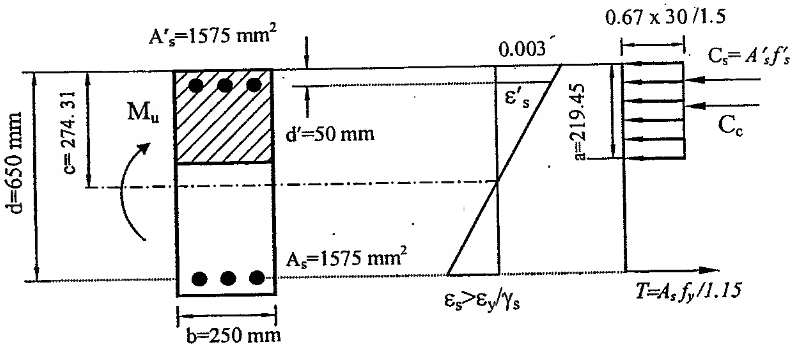
Check strength reduction factors

$$\frac{e}{t} = \left(\frac{M_u}{P_u \times t}\right) = \frac{533.16}{1102.7 \times 0.7} = 0.69$$

Since e/t > 0.5 our assumption that γ_c=1.5 and γ_s=1.15 is correct

Point 4 (tension failure)

For tension failure to occur, the neutral axis should be less than the balanced point. Let us assume the neutral axis position at c=2/3 c_b (position for maximum reinforcement in case of pure bending)



$$c = \frac{2}{3} c_b = \frac{2}{3} \times 411.47 = 274.31\ mm$$ →a= 0.8 c= 219.45 mm

γ_c=1.5 and γ_s=1.15 (no need to check this assumption, because it was valid at the balanced point)

The point is below the balanced load thus $f_s = \frac{f_y}{\gamma_s} = \frac{400}{1.15} = 347.83\ \ N/mm^2$

$$T = A_s \times f_s = 1575 \times 347.83 = 547.83\ kN$$

$$f_s' = 600 \frac{274.31-50}{274.31} = 490.6 > \frac{f_y}{\gamma_s}$$

$$\therefore f_s' = \frac{f_y}{\gamma_s} = \frac{400}{1.15} = 347.83\ N/mm^2$$

$$C_s = A_s' \times f_s' = 1575 \times 347.83 = 547.83\ kN$$

$$C_c = \frac{0.67 \times f_{cu} \times b \times a}{\gamma_c} = \frac{0.67 \times 30 \times 250 \times 219.45}{1.50} = 735.16\ \ kN$$

$P_u = C_c + C_s - T$ = 735.16+547.83-547.83=735.16 kN

Taking moment about plastic centroid (*i.e. c.g due to symmetry*)

$$M_u = C_c(\frac{t}{2} - \frac{a}{2}) + C_s(\frac{t}{2} - d') + T(\frac{t}{2} - d')$$

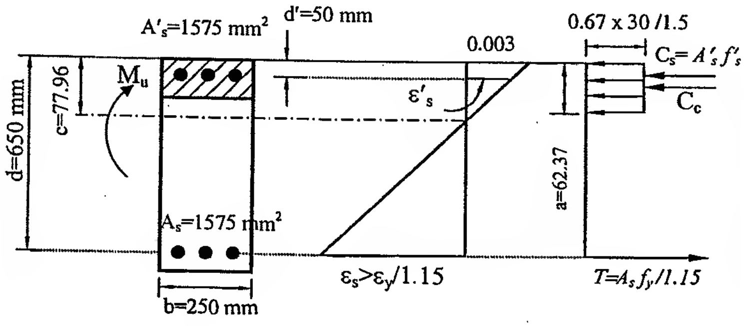
$$M_u = 735.16 \times (350 - \frac{219.45}{2}) + 547.83 \times (350-50) + 547.83 \times (350-50) = 505.34\ kN.m$$

Point 5 (Pure bending)

In the case of sections subjected to pure bending, locating the neutral axis must be performed by applying the equilibrium equation

$P_u = 0 = C - T$

γ_c=1.5 and γ_s=1.15 (no need to check this assumption, because at the balanced point this assumption was valid). Assume that compression steel did not yield.



$$f'_s = 600\frac{c-d'}{c} = 600\frac{c-50}{c}$$

$C_c + C_s - T = 0$ or $C_c + C_s = T$

$$\frac{0.67 f_{cu} \times b \times a}{1.5} + A'_s \times f'_s = \frac{A_s \times f_y}{1.15}$$

$$\frac{0.67 \times 30 \times 250 \times (0.8 \times c)}{1.5} + 1575 \times 600\frac{c-50}{c} = \frac{1575 \times 400}{1.15}$$

The previous equation is a second order equation in c, its solution results in c=77.96 mm $\to\to$ a=0.8 c = 62.37 mm

$$f'_s = 600\frac{c-d'}{c} = 600\frac{77.96-50}{77.96} = 215.18 < \frac{400}{1.15} \ldots o.k\ (\textit{did not yield as assumed})$$

$$T = A_s \times f_s = 1575 \times 347.83 = 547.83\ kN$$

$$C_s = A'_s \times f'_s = 1575 \times 215.18 = 338.90\ kN$$

$$C_c = \frac{0.67 \times f_{cu} \times b \times a}{\gamma_c} = \frac{0.67 \times 30 \times 250 \times 62.37}{1.50} = 208.93\ kN$$

Computing moment at the c.g.

$$M_u = C_c(\frac{t}{2} - \frac{a}{2}) + C_s(\frac{t}{2} - d') + T(\frac{t}{2} - d')$$

$$M_u = 208.93 \times (350 - \frac{62.37}{2}) + 338.90 \times (350 - 50) + 547.83 \times (350 - 50) = 332.63\ kN.m$$

$P_u = C_c + C_s - T = 0$

Point 6 (pure axial tension)

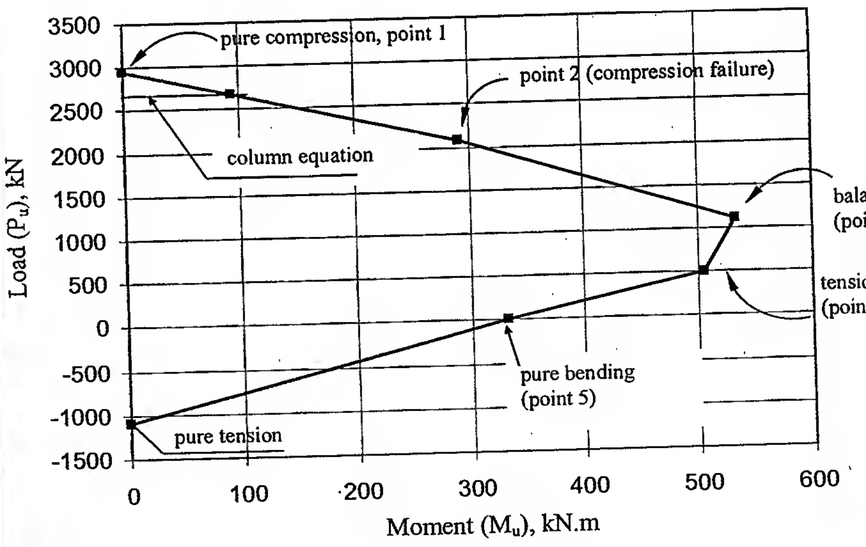
The final loading case to be considered in this example is the concentric tension. The strength of the concrete is completely neglected and the steel is in pure tension. Noting that the tension force is negative, P_u equals

$$P_u = \frac{-f_y}{1.15}(A_s + A'_s)$$

$$P_u = \frac{-400}{1.15}(1575 + 1575) = -1095.7\ kN$$

M_u=0 because the section is symmetrical

The interaction diagram for the previous column is drawn in the following Figure.



Interaction diagram for Example 7.3

7.3 Sections Subject to Eccentric Compression Forces

To design an eccentric section, one has to know the applied forces, moments and material properties as shown in the frame below.

Design Problem

Given : P_u, M_u, f_{cu}, f_y
Required : b, t, A_s, A'_s

Assume : b, t
Determine : A_s, A'_s

Three approaches are available for the design of sections subjected to eccentric compression force:

1. Interaction diagrams
2. M_{us} approach.
3. Design using curves

A trial section (b, t) is assumed and the reinforcement areas (A_s, A'_s) are determined. Selecting a trial cross section for members subjected to eccentric forces is not an easy task. However, when the eccentricity of the member is small, an approximate trial area can be established by designing the section as if it is subjected to axial loads only. On the other hand, when the eccentricity of the member is large, the trial area can be obtained by designing the section as if it is subjected to bending moments only.

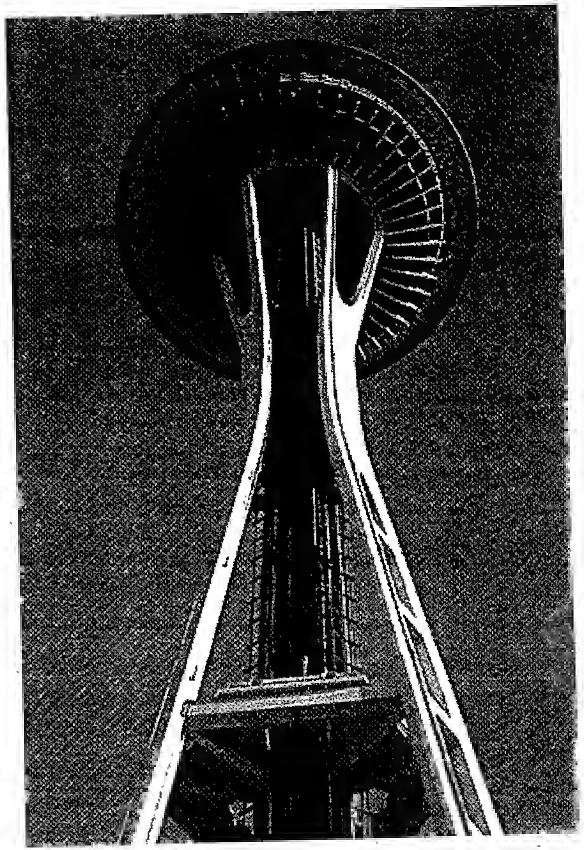


Photo 7.3 Eccentrically loaded section in a reinforced concrete tower

7.3.1 Design Using Interaction Diagrams

Interaction diagrams can also be used as design tools when prepared in special forms. The horizontal axis in the design interaction diagrams represents normalized (*non-dimensional*) bending moment $M_u/f_{cu}b\ t^2$, while the vertical axis represents the normalized axial load $P_u/f_{cu}\ b\ t$. These interaction diagrams are valid for designing members subjected to eccentric compression forces, whether it fails in compression or tension. However, they were prepared for sections with compression steel of more than 40% of the tension steel (i.e:α= 0.40, 0.60, 0.80, 1). Thus if the designer need to use lower ratio of the compression reinforcement *(especially in tension failure zone)*, the use of the interaction diagram is not attainable.

The interaction diagram is usually divided into four zones as follows (refer to Fig. 7.6)

1- Zone A ($e/t<0.05$): In this zone, the design of sections is attained by applying the equation for tied columns and the applied moment is ignored.

2- Zone B (compression failure $P_u>P_b$): The design of sections in this zone is performed by using the interaction diagrams directly.

3- Zone C (tension failure $P_u<P_b$) : The design of sections can be performed according to the ratio of the compression reinforcement as follows:

   a) If the ratio of the compression steel is equal or greater than 0.4, the use of interaction diagrams is preferred.

   b) If the ratio of the compression steel is smaller than 0.4, (1)approximate methods (M_{us})can be used (as explained later) or (2) eccentric design charts. The use of the approximate methods (M_{us}) should be limited to sections subjected to relatively small compression forces otherwise it may lead to unsafe designs *(as explained in the next section).*

4- Zone D ($P_u/f_{cu}\ b\ t<0.04$): In this zone, the compression force is completely ignored and the section may be designed as if it is subjected to moment only.

$$\xi=\frac{t-2\times\text{cover}}{t}=\frac{d-d'}{t}$$

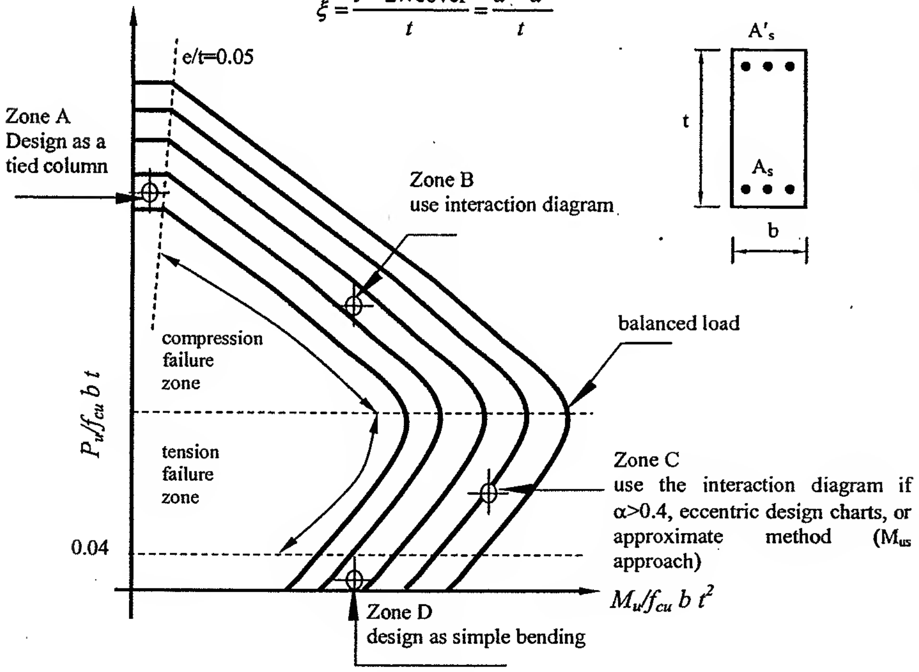


Fig. 7.6 Using interaction diagrams and design zones

The use of the interaction diagrams can be summarized in the following steps

1) Estimate the cross section dimensions if not given using a trial section
2) Determine the required diagram with the given f_y, α and ζ

$$if\ \frac{e}{t}\le 0.2 \qquad use\ uniform\ steel\ chrats$$

$$if\ \frac{e}{t}(0.2-0.5) \qquad use\ top\ and\ bottom\ steel\ \alpha=1$$

$$if\ \frac{e}{t}>0.5 \qquad use\ top\ and\ bottom\ steel\ \alpha=0.4-1$$

3) Calculate the following terms

$$\frac{P_u}{f_{cu}\ b\ t},\ \frac{M_u}{f_{cu}\ b\ t^2}$$

4) Locate the reinforcement ratio ρ, and interpolate if required.
5) Calculate area of steel using

$$\mu=\rho\times f_{cu}\times 10^{-4},\quad A_s=\mu\times b\times t \text{ or } A_{s,total}=\mu\times b\times t$$

6) Compute the area of the compression steel *(case of top and bottom steel)*

$$A_s'=\alpha\, A_s$$

7) Check that the total area of steel is higher than code minimum requirement

$$A_s'+A_s>0.008\, b\times t \qquad \text{(columns)}$$

$$A_s>0.225\sqrt{f_{cu}}/f_y\ b\times d \text{ and } (1.3\ A_s) \qquad \text{(beams)}$$

If the point falls inside the interaction diagram (point a), the minimum area of steel should be used as shown in Fig. 7.7. If the point falls within the diagram (point b), the design of the cross-section should be carried out normally. However, if the point falls outside the boundaries of the interaction diagram (point c), this indicates that the cross-section dimensions are inadequate and must be increased.

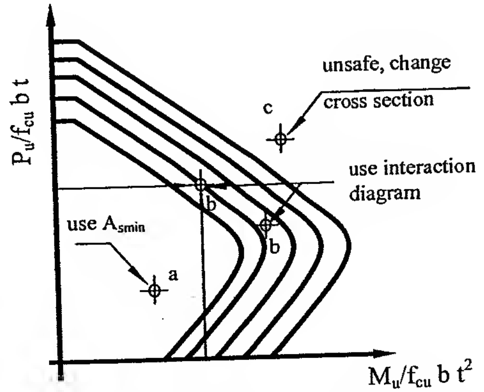


Fig. 7.7 The use of the interaction diagram

Appendix B contains interaction diagrams that can be used in design of eccentrically loaded sections with top and bottom reinforcement. An example of such diagrams is shown in Fig. 7.8. Appendix C contains interaction diagrams for sections with uniform reinforcement.

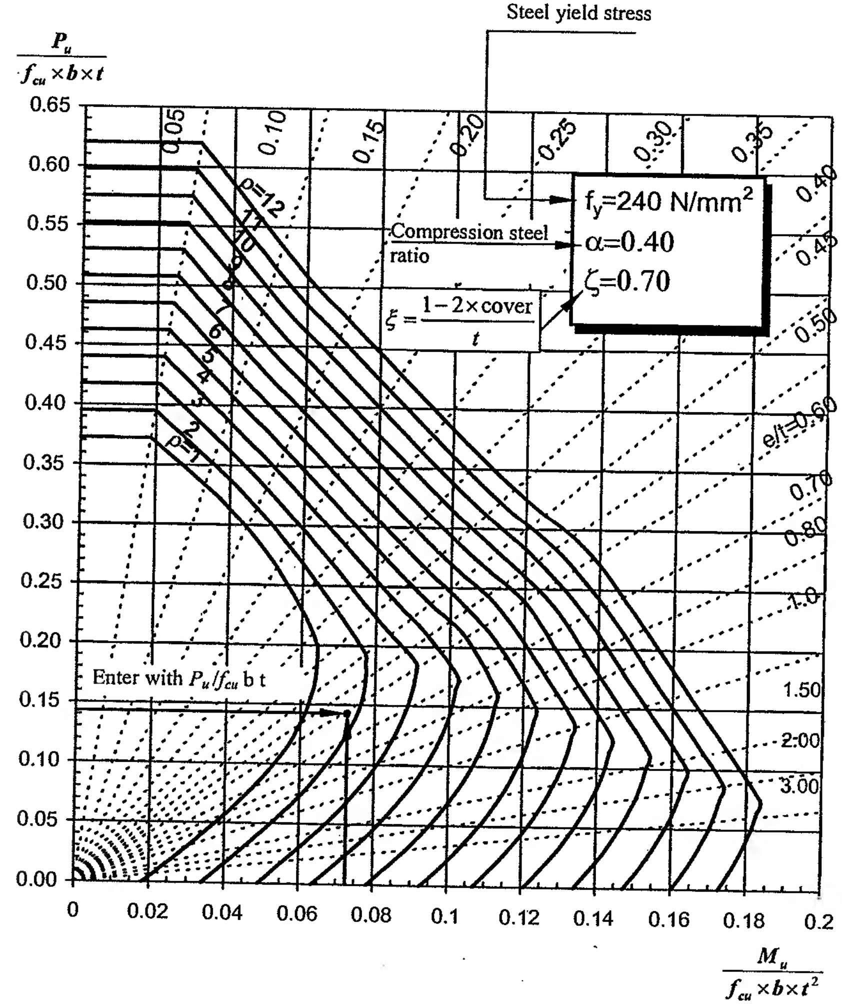


Fig. 7.8 An example of an interaction diagram with top and bottom reinforcement(Appendix B)

Example 7.4

Design a reinforced concrete column using interaction diagrams knowing that it is subjected to the following straining actions

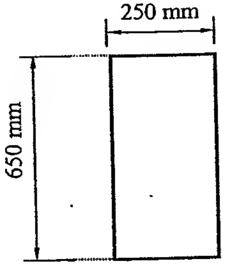
P_u=1400 kN.

M_u=295 kN.m

The material properties are

f_{cu} =30 N/mm^2

f_y =280 N/mm^2



Solution

Step 1: Determine the suitable design interaction diagram

$$\frac{e}{t}=\frac{M_u}{P\times t}=\frac{295}{1400\times 0.65}=0.32$$

since e/t =0.2→0.5 use α=1.0

Assume cover=40 mm

$$\xi=\frac{t-2\times cover}{t}=\frac{650-2\times 40}{650}\cong 0.9$$

Using chart in Appendix B with ζ=0.9, f_y=280 N/mm^2

Step2: Calculate the non-dimensional terms

$$\frac{P_u}{f_{cu}\,b\,t}=\frac{1400\,x\,10^3}{30\times 250\ \times 650}=0.287$$

$$\frac{M_u}{f_{cu}\,b\,t^2}=\frac{295\times 10^6}{30\times 250\ \times 650^2}=0.093$$

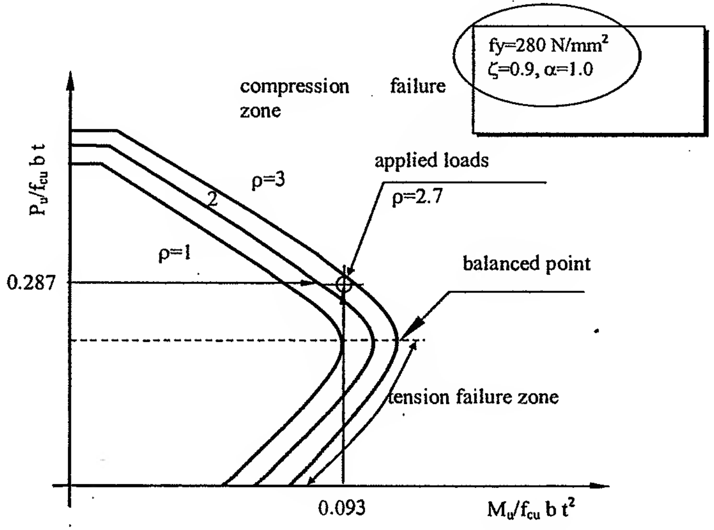
Step3: Calculate As, A's

Locating the point in the chart and determining ρ =2.8, (compression failure)

$\mu=\rho\ f_{cu}\ 10^{-4}=2.8\times 30\times 10^{-4}=0.0084$

$A_s=\mu\ b\ t=0.0084\times 250\times 650=1365$ mm^2 →(4ϕ22)

$A'_s=\alpha\ A_s=1\times 1365=1365$ mm^2 →(4ϕ22)

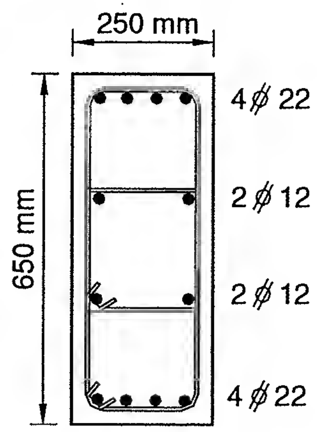


Step 4: Check A_{smin}

The minimum area steel for short columns is 0.008

$A_{s,\min} = 0.008 \times b \times t = 0.008 \times 250 \times 650 = 1300\ mm^2$

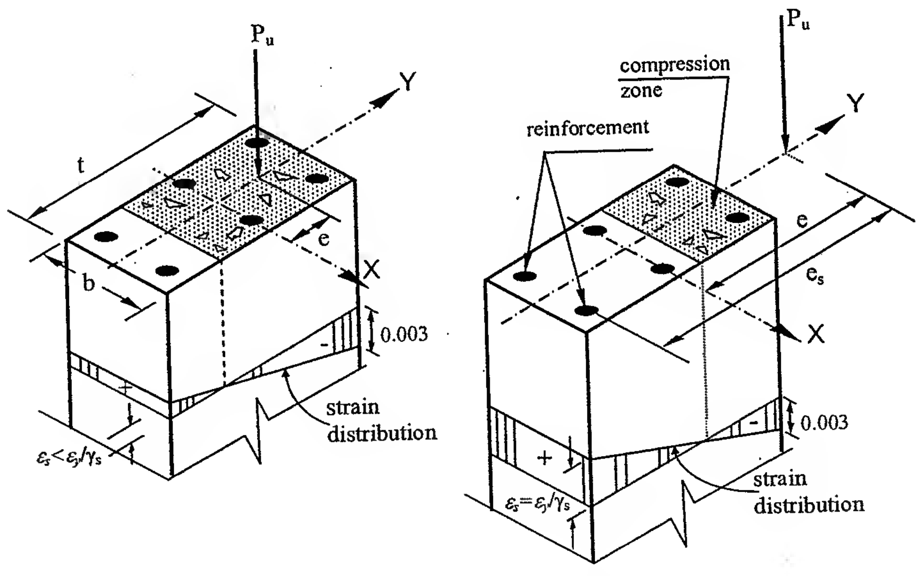
$A_{s,total} = A_s + A'_s = 1365 + 1365 = 2730\ mm^2 > A_{s,min}$ ....o.k



7.3.2 Design Using M_{us} Approach

If the eccentricity of the applied compression load is relatively small (above the balanced point), the tension steel does not yield and the section lies on the compression failure zone as shown in Fig. 7.9.a. However, all points lower than the balanced failure point represent a case in which the section is partially cracked and the strain in the tension steel is greater than the yield strain as shown in Fig 7.9.b. This is defined as the tension failure zone. Although it is possible to derive a family of equations to evaluate the strength of sections subjected to combined bending and axial force, these equations are tedious to use and cumbersome. Interaction diagrams can be used to design sections in the tension failure zone. The available interaction diagrams were prepared for sections having equal amounts of steel on both sides or with a maximum difference of 40 percent between area of tension steel and compression ($\alpha = A'_s/A_s = 0.4$). For sections in the tension failure zone, it is more economical to place most of the reinforcement in the tension side.

There is analytical method for designing such type of sections called the M_{us} approach. The approximation in this method comes from neglecting the compression steel contribution in the calculation.



a- Compression failure

b-Tension failure

Fig. 7.9 Strain distributions of sections subjected to eccentric compression

In the M_{us} approach, the moment is taken about tension steel and called M_{us} as given in Eq. 7.10. It should be emphasized that the balanced load has to be evaluated to verify that the type of failure is a tension failure.

Referring to Fig. 7.10, the external moment M_{us} about the tension steel equals to:

$$M_{us} = P_u \times e_s \qquad \text{...} (7.10)$$

where

$$e_s = e + \frac{t}{2} - \text{cover} \qquad \text{..} (7.11)$$

or alternatively M_{us} can be written as

$$M_{us} = M_u + P_u \ (\frac{t}{2} - \text{cover}) \qquad \text{...............................} (7.12)$$

Taking moment of the internal forces about the tension steel and equating it to the external moment M_{us}

$$M_{us} = \frac{0.67 f_{cu}\, b\, a}{1.50}(d - \frac{a}{2}) + A'_s \frac{f_y}{1.15}(d - d') \qquad \text{.................} (7.13)$$

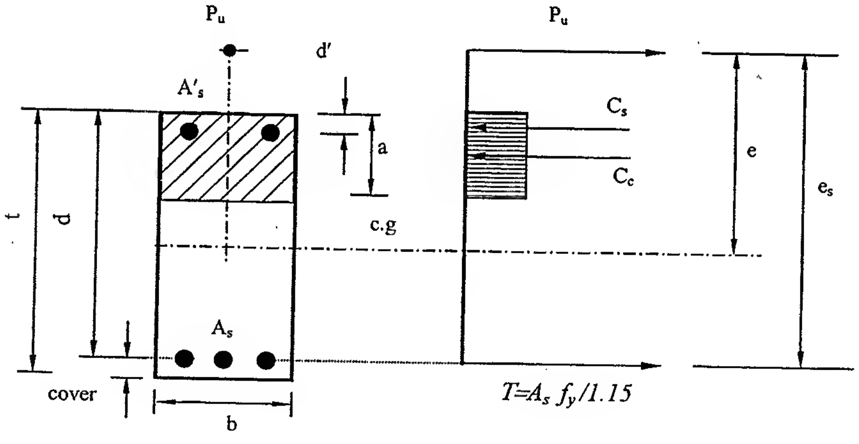


Fig. 7.10 Design of sections subjected to tension failure

Equilibrium of internal forces gives

$$A_s \frac{f_y}{1.15} = \frac{0.67 f_{cu}\, b\, a}{1.50} - P_u + A'_s \frac{f_y}{1.15} \qquad \text{..} (7.14)$$

Substituting with M_{us} expression in the previous equation gives

$$A_s = \frac{M_{us}}{(d - \frac{a}{2})\, f_y/1.15} - \frac{P_u}{f_y/1.15} + A'_s\left(1 - \frac{(1 - d'/d)}{(1 - a/2d)}\right) \qquad \text{....................} (7.15)$$

To simplify the previous expression, the third term in Eq. 7.15 is neglected

$$A_s = \frac{M_{us}}{(d - \frac{a}{2})\, f_y/1.15} - \frac{P_u}{f_y/1.15} \qquad \text{..............................} (7.16)$$

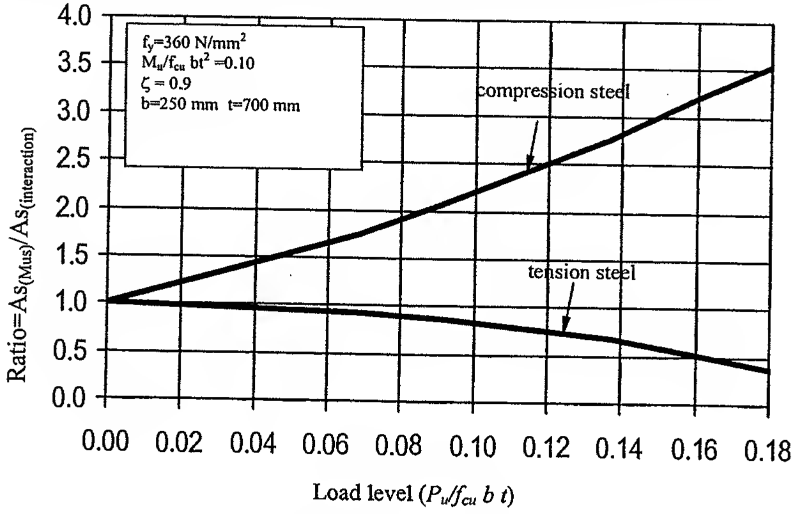
The first term in the equation can be determined using design curves for section subjected to pure bending such as (C1-J or R-ω).

Equation 7.16 (M_{us})gives the same solution as the strain compatibility in case of A'_s=0. Furthermore, equation 7.16 gives a very close solution if the applied compression force is relatively small (P_u<<P_{ub}) and consequently the values of α are usually small. On the other hand, as the applied compression force increases and gets closer to the balanced load, the assumption of ignoring the effect of the compression reinforcement in Eq. 7.15 becomes unconservative especially with high ratios of compression steel.

Figure 7.11 presents the ratios of the required area of the steel when using the M_{us} approach and those when using of strain compatibility method. It is clear that the calculated area of tension steel using the M_{us} approach is much less than that calculated using the strain compatibility method; leading to unconservative results.

The area of the tension steel using the M_{us} approach can be ½ that of the strain compatibility method (interaction diagram). On the other hand, the area of the compression steel obtained using the M_{us} approach can be as high as three times that obtained using the interaction diagrams. Thus, the use of M_{us} approach should be limited to sections with relatively small compression forces in which the values of α are usually very small.

It is clear from Fig. 7.11 that deviation between the area of steel obtained using M_{us} approach and the interaction diagrams increases when the load level increases.



.Fig. 7.11 Comparison between area of steel calculated using the M_{us} approach and that calculated from the interaction diagram for sections in tension failure zone

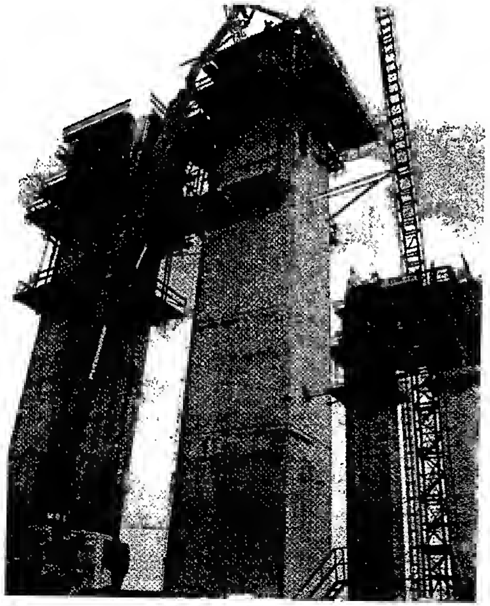
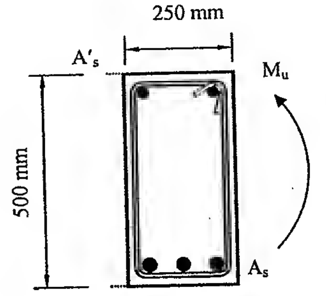


Photo 7.4 Bridge column during construction

Example 7.5

Design the concrete section shown in figure below if it is subjected to an ultimate compression force of 220 kN and ultimate bending moment of 150 kN.m

f_{cu} =30 N/mm² f_y =360 N/mm² α=0.2



Solution

Step 1: Assume failure condition

Since we do not have the area of steel we have to assume the failure condition. Assume that $P_u<P_b$ ....tension failure (the bending moment is relatively high)

$$e=\frac{M_u}{P_u}=\frac{150}{220}=0.682\,m$$

Step 2: Determine area of steel

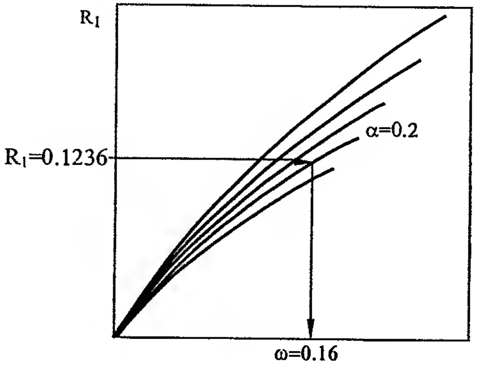
Using the approximate method (tension failure), calculate the moment about tension steel M_{us}. Assume that the concrete cover is 40 mm

$$d=t-\text{cover}=500-40=460\ mm$$

$$M_{us}=M_u+P_u\ (\frac{t}{2}-\text{cover})=150+220\times\left(\frac{500}{2}-40\right)\times\frac{1}{1000}=196.2\ kN.m$$

$$\left.\begin{aligned}&e_s=e+\frac{t}{2}-\text{cover}=0.682+\frac{0.50}{2}-0.04=0.892\ m\\&M_{us}=P_u\times e_s=220\times0.982=196.2\ kN.m\end{aligned}\right]\text{—— Another method to calculate } M_{us}$$

$$R_1'=\frac{M_{us}}{f_{cu}\,b\,d^2}=\frac{196.2\ x\ 10^6}{30\ x\ 250\ \ x\ 460^2}=0.1236$$



Using simple bending curves with compression steel in Appendix A, with (α=0.2) and d′/d = 0.1, it can be determined that:

$\omega = 0.16$

$$A_s = \omega \frac{f_{cu}}{f_y} b \times d - \frac{P_u}{f_y / \gamma_s} = 0.16 \frac{30}{360} 250 \times 460 - \frac{220 \times 1000}{360/1.15} = 830.56\ mm^2$$

$$A'_s = \alpha\, \omega \frac{f_{cu}}{f_y} b \times d = 0.2 \times 0.16 \frac{30}{360} \times 250 \times 460 = 306.67\ mm^2$$

Note: The strain compatibility method *(calculations not shown)* gives

A_s=841 mm^2 and A'_s=310.5 mm^2, Which is very close to the approximate solution because the applied load is small and far from the balanced load and the compression steel ratio α is small.

Step 3: Verify failure condition

$$c_b = \frac{690}{690 + f_y} d = \frac{690}{690 + 360} 460 = 302.285\ mm$$

Applying the equilibrium equation

$$P_{ub} = \frac{0.67 \times f_{cu}\ b\ (0.8\ c_b)}{1.5} + \frac{A'_s \times f_y}{1.15} - \frac{A_s \times f_y}{1.15}$$

$$P_{ub} = \frac{0.67 \times 30 \times 250\ (0.8 \times 302.285)}{1.5} + \frac{306 \times 360}{1.15} - \frac{830 \times 360}{1.15} = 645\ kN > P_u$$

Since $P_u < P_{ub}$, This it is a tension failure mode as assumed.

7.3.3 Design Curves For Eccentric Sections

As mentioned before, M_{us} approach for designing eccentric compression sections should be limited to sections subjected to tension failure and with relatively small compression force. This section presents *new* design charts prepared based on the strain compatibility principle and the equilibrium of forces and can be used to accurately design sections subjected to tension failure whatever the value of the compression force.

The following procedure is adopted to develop the charts. The neutral axis depth is first assumed and the corresponding strain in the steel is calculated for a concrete compressive strain of 0.003. To achieve equilibrium, a trial and adjustment procedure is performed. If the summation of the tension and compression forces is not equal to the desired load level, another value for the neutral axis depth is assumed and iteration is performed. This procedure is continued until the equilibrium is achieved.

Consider the design interaction diagram shown in Fig. 7.12.a which were prepared for certain value of f_y and α, ζ. Each curve in the chart is equivalent to cutting the design interaction diagrams by a horizontal line at a certain load level ($R_b = P_u / f_{cu}$ b t). This gives a group of points with different values of the normalized moment (M_u / f_{cu} bt^2) and the reinforcement index ρ. The normalized moment values were plotted on the vertical axis and the corresponding values of ω are plotted on the horizontal axis and are calculated using the following relation.

$$\omega = \rho \times f_y \times 10^{-4}$$

These charts fill the gap that was not covered by the interaction diagram (from α=0 to α=0.6).

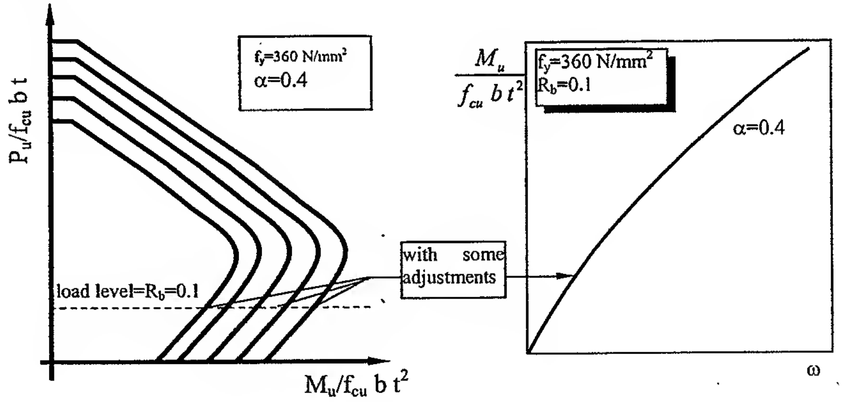


Fig. 7.12.a Development of the design charts

Appendix D contains design charts that can be used in the design of eccentrically loaded sections. An example of such diagrams is shown in Fig. 7.12.b.

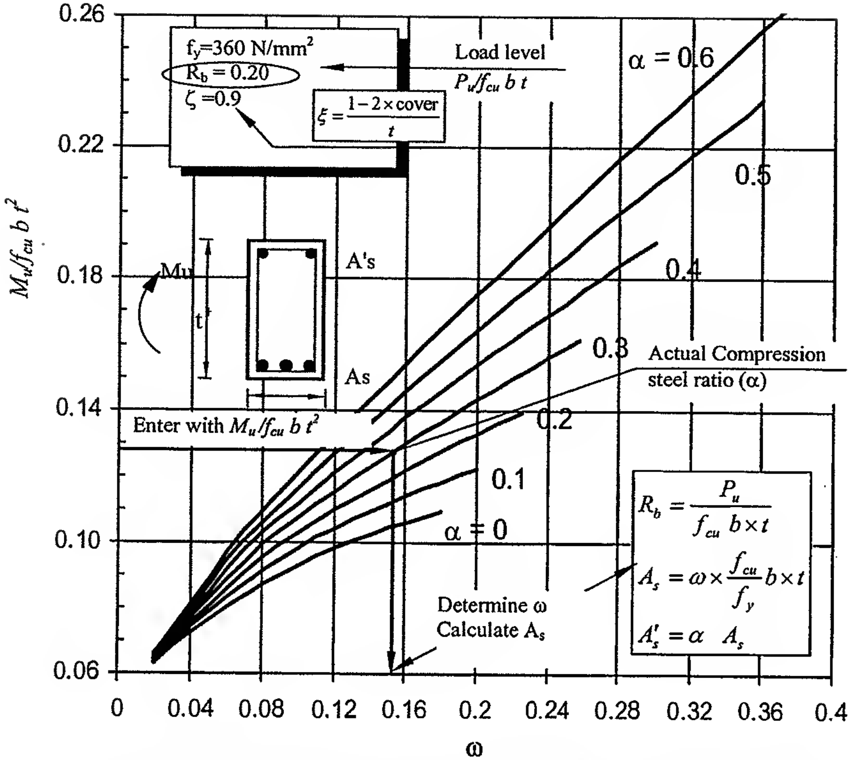


Fig. 7.12.b Eccentric design curve for load level of 0.2 (Appendix D)

The design steps for section subjected to eccentric forces in the tension failure zone using the proposed charts can be summarized in the following steps

Design steps for using eccentric design charts

1. Determine the appropriate design curves by calculating the load level R

$$R_b = \frac{P_u}{f_{cu}\, b\, t}$$

   P_u is positive for compression force and negative for tension force
2. Enter the chart with the applied normalized moment capacity ($M_u/f_{cu}\, b\, t^2$) and choose the appropriate value for α
3. Locate the steel index value ω (interpolate if necessary)
4. Calculate A_s and A'_s from

$$A_s = \omega \frac{f_{cu}}{f_y} b\, t \qquad \text{and} \qquad A'_s = \alpha\ A_s$$

Example 7.6

Design a reinforced concrete section if it is subjected to an eccentric compression force using

A- Interaction Diagram
B-Approximate method (M_{us} approach)
C-Design curves for eccentric sections

Data

M_u=250 kN.m	P_u=700 kN	b=250 mm	t=700 mm
f_{cu} =25 N/mm²	f_y =360 N/mm²	ζ=0.9	α=0.6

Solution

A-Interaction Diagram

Using interaction diagram for f_y=360 N/mm² , α=0.6, and ζ=0.9

The point is below the balanced point, thus it is a **tension failure**

$$\frac{P_u}{f_{cu}\, b\, t} = \frac{700\ x\ 10^3}{25 \times 250\ \times 700} = 0.16$$

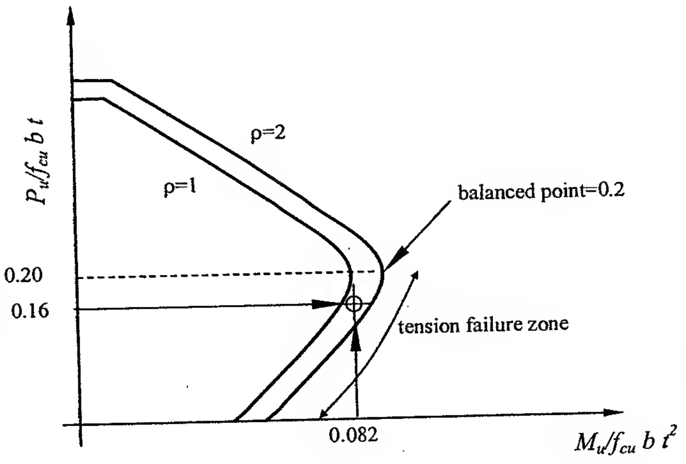
$$\frac{M_u}{f_{cu}\, b\, t^2} = \frac{250 \times 10^6}{25 \times 250\ \times 700^2} = 0.082$$

Locating the point in the chart and determining ρ =1.30

$\mu = \rho\ f_{cu}\ 10^{-4} = 1.3 \text{ x } 25 \text{ x } 10^{-4} = 0.00325$

$A_s = \mu\ b\ t = 0.00325 \text{ x } 250 \text{ x } 700 = 570 \text{ mm}^2$

$A'_s = \alpha\ A_s = 0.6 \text{ x } 570 = 341 \text{ mm}^2$



B- Approximate Method (M_{us})

Since it is a tension failure mode the approximate method (M_{us})can used

$$cover = \frac{1-\xi}{2} \times t = \frac{1-0.9}{2} \times 700 = 35\ mm$$

d = 700- 35 =665 mm

$$e_s = e + \frac{t}{2} - cover = \frac{250}{700} + \frac{0.7}{2} - 0.035 = 0.672\ m$$

$$M_{us} = P_u \times e_s = 700 \times 0.672 = 470.5\ kN.m$$

$$R1 = \frac{M_{us}}{f_{cu}\ b\ d^2} = \frac{470.5 \times 10^6}{25 \times 250\ \times 665^2} = 0.17$$

$$\frac{d'}{d} = \frac{35}{655} \cong 0.05$$

Using simple bending curves (R1-ω) in Appendix A, with compression steel (α=0.6) and d'/d=0.05 →→ω = 0.21

$$A_s = \omega \frac{f_{cu}}{f_y} b \times d - \frac{P_u}{f_y/\gamma_s} = 0.21 \frac{25}{360} 250 \times 665 - \frac{700 \times 1000}{360/1.15} = 188\ mm^2$$

$$A_s' = \alpha\ \omega \frac{f_{cu}}{f_y} b \times d = 0.6 \times 0.21 \frac{25}{360} \times 250 \times 665 = 1455\ mm^2$$

It can be concluded from this example that the approximate method did not give a valid reinforcement area of steel solution as the compression reinforcement is greater than the tension reinforcement, and the area of the tension steel (188 mm^2) is less than obtained from the interaction diagrams (570 mm^2). The main reason for this big difference is that R_b (0.17) is very close to the balanced load level[1] (0.20)

C-Using Design curves for eccentric sections

$$R_b = \frac{P_u}{f_{cu}\ b\ t} = \frac{700 \times 10^3}{25 \times 250\ \times 700} = 0.16$$

$$\frac{M_u}{f_{cu}\ b\ t^2} = \frac{250 \times 10^6}{25 \times 250\ \times 700^2} = 0.082$$

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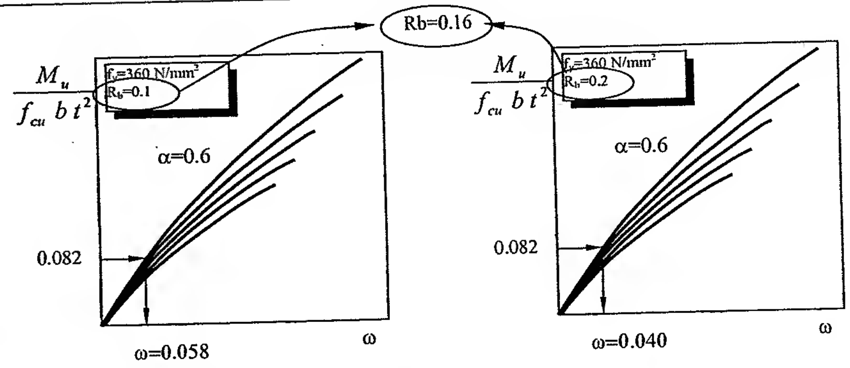
[1] Obtained from the interaction diagram (refer to the figure in the previous page)



Using charts in Appendix D with fy= 360 N/mm^2, (α=0.6)

For load level R_b=0.1 and R_b=0.2, ω can be obtained as follows:

$\omega_{Rb=0.1}$= 0.058 and $\omega_{Rb=0.2}$= 0.040

Using interpolation $\omega_{Rb=0.16}$= 0.0472

$$A_s = \omega \frac{f_{cu}}{f_y} b \times t = 0.0472 \frac{25}{360} 250 \times 700 = 574\ cm^2$$

A'$_s$= α A$_s$= 0.6 x 574 = 344 cm^2

It is clear from Fig. 7.13 that the area of steel calculated from the charts is exactly as the interaction diagrams and the approximate method (M_{us}) gives unconservative tension area of steel.

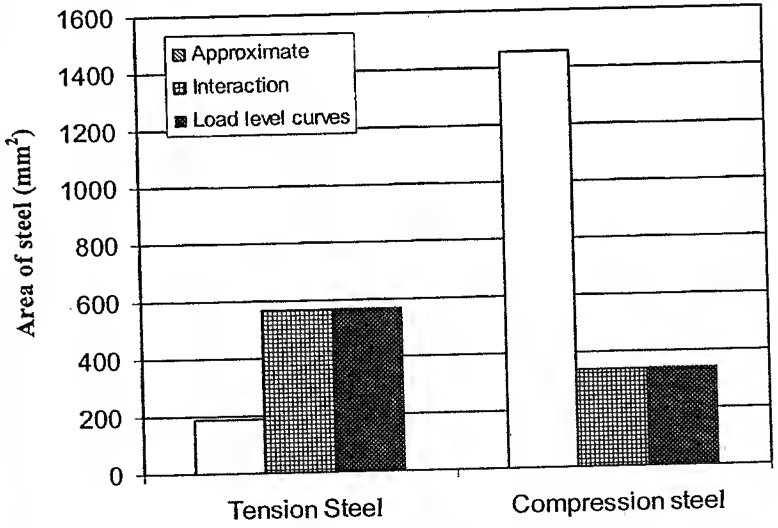
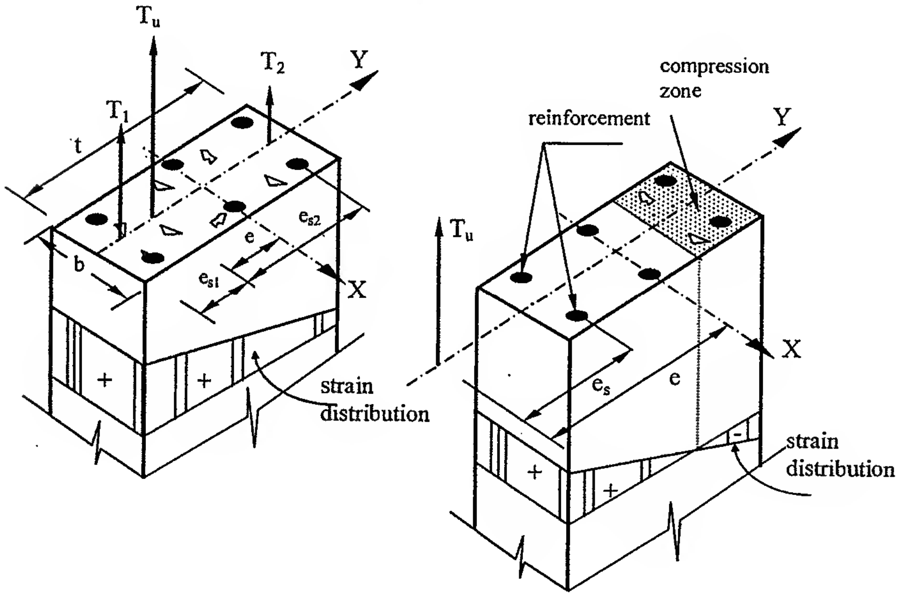


Fig. 7.13 Comparison between the approximate method, interaction diagram and Eccentric load curves in example 7.6

7.4 Sections Subjected to Eccentric Tension Forces

Sections subjected to tension force are some times encountered in frames, and tension members. The design of these types of members depends on the amount of the eccentricity. The ECP 203 states that concrete strength must be completely ignored if the applied tension force is inside the cross section. In this case the tension force is carried solely by the reinforcement as shown in Fig. 7.14.a. However, if the tension force, T_u, lies outside the cross-section, part of the section will be subjected to compression as shown in Fig. 7.14.b.



a- Small eccentric tension

$$e \le \frac{d-d'}{2}$$

See section 7.4.1

b- Big eccentric tension

$$e \ge \frac{d-d'}{2}$$

See section 7.4.2

Fig. 7.14 Sections subjected to eccentric tension force.

7.4.1 Sections Subject to Small Eccentric Tension Forces

In members subjected to small eccentric force, the whole section is subjected to tensile strain and the concrete strength is completely ignored. The eccentricity of the applied tension force and moment must be within the cross section, or

$$e \le \frac{d-d'}{2} \quad \text{.........(7.17)}$$

Only the steel reinforcement act against the applied tension force with no concrete contribution as shown in Fig. 7.15. The developed force in each layer of steel is calculated according to its distance from the applied load and is given by

$$e_{s1} = \frac{d-d'}{2} - e \quad \text{.........(7.18)}$$

$$e_{s2} = \frac{d-d'}{2} + e \quad \text{.........(7.19)}$$

Calculate the developed tension forces T_1 and T_2 (T_1 is always > T_2). To calculate T_1, take moment of forces about point o in Fig. 7.15 as follows:

$$T_1 = T_u \times \frac{e_{s2}}{d-d'} \quad \text{.........(7.20)}$$

$$T_2 = T_u - T_1 \quad \text{.........(7.21)}$$

Calculate A_{s1}, A_{s2}

$$A_{s1} = \frac{T_1}{f_y/1.15} \quad \text{.........(7.22)}$$

$$A_{s2} = \frac{T_2}{f_y/1.15} \quad \text{.........(7.23)}$$

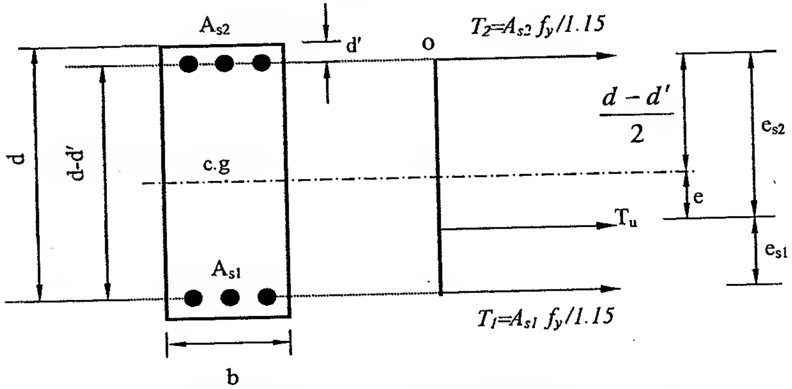


Fig. 7.15 Equilibrium of a section subjected to a small eccentric tension force

Example 7.7

Calculate the reinforcement required for a concrete section of (250x600 mm), if it is subjected to M_u=30 kN.m and T_u= 300 kN. Knowing that f_y=400 N/mm^2 and f_{cu}=30 N/mm^2

Solution

Step 1: Calculate e_{s1} and e_{s2}

$$e=\frac{M_u}{T_u}=\frac{30}{300}=0.1\ m$$

Assume that the distance from the centerline of steel to concrete is 50 mm

d'=50mm

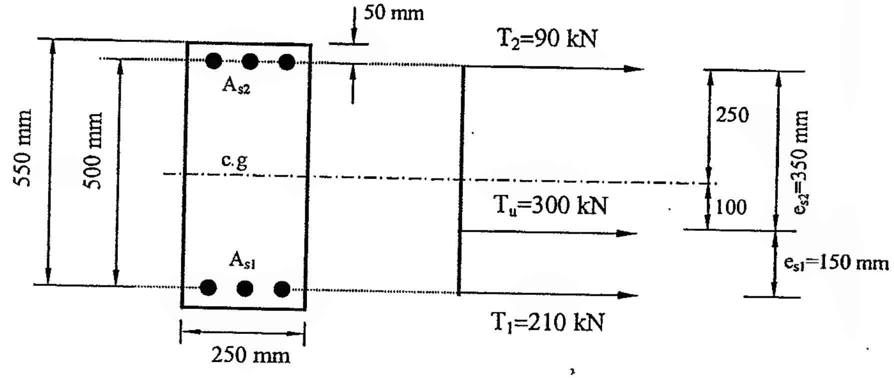
d=600-50 = 550 mm

d-d'=550-50=500 mm=0.5 m

since $e<\frac{d-d'}{2}<\frac{0.5}{2}<0.25$, (small eccentric tension)

Since the member is small eccentric section, concrete strength is completely neglected and all the tension force is resisted by the reinforcement.

$$e_{s2}=\frac{d-d'}{2}+e=\frac{0.5}{2}+0.1=0.35\ m$$



Step 2 Calculate the developed tension force

$$T_1=T_u\times\frac{e_{s2}}{d-d'}=300\times\frac{0.35}{0.5}=210\ kN$$

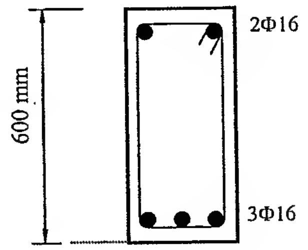
$$T_2=T_u-T_1=300-210=90\ kN$$

Step 3: Calculate A_{s1}, A_{s2}

$$A_{s1}=\frac{T_1}{f_y/1.15}=\frac{210\times1000}{400/1.15}=603.75\ mm^2 \quad (3\Phi16)$$

$$A_{s2}=\frac{T_2}{f_y/1.15}=\frac{90\times1000}{400/1.15}=258.75\ mm^2 \quad (2\Phi16)$$

$$A_{s\,min}=smaller\ \ of\begin{cases}\frac{0.225\sqrt{30}}{400}250\times600=462\ mm^2\\ 1.3\times603=783\ mm^2\end{cases}=462\ mm^2<A_{s1}\cdots o.k$$



Final design

7.4.2 Sections Subjected to Big Eccentric Tension Forces

This case is usually found in reinforced concrete tanks, tunnels and aqueducts, in which the eccentric tension force lies outside the cross section "big eccentricity", creating tension on the near side and compression on the far side. Thus, the concrete contributes to the strength of the section. This part is represented by the part E-F in the interaction diagram shown in Fig 7.3. However, since this part is not represented in the interaction diagram as explained before, the same approximate method (M_{us}) used in sections with compression forces is used here with minor modifications The distance e_s shown in Fig. 7.16 is given by.

$$e_s = e - \frac{t}{2} + \text{cover} \quad \text{...(7.24)}$$

where $e = \dfrac{M_u}{T_u}$

The moment about the tension steel M_{us} equals

$$M_{us} = T_u e_s = M_u - T_u\left(\frac{t}{2} - \text{cover}\right) \quad \text{..............................(7.25)}$$

The problem can be solved using curves with double reinforcement (R1-ω)

Compute $\dfrac{M_{us}}{f_{cu}\, b\, d^2}$

Locate ω from curves using the desired compression steel ratio α and calculate A_s

$$A_s = \omega \frac{f_{cu}}{f_y} b\, d + \frac{T_u}{f_y/1.15} \quad \text{.................................(7.26)}$$

$$A'_s = \alpha\, \omega \frac{f_{cu}}{f_y} b\, d \quad \text{..(7.27)}$$

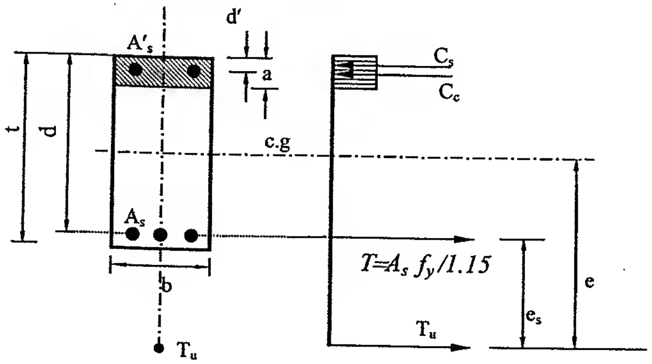


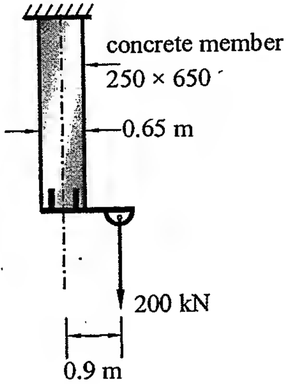
Fig. 7.16 Design of sections subjected to big eccentric tension force

Example 7.8

Design a tension member (250 × 650 mm) if it is subjected to eccentric axial tension force of 200 kN as shown in figure.

f_y=400 N/mm^2

f_{cu}=35 N/mm^2



Solution

T_u=200 kN (tension)

e=0.9 m (outside the cross section → big eccentric tension)

M_u =0.9 × 200 =180 kN.m

Assume concrete cover of 50 mm → d=650-50=550 mm $\rightarrow d'/d = 50/600 \simeq 0.10$

$$e_s = e - \frac{t}{2} + \text{cover} = 0.9 - \frac{0.65}{2} + 0.05 = 0.625\ m$$

$$M_{us} = T_u . e_s = 200 \times 0.625 = 125\ kN.m$$

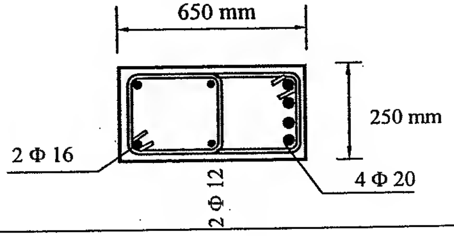
Compute $\dfrac{M_{us}}{f_{cu}\, b\, d^2} = \dfrac{125 \times 10^6}{35 \times 250 \times 600^2} = 0.0397$

From doubly reinforced tables or curves with α=0.4 d'/d=0.10, and R=0.0397

From the curves → ω=0.048

$$A_s = \omega \frac{f_{cu}}{f_y} b\, d + \frac{T_u}{f_y/1.15} = 0.048 \frac{35}{400} \times 250 \times 600 + \frac{200 \times 1000}{400/1.15} = 1205\ mm^2 \quad \rightarrow (4\Phi 20)$$

$$A'_s = \alpha\, \omega \frac{f_{cu}}{f_y} b\, d = 0.4 \times 0.048 \frac{35}{400} \times 250 \times 600 = 252\ mm^2 \quad \rightarrow (2\Phi 16)$$



7.5 T-Sections Subjected To Eccentric Forces

T-sections subjected to eccentric forces are often encountered in the girders of framed structures. Design interaction diagrams are available in design format only for rectangular sections. Preparations of such diagrams for T-section would be unrealistic because many variables are encountered such as B/b and t_s/t ratios. The approximate method can be used to transform the T- shaped sections to members subjected to bending only using M_{us} approach. The design steps are the same as those explained before in section 7.3, however, the designer has to check the location of the neutral axis (a=0.8 c), and determine the area of steel using the (C1-J) as follows.

$$A_s = \begin{cases} a \le t_s & A_s = \dfrac{M_{us}}{f_y\ J\ d} \pm \dfrac{P_u}{f_y/1.15} \\ a > t_s & A_s = \dfrac{M_{us}}{f_y/1.15\,(d - t_s/2)} \pm \dfrac{P_u}{f_y/1.15} \end{cases} \qquad \text{.........................(7.28)}$$

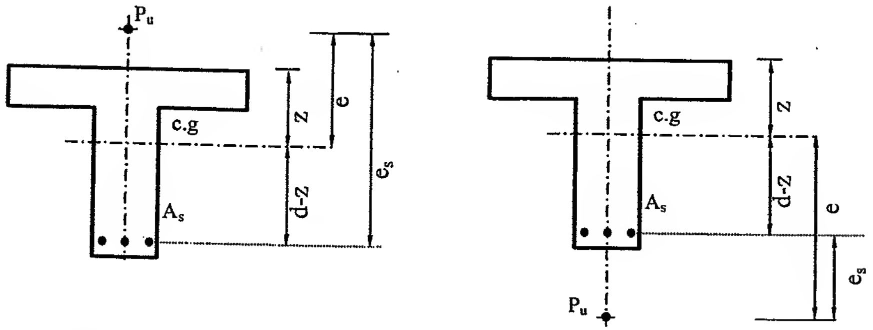
where t_s is the slab thickness and P_u is the applied axial force *(negative in case of compression and positive in case of tension)*

where e_s is measured from the c.g. of the T-section as shown in Fig. 7.17 and determined from the following relations:

$$M_{us} = P_u \times e_s \quad \text{..(7.29a)}$$

$$e_s = e + (d - z) \ \text{(compression)} \text{.................................(7.29b)}$$

$$e_s = e - (d - z) \ \text{(tension)} \text{..(7.29c)}$$



$e_s = e + (d - z)$

a-Eccentric compression

$e_s = e - (d - z)$

b-Eccentric tension

Fig. 7.17 Definition of e_s in T-sections

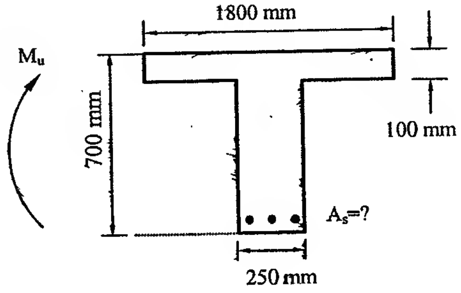
Example 7.9

Design the T-section shown in figure if it is subjected to the following straining actions.

P_u =600 kN (compression)
M_u =450 kN.m

The material properties are

f_{cu} =25 N/mm^2
f_y =240 N/mm^2



Solution

Since the member is T-section, we can not use interaction diagrams and the approximate method should be used

d = 700- 50 =650 mm

$$e = \frac{M_u}{P_u} = \frac{450}{600} = 0.75 \ m$$

$$z = \frac{100\times1800\times50 + 600\times250\times(600/2+100)}{100\times1800 + 600\times250} = 209.1 \ mm$$

$$e_s = e + (d - z) = 0.75 + (0.65 - 0.209) = 1.19 \ m$$

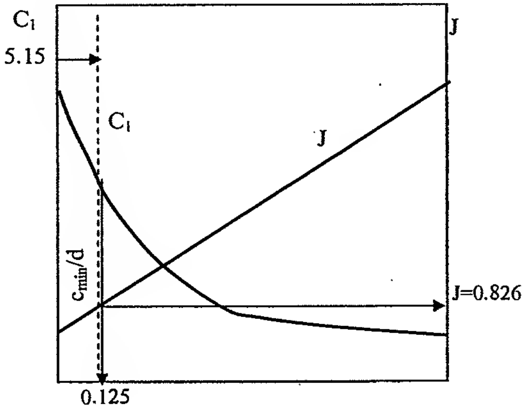
$$M_{us} = P_u \times e_s = 600\times1.19 = 714.5 \, kN.m$$

$$d = C1\sqrt{\frac{M_{us}}{f_{cu}\ B}}$$

$$650 = C1\sqrt{\frac{714.5\times10^6}{25\times1800}} \qquad C1=5.15$$

Using the (C1-J), determine c/d ratio

$$\because \frac{c}{d} < \left.\frac{c}{d}\right)_{min} \quad \therefore \left.\frac{c}{d}\right)_{min} = 0.125$$



c=0.125 × 650=81.25 mm

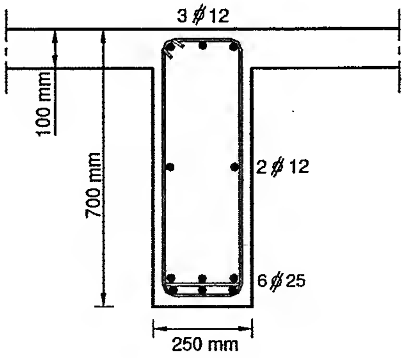
a= 0.8 c = 65 mm

since a< ts(100 mm), get J from the curve using c/d=0.125

J=0.826

$$A_s = \frac{M_{us}}{f_y\ J\ d} \pm \frac{P_u}{f_y\ /1.15} = \frac{714.5\times10^6}{240\times0.826\times650} - \frac{600\times1000}{240/1.15} = 2670\ mm^2$$

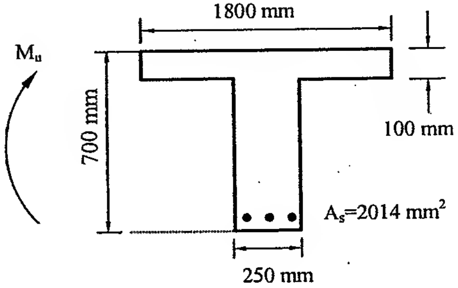
$A_{s,chosen} = 6\ \Phi\ 25$



Example 7.10

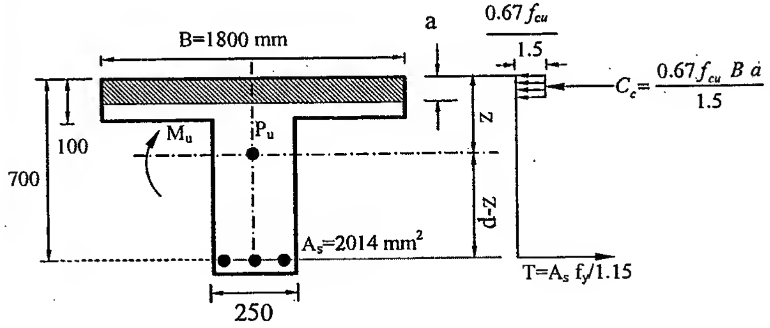
For the cross section shown in the figure below calculate the moment capacity from the first principles. The material properties are f_{cu}=25 N/mm^2 and f_y=240 N/mm^2

P_u =600 kN (compression)



Solution

Assume that a<t_s, apply the equilibrium equation, and assume that γ_c=1.5 and γ_s=1.15



$$P_u = \frac{.67\,f_{cu}\times B\times a}{1.5} - \frac{A_s\ f_y}{1.15}$$

$$600\times1000 = \frac{.67\times25\times1800\times a}{1.5} - \frac{2014\times240}{1.15}$$

a=50.67 mm < t_s, our assumption is correct

The axial force P_u is located at a distance e from the plastic centroid. For simplicity it shall be assumed that the plastic centroid coincides with the c.g.

$$z = \frac{100\times1800\times50 + 600\times250\times(600/2+100)}{100\times1800+600\times250} = 209.1\ mm$$

The moment capacity of the section about the c.g equals

$$M_u = C_c(z-\frac{a}{2}) + T\times(d-z) = \frac{0.67\,f_{cu}B\,a}{1.5}(z-\frac{a}{2}) + \frac{A_s\times f_y}{1.15}\times(d-z)$$

$$M_u = \frac{0.67\times25\times1800\times50.67}{1.5}\left(209.1-\frac{50.67}{2}\right) + \frac{2014\times240}{1.15}\times(650-209.1) = 372.8\,kN.m$$

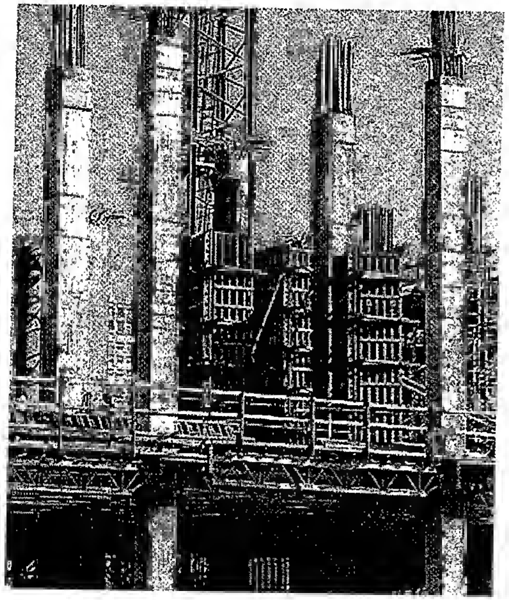


Photo 7.5 Reinforced concrete columns during construction

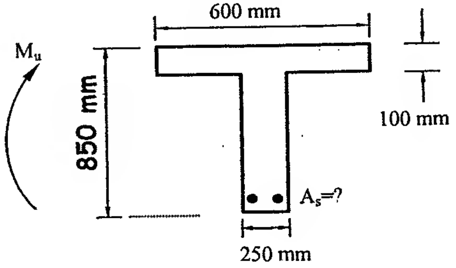
Example 7.11

Design the T-section shown in figure if it is subjected to the following straining actions

P_u=150 kN (Tension) M_u=850 kN.m

The material properties are

f_{cu}=30 N/mm^2 f_y=400 N/mm^2



Solution

Since the section is T-section and subjected to tension force, transformation using the approximate method must be made

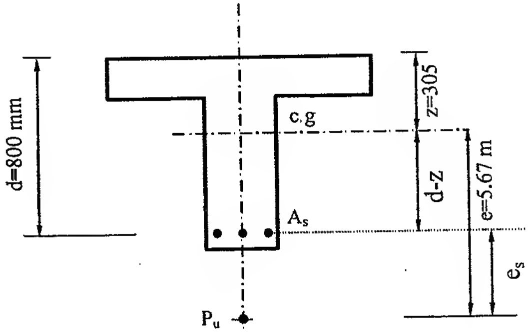
d = 850 - 50 =800 mm

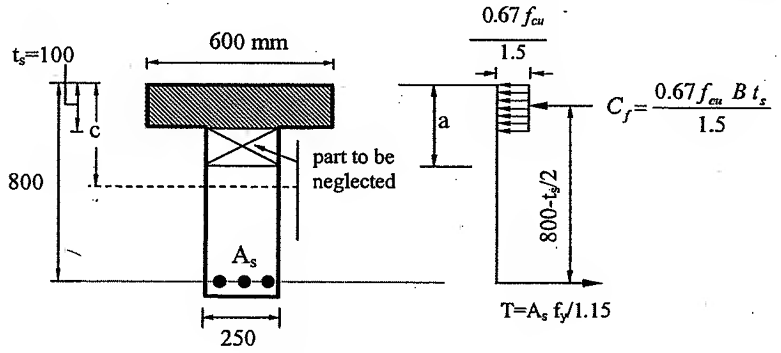
$$e = \frac{M_u}{P_u} = \frac{850}{150} = 5.667\,m$$

calculate the c.g of the section

$$z = \frac{100\times600\times50 + 750\times120\times(750/2+100)}{100\times600 + 750\times120} = 305\ mm$$

$$e_s = e-(d-z) = 5.667-(0.8-0.305) = 5.172\ m$$





$M_{us} = P_u \times e_s = 150 \times 5.172 = 775.75$ kN.m

$$d = C1\sqrt{\frac{M_{us}}{f_{cu}\ B}}$$

$$800 = C1\sqrt{\frac{775.75\times10^6}{30\times600}} \qquad \rightarrow \text{C1=3.85}$$

Using the (C1-J), determine c/d ratio $\rightarrow \frac{c}{d} = 0.21$

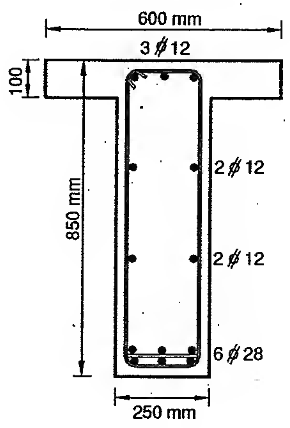
c=0.21 x 800=168 mm

a= 0.8 x 168 = 134.4 mm

Since a(134.4)> t_s(100 mm), neglect the part in the web and calculate A_s using

$$A_s = \frac{M_{us}}{f_y/1.15\,(d - t_s/2)} \pm \frac{P_u}{f_y/1.15} = \frac{775.75\times10^6}{400/1.15\times(800-100/2)} + \frac{150\times1000}{400/1.15} = 3404\,mm^2$$

Choose 6Φ 28 (3695 mm²)



7.6 Analysis of Irregular Sections

7.6.1 General

Reinforced concrete sections can take any shape. Irregular cross-sections are usually encountered in shear walls where irregularity comes from either the shape of the cross-section or the distribution of the reinforcement. For these sections, the development of the interaction diagrams follows the previously mentioned procedure.

Referring to Fig. 7.18, any point falling inside the interaction diagram is considered safe (point A), while any point falling outside the diagram is considered unsafe (point B).

The adequacy of the section is satisfied by ensuring that for the same axial load, the calculated moment capacity is greater than the applied moment. Thus comparing points C & F indicates that point C is considered safe because P_u=P_n and M_n>M_u

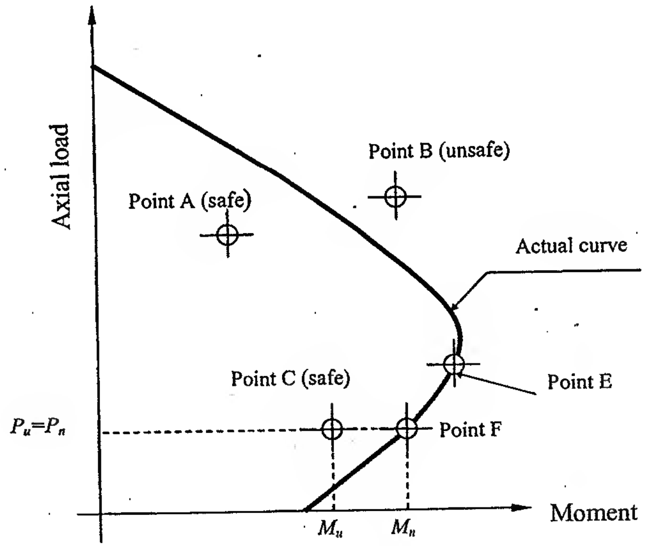


Fig. 7.18 Analysis of irregular sections

7.6.2 Strength of Shear Walls

Shear walls are usually encountered in tall buildings to resist lateral loads initiated by wind or earthquake. The analysis of these walls should be based on strain compatibility and equilibrium of forces as shown in Fig. 7.19. The calculation can be carried out in tabulated form or using spreadsheet like EXCEL. The design procedure can be summarized in the following steps:

1. The shape of the cross-section is usually chosen to fit the architectural requirements of the structure.
2. The structural engineer usually assumes the concrete dimensions and the reinforcement distribution. Some codes of practice give recommendations regarding these issues.
3. The neutral axis distance is assumed and the internal forces are evaluated. The stress in each steel bar is given by

$$f_{si} = 600 \times \frac{c - d_i}{c} \le \frac{f_y}{\gamma_s} \quad \text{...(7.30)}$$

The force in each layer of steel equals the stress multiplied by its area

$$F_{si} = f_{si} \times A_{si} \quad \text{(positive if compression) ....(7.31)}$$

4. The assumption of equivalent stress block is used, and the distance of the compression block is evaluated as $(a=0.8\ c)$. The concrete force is divided into areas A_{ci} then multiplied by the stress $(0.67\,f_{cu}/\gamma_c)$.

$$F_{ci} = \frac{0.67 \times f_{cu}}{\gamma_c} A_{ci} \quad \text{..(7.32)}$$

5. The section nominal ultimate axial capacity P_n is the sum of concrete and steel contributions as follows:

$$P_n = \sum_{i=1}^{i=nc} F_{ci} + \sum_{i=1}^{i=ns} F_{si} \quad \text{..(7.33)}$$

This procedure is repeated until the condition of $P_n = P_u$ is satisfied.

6. The nominal moment capacity M_n is computed by summing the moments of all the internal forces from its c.g to the plastic centroid of the section (X_p). Thus the moment capacity equals:

$$M_n = \sum_{i=1}^{i=nc} F_{ci} \times y_i + \sum_{i=1}^{i=ns} F_{si}\ (X_p - d_i) \quad \text{..............................(7.34)}$$

7. The section will be considered safe if the calculated axial capacity P_n equals the applied force P_u (within 5%) and calculated moment is greater than the applied:

$$P_n = P_u \quad \text{and } M_n \ge M_u \quad \text{..(7.35)}$$

8. If the calculated force is less than applied force, increase the neutral axis distance to increase the axial load capacity.
9. If Pn=Pu but the calculated moment is less than the applied moment (Mn<Mu) increase either section dimension or reinforcement or both

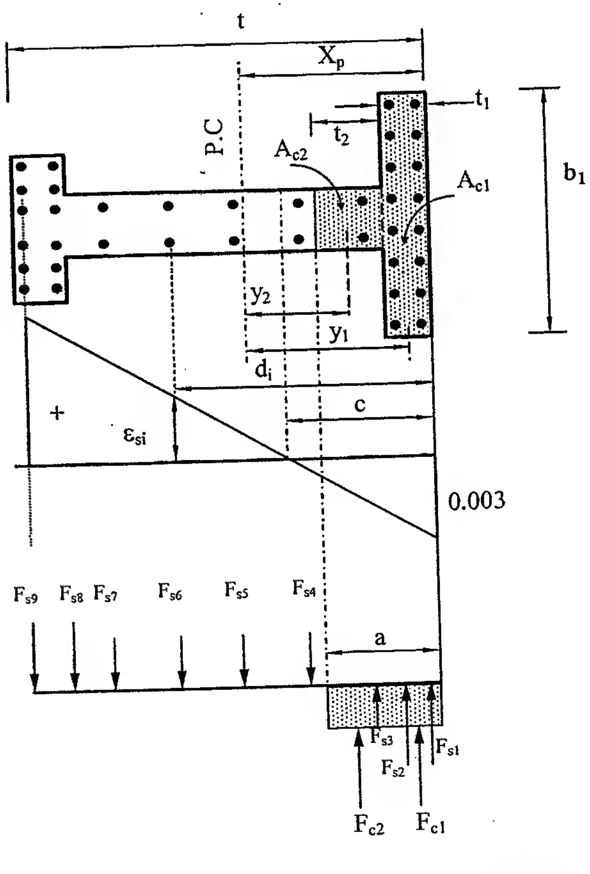


Fig. 7.19 Forces and strains in the shear wall at ultimate

Example 7.12

The shear wall shown in figure is subjected to the following straining actions

P_u=12600 kN

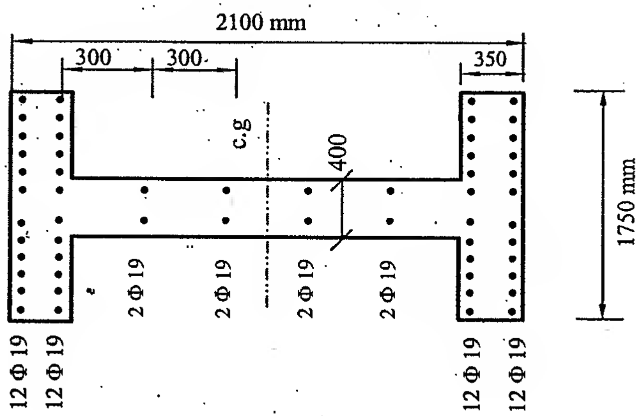
M_u=7520 kN.m

The material properties are

f_{cu}=30 N/mm^2

f_y=400 N/mm^2

Determine the adequacy (Safety) of the cross section



Solution

Step 1: Find the location of the N.A.

Since the location of the neutral axis is unknown, a trial and adjustment procedure is carried out. As a first trial, assume that the neutral axis distance c=700 mm

$\therefore a = 0.8c = 0.80 \times 700 = 560\,mm$

Step 1.1 Forces and moments in steel

Since the section is symmetrical, the c.g. and the plastic centroid coincides.

Xp=t/2=1050 mm

Calculate the applied eccentricity e_u

$$e_u = \frac{M_u}{P_u} = \frac{7520}{12600} = 0.5968\ m$$

Calculate the strength reduction factors γ_c and γ_s.

$$\gamma_c = 1.5 \times \left(\frac{7}{6} - \frac{0.5968}{3}\right) = 1.45 < 1.5 \ldots\ldots \gamma_c = 1.5$$

$$\gamma_s = 1.15 \times \left(\frac{7}{6} - \frac{0.5968}{3}\right) = 1.11 < 1.15 \ldots\ldots \gamma_s = 1.15$$

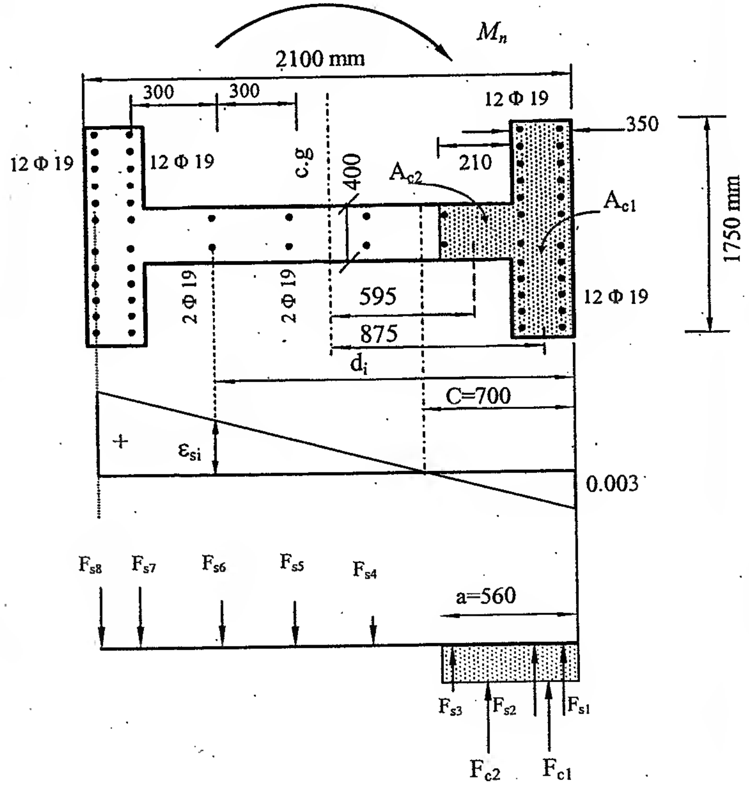
The stress at each bar at distance d_i from the compression face equals

$$f_{si} = 600 \times \frac{c - d_i}{c} = 600 \times \frac{700 - d_i}{700} \le \frac{400}{1.15} \le 347.8 N/mm^2$$

The force in each layer of steel equals the stress multiplied by its area by number of bars in the layer (positive in compression)

The area of one bar Φ 19 A_{si} is 283.53 mm^2

$$F_{si} = f_{si} \times (n_i \times A_{si}) \frac{1}{1000} \quad (kN)$$



The following table summarizes the results

Layer	No of bars, n_i	d_i mm	f_{si} N/mm^2	F_{si} kN
1	12	50	347.8	1183.4
2	12	300	342.9	1166.5
3	2	600	85.7	48.6
4	2	900	-171.4	-97.2
5	2	1200	-347.8	-197.2
6	2	1500	-347.8	-197.2
7	12	1800	-347.8	-1183.4
8	12	2050	-347.8	-1183.4
Total				P_s=-459.99

Step 1.2: Force and moment in concrete

a= 0.8 × c = 560 mm

Since a > flange thickness, divide the compression zone into two zones, the flange and the web. The force developed in each layer equals

$$F_{ci}=\frac{0.67\times f_{cu}}{\gamma_c}A_{ci}\frac{1}{1000}=\frac{0.67\times 30}{1.5}A_{ci}\frac{1}{1000}=0.0134\,A_{ci}$$

$$t_2=560-350=210\,mm$$

The following table summarizes the calculations

No	t_i mm	b_i mm	A_{ci} mm^2	F_{ci} kN
1	350	1750	612500	8207.5
2	210	400	84000	1125.6
Total				P_c=9333.1

$$P_n=\sum_{i=1}^{ns}F_{si}+\sum_{i=1}^{nc}F_{ci}=P_s+P_c=9333.1-459.99=8873.11\text{kN}$$

Since P_n (8873.11 kN) is less than the applied load P_u (12600 kN), thus location of the N.A. must be adjusted.

Note: The corresponding M_u=12106 kN.m (calculations not shown)

Step 2: Adjust c, and Recalculate P_n

Since the calculated normal force is less than the applied, try increasing the neutral axis distance c. After several trials, it can be found that the neutral axis distance that gives the axial force of 12600 kN is 1303 mm

$$c=1303\ mm$$

Step 2.1: Forces and moments in steel

The stress in each bar f_{si} equals

$$f_{si}=600\,\frac{1303-d_i}{1303}\le 347.8\ N/mm^2$$

$$F_{si}=f_{si}\times(n_i\times A_{si})\frac{1}{1000}\quad(kN)$$

The bending of each bar about the c.g equals

$$M_{si}=F_{si}\,(\frac{t}{2}-d_i)\times\frac{1}{1000}\quad(kN.m)$$

The following table summarizes the results

Layer	No of bars, n_i	A_{si} mm^2	d_i mm	f_{si} N/mm^2	F_{si} kN	$t/2-d_i$ mm	M_{si} kN.m
1	12	283.53	50	347.8	1183.4	1000	1183.4
2	12	283.53	300	347.8	1183.4	750	887.6
3	2	283.53	600	323.7	183.6	450	82.6
4	2	283.53	900	185.6	105.2	150	15.8
5	2	283.53	1200	47.4	26.9	-150	-4.0
6	2	283.53	1500	-90.7	-51.4	-450	23.1
7	12	283.53	1800	-228.9	-778.6	-750	584.0
8	12	283.53	2050	-344.0	-1170.3	-1000	1170.3
Total					P_s=682.2		3942.8

Step 2.2: Forces and Moments in Concrete

$a = 0.8\times c = 1042{,}4$ mm $\qquad t_2 = 1042.4 - 350 = 692.4\ mm$

The force developed in each layer equals

$$F_{ci} = \frac{0.67\times f_{cu}}{\gamma_c} A_{ci}\frac{1}{1000} = \frac{0.67\times 30}{1.5} A_{ci}\frac{1}{1000} = 0.0134\, A_{ci}$$

$$M_{ci} = F_{ci}\times y_i$$

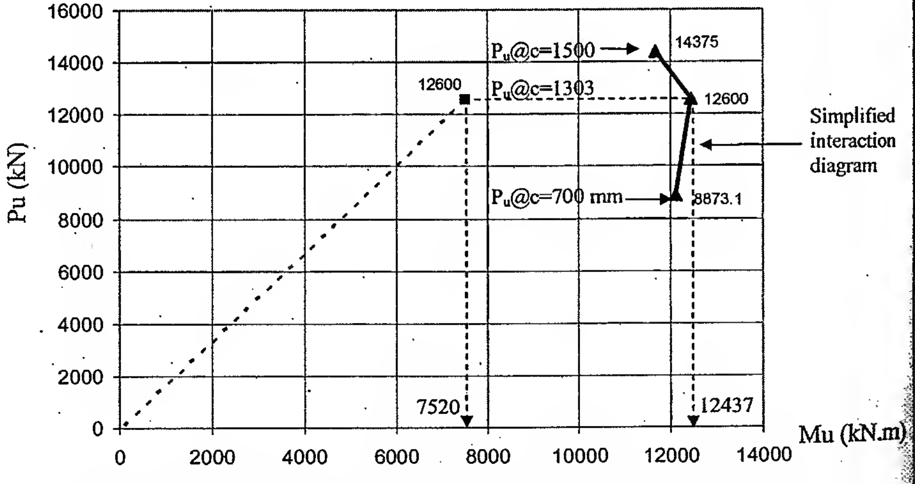
No	t_i mm	b_i mm	A_{ci} mm²	F_{ci} kN	y_i mm	M_{ci} kN.m
1	350	1750	612500	8207.5	875	7181.56
2	692.4	400	276960	3711.26	353.8	1313.05
Total				P_c=11918.8		8494.61

$$P_n = \sum_{i=1}^{ns} F_{si} + \sum_{i=1}^{nc} F_{ci} = P_s + P_c = 682.2 + 11918.8 = 12600\text{kN}$$

$$M_n = \sum_{i=1}^{ns} M_{si} + \sum_{i=1}^{nc} M_{ci} = 3942.8 + 8494.6 = 12437.4\text{kN.m}$$

Step 3: Conclusion

Since P_u=P_n and $\quad M_n(12437) > M_u(7520)\quad$, the section is considered safe [1]



Simplified interaction diagram for shear wall presented in example 7.12

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[1] Calculation for c=1500 was not shown

7.7 Interaction Diagrams For Circular Columns

The same procedure used in developing the capacity of rectangular columns is used for circular ones. However, the ECP-203 does not permit the use of the equivalent stress block in developing circular section capacity. The depth of the neutral axis is assumed and the resulting compression zone is a segment of a circle. Since the stress-strain curve is parabolic and the width of the cross section varies along the depth of the neutral axis, an integration procedure must be followed. In order to calculate the compressive force resisted by the concrete, the compressed zone is divided to n segments with height t_i and width w_i as shown in Fig. 7.20. The width of the segment w_i is given by

$$w_i = 2\sqrt{h_i\times(2r - h_i)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.36)$$

Where h_i is the height of the segment from the top

The corresponding concrete force at the center of gravity of each segment is evaluated as follows.

$$C_c = \sum_{i=n}^{i=1} w_i\times t_i\times f_{ci} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.37)$$

where f_{ci} is the concrete stress at the c.g of the segment.

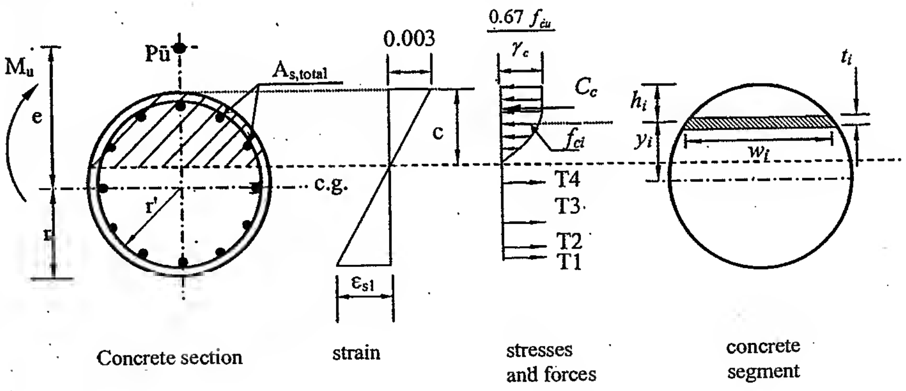


Fig. 7.20 Internal forces and strains distribution in circular columns

The moment of this segment is determined by multiplying the force by the distance to the column center of gravity y_i.

$$M_{uc} = \sum_{n}^{i=1} w_i\times t_i\times f_{ci}\times y_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.38)$$

Summing all the forces and the moments of all segments gives the total concrete force and moments about the column center of gravity. The number of bars in the column affects the shape of the interaction diagram. This is because the position of both compression and tension steel varies according to bar arrangement with column height giving different strain distribution. Therefore interaction diagrams for circular sections are computed by assuming a continuous ring. Placing 8 bars in the column or more is sufficient to validate this assumption. The interaction diagram is normalized to concrete column radius rather than the total area of the column (πr^2) as follows

$$\frac{P_u}{f_{cu}\ r^2} \text{ and } \frac{M_u}{f_{cu}\ r^3}$$

The reinforcement area is obtained by multiplying the column area with the reinforcement ratio as follows

$$A_{s,total} = \mu \left(\pi\ r^2\right) \qquad (7.39a)$$

Appendix E contains interaction diagrams for circular column based on the above procedure. Example of the developed interaction diagrams is given in Fig. 7.21.
Similarly, interaction diagrams for hollow circular sections can be prepared as shown in Appendix F. The reinforcement area in this case is obtained by multiplying the net column area with the reinforcement ratio as follows:

$$A_{s,total} = \mu\, A_c \qquad (7.39b)$$

$$A_c = \pi \left(r^2 - r_i^2\right) \qquad (7.39c)$$

Where r and r_i are the external and internal radius respectively.

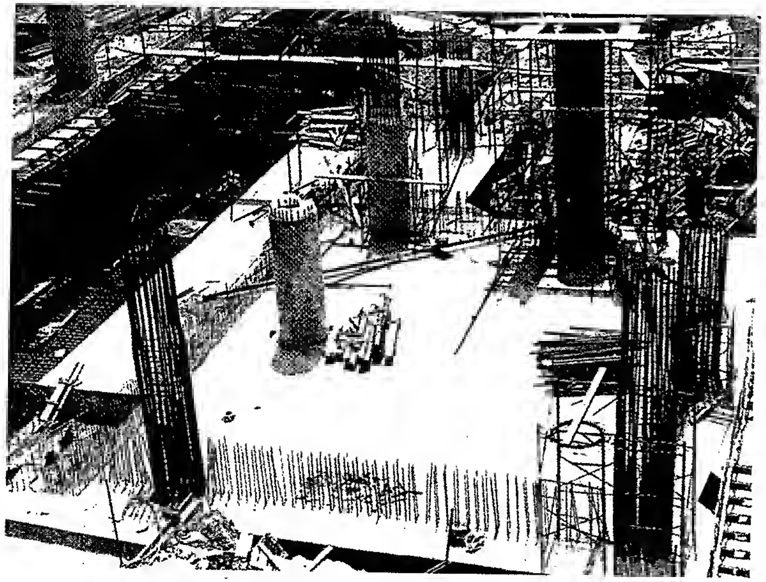


Photo 7.6 Circular columns during construction

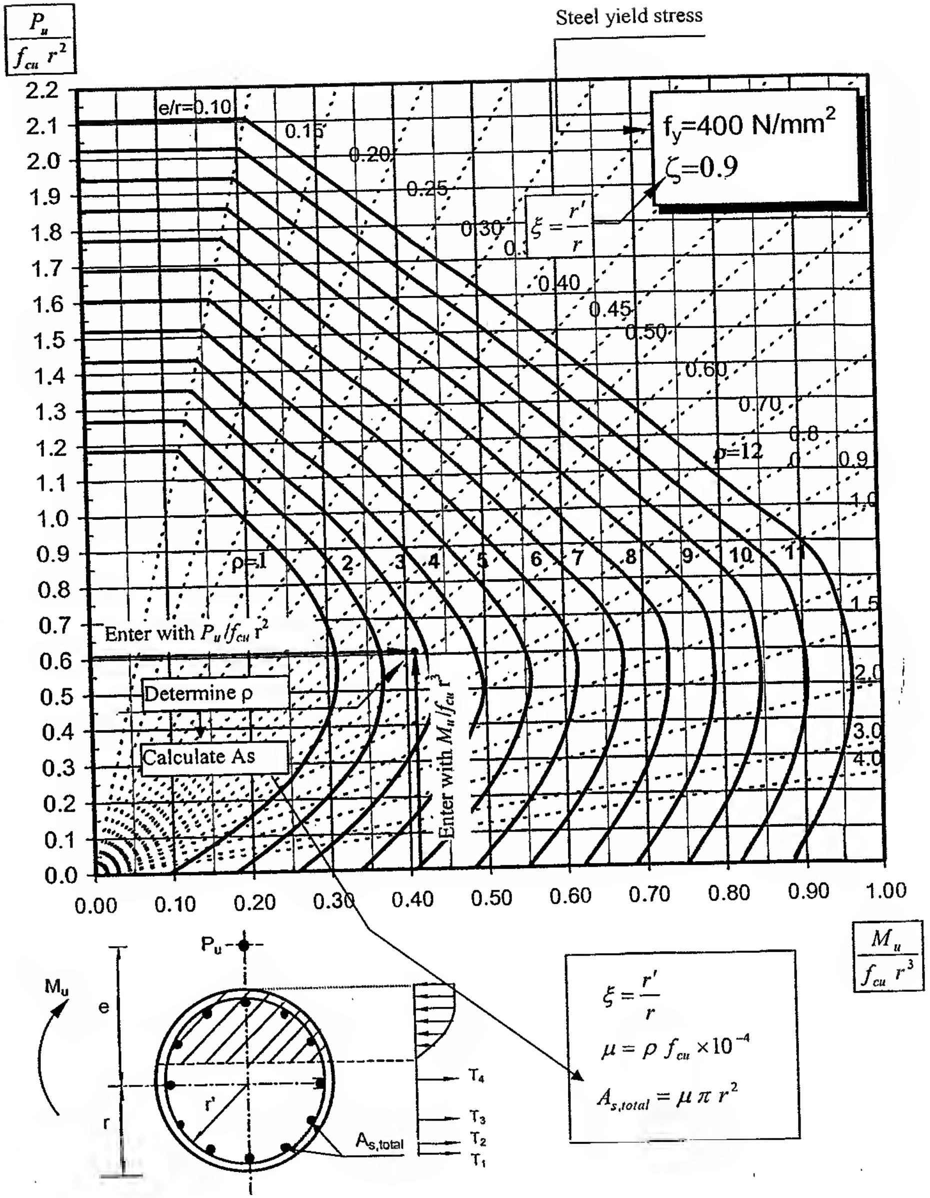


Fig. 7.21 Interaction diagrams for circular sections (appendix E)

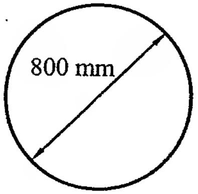
Example 7.13

Design a circular column to resist the following straining actions:

P_u=5600 kN
M_u=830 kN.m

The material properties are as follows:

f_{cu} =40 N/mm^2
f_y =360 N/mm^2



Solution

Since the section is circular and is subjected to eccentric force, use interaction diagrams for circular sections.

$$r = \frac{D}{2} = \frac{800}{2} = 400\ mm$$

Calculate the following terms

$$\frac{P_u}{f_{cu}\ r^2} = \frac{5600 \times 1000}{40 \times 400^2} = 0.875$$

$$\frac{M_u}{f_{cu}\ r^3} = \frac{830 \times 10^6}{40 \times 400^3} = 0.324$$

Assuming 40 mm concrete cover, r′=400-40=360 mm

$$\xi = \frac{r'}{r} = \frac{360}{400} = 0.9$$

Using the interaction diagram with ξ=0.9, f_y=360 N/mm^2

ρ=2.5

$$\mu = \rho \times f_{cu} \times 10^{-4} = 2.5 \times 40 \times 10^{-4} = 0.010 > \mu_{min}\ (0.008).....o.k$$

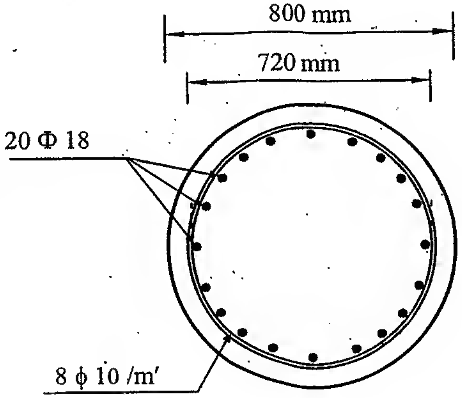
$$A_{s,total} = \mu\left(\pi\ r^2\right) = 0.010 \times \pi \times 400^2 = 5026\ mm^2 \quad (20\Phi 18,\ 5089\ mm^2)$$

Assume that we shall use 8ϕ10 stirrups per meter and A_{sp} for ϕ 10mm =78.5 mm^2

The volume of the stirrups in 1 meter equals

$$V_s = n \times A_{sp} \times (\pi \times D_s) = 8 \times 78.5 \times (\pi \times 720) = 1420502\ \ mm^3$$

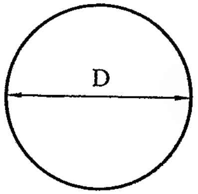
$$V_{s,min} = \frac{0.25}{100} \times \frac{\pi}{4} \times 800^2 \times 1000 = 1256637\ mm^3 < V_s \ldots o.k$$



Example 7.14

Design a spirally reinforced column that is subjected to a normal force of 1100 kN and bending moment of 180 kN (factored values). The material properties are as follows:

$f_{cu} = 30$ N/mm^2
f_y =400 N/mm^2
$f_{yp} = 240$ N/mm^2



Solution

Step1: Estimate cross section diameter

Interaction diagrams for circular sections is used to calculate the capacity of the spiral column by neglecting the contribution of the spiral under eccentric loading.. The spiral reinforcement will be added after using the interaction diagram for confinement only. Since the column dimension is not given, assume a middle point on the interaction diagram such as $P_u/f_{cu}\,r^2$=0.9

$$\frac{1100 \times 1000}{30 \times r^2} = 0.9$$

r=201.8 mm, try r=250 mm and D=500mm

Step2: Calculate the following terms

$$\frac{P_u}{f_{cu}\,r^2} = \frac{1100 \times 1000}{30 \times 250^2} = 0.5866$$

$$\frac{M_u}{f_{cu}\,r^3} = \frac{180 \times 10^6}{30 \times 250^3} = 0.384$$

Assume concrete cover of 30 mm, r′=250-30=220 mm

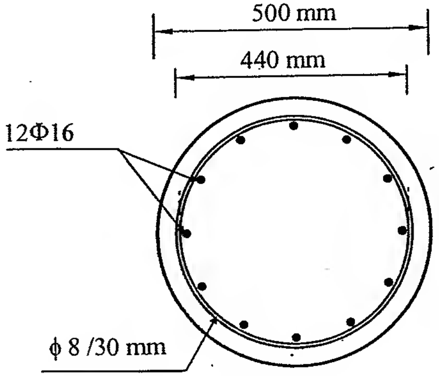
$$\xi = \frac{r'}{r} = \frac{220}{250} = 0.88$$

Using the interaction diagram twice with ξ=0.9 and ξ=0.8

$\rho_{\zeta=0.9}$=2.3, and $\rho_{\zeta=0.8}$=2.8

$\rho_{\zeta=0.88}$=2.4

$\mu = \rho \times f_{cu} \times 10^{-4} = 2.4 \times 30 \times 10^{-4} = 0.0072 < \mu_{min}(0.01A_c\ or\ 0.012\ A_k)$



$$A_{s,min} = \begin{cases} 0.01 \times \pi \times 250^2 & = 1963\ mm^2 \\ 0.012 \times \pi \times 220^2 & = 1824\ mm^2 \end{cases}$$

Choose 12 Φ 16 (2400 mm^2)

Step 3: Spiral design

The minimum volume of stirrup for spiral column is used to calculate the required pitch *p*.

$$V_{sp,min} = 0.36 \times \frac{f_{cu}}{f_{yp}}(A_c - A_k) = 0.36 \times \frac{30}{240}(\pi \times 250^2 - \pi \times 220^2) = 1993\ \ mm^2$$

Using 8mm spiral with A$_{sp}$=50 mm^2

$$p = \frac{\pi\ A_{sp}\ D_k}{V_{sp}} = \frac{\pi \times 50 \times 440}{1993} = 34.67\ mm$$

Use ϕ 8 /30 mm

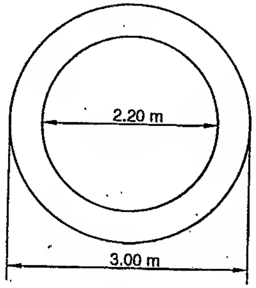
Example 7.15

Design a hollow circular core shown below to resist the following straining actions:

$P_u = 30000$ KN
$M_u = 19000$ KN.m

The material properties are as follows:

$f_{cu} = 30$ N /mm^2.
$f_y = 360$ N /mm^2.



Solution:

Step No.1: Calculate the following terms:

Since the core is hollow circular section and is subjected to eccentric force, use interaction diagrams for hollow circular sections.

$$r = \frac{D}{2} = \frac{3000}{2} = 1500\ mm$$

$$A_c = \pi\left[(1500)^2 - (1100)^2\right] = 3.27\times10^6\ mm^2 = 3.27 m^2$$

$$\frac{P_u}{f_{cu}\,A_c} = \frac{30000\times1000}{30\times(3.27\times10^6)} = 0.306$$

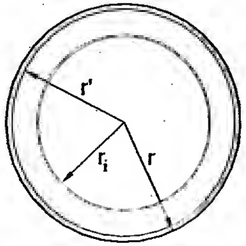
$$\frac{M_u}{f_{cu}\,A_c r} = \frac{19000\times10^6}{30\times(3.27\times10^6)\times1500} = 0.129$$

Step No.2: Design of the reinforcement:

Assuming 40 mm concrete cover, r′=1500-40=1460 mm

$$\zeta = \frac{r'}{r} = \frac{1460}{1500} = 0.97$$

$$\frac{r_i}{r} = \frac{1100}{1500} = 0.75$$



Using the interaction diagram for hollow circular sections (Appendix F)

$f_y = 360$ N /mm^2, $\zeta = 0.95$, $r_i/r = 0.75$
Therefore $\rho = 3$

$$\mu = \rho\times f_{cu}\times10^{-4} = 3\times30\times10^{-4} = 0.009 > \mu_{min}\ (0.008)\ldots..\text{o.k}$$

$$A_{s\,total} = \mu\left(A_c\right) = 0.009\times(3.27\times10^6) = 29340\ mm^2 \quad (96\Phi 20,\ 30159\,mm^2)$$

Step No.3: Stirrups design:

Assume that we shall use 7 ϕ10/m' stirrups perimeter and A_{sp} for ϕ 10 mm = 78.5 mm^2
The volume of the stirrups in 1 meter equals:
For outer diameter:

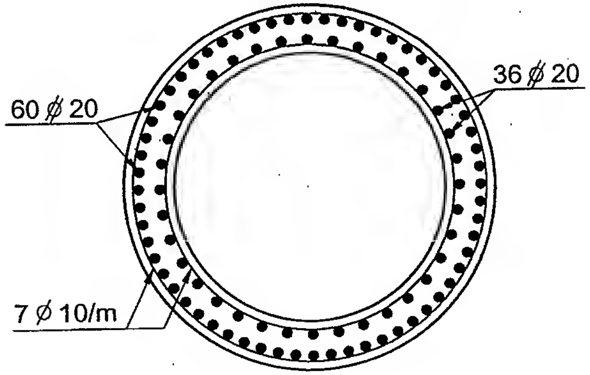
$$V_s = n\times A_{sp}\times(\pi\times D_s) = 7\times78.5\times(\pi\times2920) = 5.04\times10^6\ mm^3.$$

For inner diameter:

$$V_s = n\times A_{sp}\times(\pi\times D_s) = 7\times78.5\times(\pi\times2120) = 3.66\times10^6\ mm^3$$

$$\therefore V_{s\,total} = V_{s\,inner} + V_{s\,outer} = 3.66\times10^6 + 5.04\times10^6 = 8.7\times10^6\ mm^3$$

$$V_{s,min} = \frac{0.25}{100}\times A_c = \frac{0.25}{100}\times(3.27\times10^6)\times1000 = 8.17\times10^6\ mm^3 < V_{s\,total}\ldots o.k$$



Note: Since the outer diameter is larger than the inner diameter, about 60% of the reinforcement is assigned to the outer diameter and about 40% of the reinforcement is assigned to the inner diameter.

7.8 Interaction Diagrams For Box Sections

The development of non-traditional interaction diagrams is performed for box section, which is frequently found in building and bridges. The construction of the diagrams is similar to those for rectangular sections. It is obvious that the capacity of the box section is greatly affected by the area of the internal void. Thus, in developing the interaction diagram the thickness of the concrete wall must be specified and is given as a ratio from the section width and depth (αt and α b) as shown in Fig. 7.22

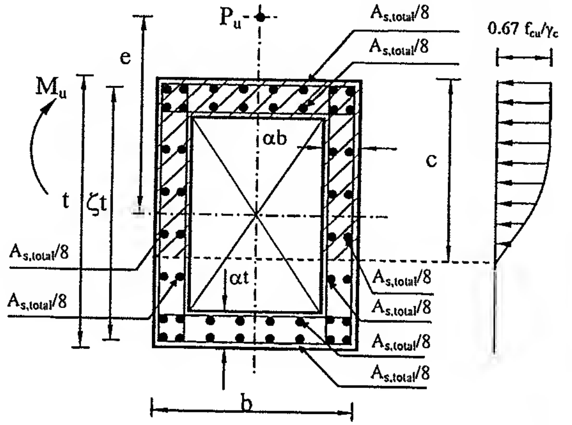


Fig. 7.22 Distribution of the reinforcement in box section

If the neutral axis is inside the top flange, the concrete force and moment can be easily evaluated exactly as rectangular sections. However, if the neutral axis is outside the top flange the area of the void must be subtracted to obtain the exact area of the compressed concrete. The ECP-203 concrete stress-strain curve was used in developing the charts. The developed compression force in the concrete C_c is evaluated by integrating the compressed area with the stress-strain curve and subtracting the void area.

The reinforcement area determined from the interaction diagram is the total area of steel and should be uniformly distributed around the cross section and in both sides of the section (i.e. each face will have 1/8 of the total area) as shown in Fig. 7.22.
The interaction diagrams are normalized to the net concrete column area A_c rather than the total area of the column as follows

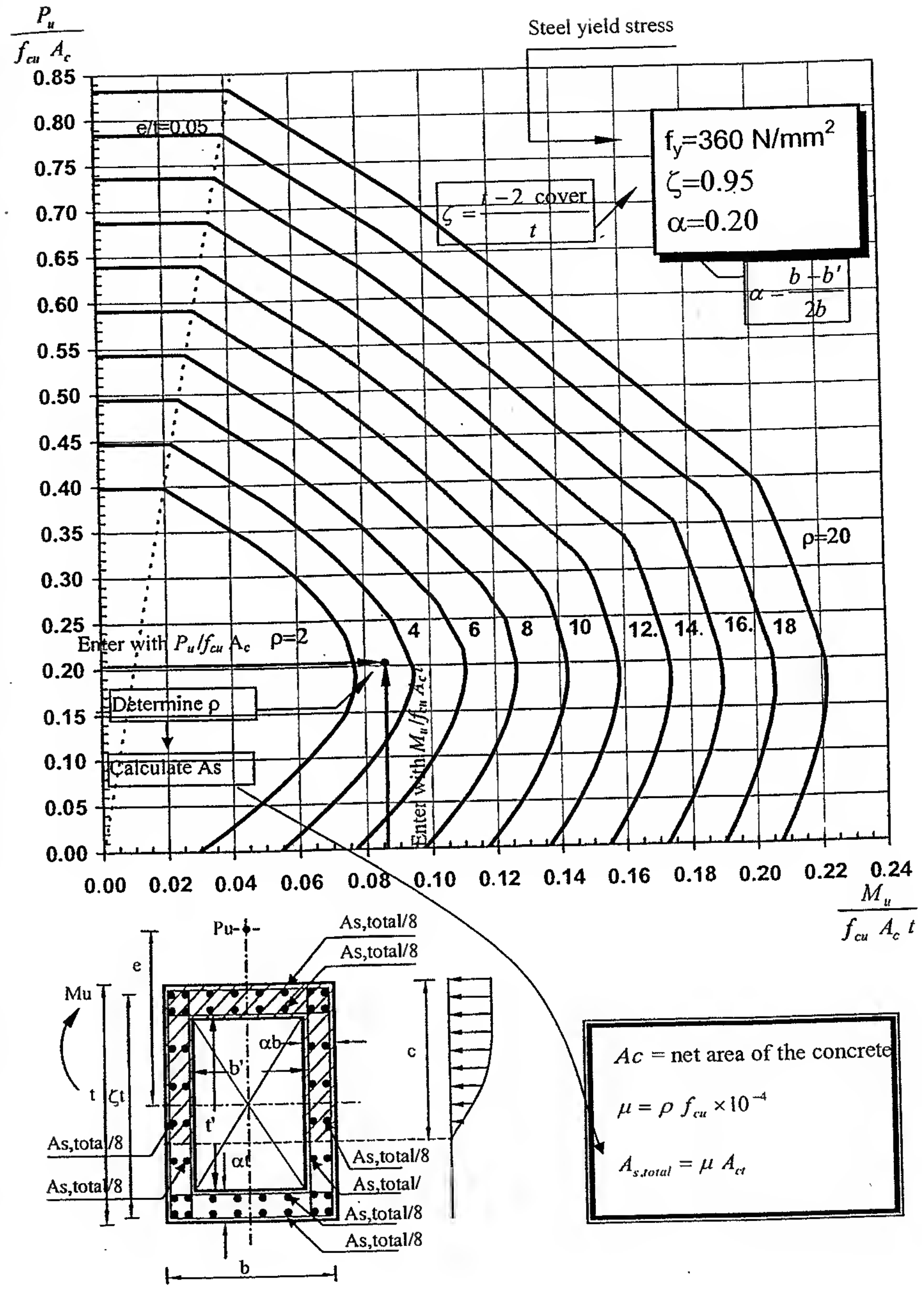
$$\frac{P_u}{f_{cu}\ A_c} \qquad \text{and} \qquad \frac{M_u}{f_{cu}\ A_c\ t}$$

The reinforcement area is given by multiplying the reinforcement ratio with the net concrete area A_c as follows:

$$A_{s\,,total} = \mu\, A_c \quad \text{..}(7.40)$$

$$A_c = b\ t\ -void\ area$$

An example of the curves is shown in Fig. 7.23 and the rest is given in Appendix G.



Ac = net area of the concrete

$\mu = \rho\ f_{cu} \times 10^{-4}$

$A_{s,total} = \mu\ A_{ct}$

Fig. 7.23 An example of an interaction diagram for box section (appendix G)

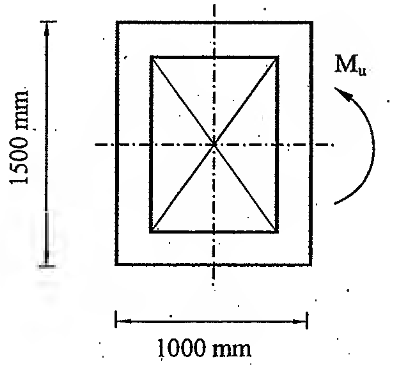
Example 7.16

Design the box section shown in figure if it is subjected to the following straining actions

P_u =9000 kN
M_u =2700 kN.m

The material properties are as follows

f_{cu} =30 N/mm^2
f_y =360 N/mm^2



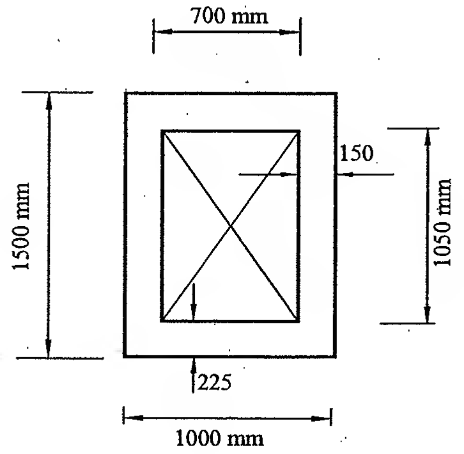
Solution

Interaction diagrams for box sections are used to design the given member.

Since the thickness of the concrete walls is not given assume α=0.15, thus

αt= 0.15 (1500)=225 mm

αb=0.15 (1000)=150 mm



The net area of the concrete A_c equals

$$A_c = 1500\times1000 - 700\times1050 = 765000\ mm^2$$

$$\frac{P_u}{f_{cu}\ A_c} = \frac{9000\times1000}{30\times765000} = 0.3921$$

$$\frac{M_u}{f_{cu}\ A_c\ t} = \frac{2700\times10^6}{30\times765000\times1500} = 0.078$$

From the diagram with f_y=360 N/mm^2, ζ=0.95, α=0.15

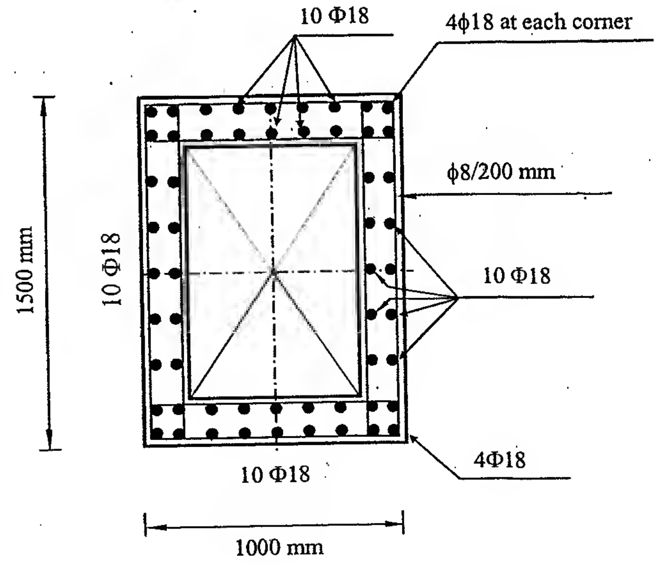
ρ=6.2

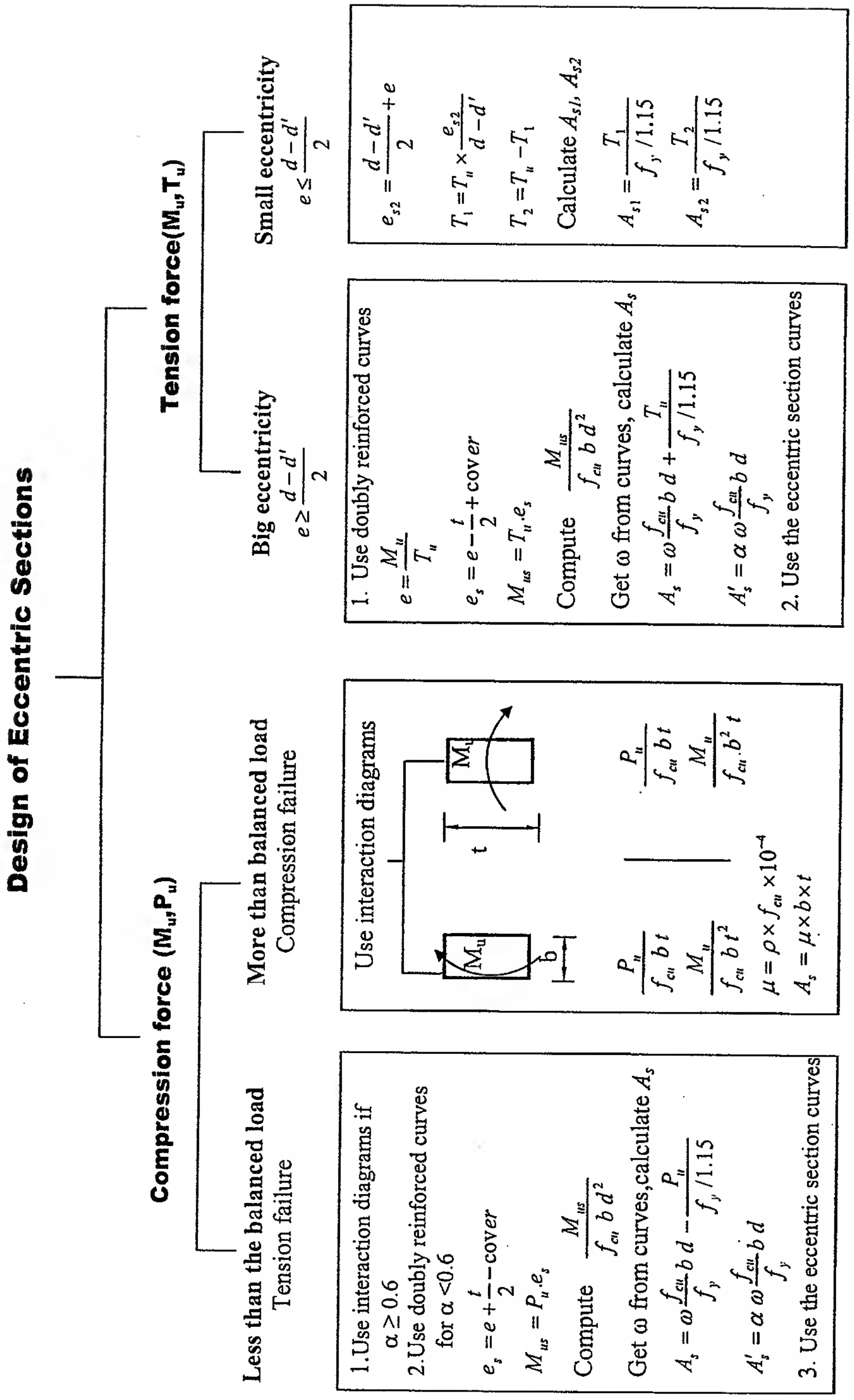
$$\mu = \rho\times f_{cu}\times10^{-4} = 6.2\times30\times10^{-4} = 0.0186$$

$$A_{s,total} = \mu\ A_c = 0.0186\times765000 = 14229\ mm^2$$

Choose the area of steel multiple of eight and more than 24 bars

Choose 56 Φ 18 (14250 mm^2)





7.9 Columns Subjected to Biaxial Bending

7.9.1 General

Designing a rectangular cross section for biaxial bending and axial load is a complicated process because the direction and the position of the neutral axis are difficult to establish. Furthermore, since the strain over the cross section varies linearly in both directions as shown in Figure 7.24, considerable computation time is required to establish equilibrium.

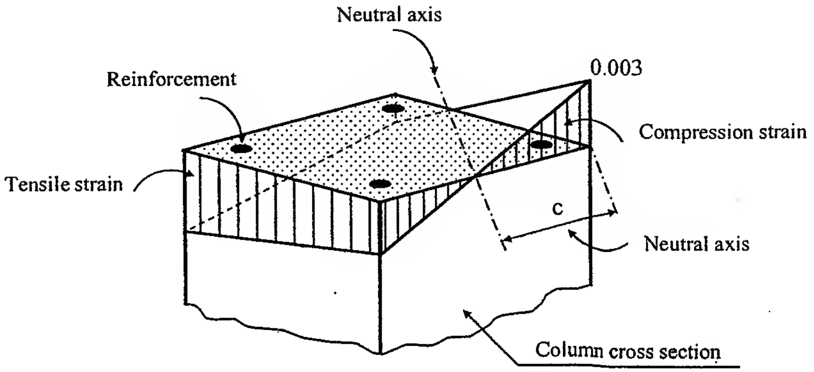


Fig. 7.24 Strain distribution for columns subjected to biaxial bending

The failure surface of sections subjected to biaxial bending is a three-dimensional surface as shown in Fig. 7.25. Any combination of P_u, M_{ux}, and M_{uy} falling inside the surface is considered safe, but any point falling outside the surface is considered unsafe. The surface consists of infinite number of interaction diagrams as follows:

1. Position 1 represents the uniaxial section capacity in x direction in which the neutral axis is parallel to x-direction (M_{ux}).
2. Position 2 represents the uniaxial section capacity in y-direction in which the neutral axis is parallel to y-direction (M_{uy}).
3. An inclined neutral axis such as position 3 represents the case of column under biaxial bending (M_{ux}, M_{uy}).

Another representation can be made by cutting the failure surface with horizontal planes called load level. This approach is adopted for preparing the design aids presented in the Appendix H of this book.

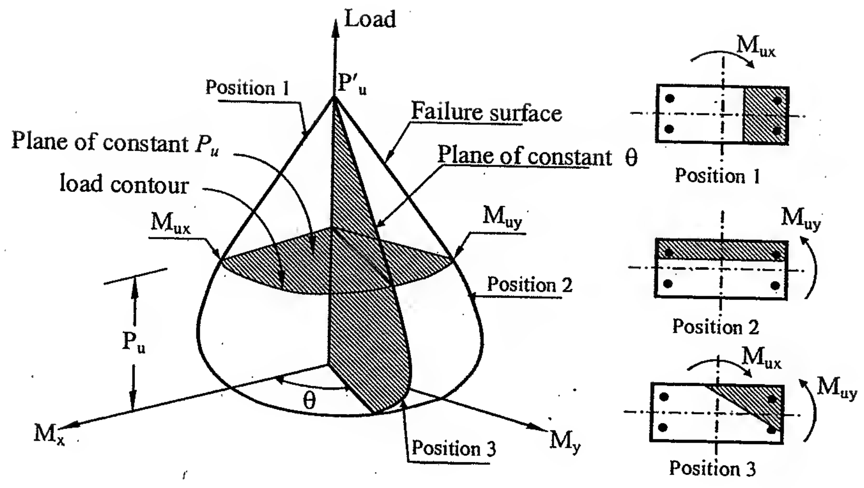


Fig. 7.25 Failure surface for columns subjected to biaxial bending

7.9.2 Exact Analysis of Biaxial Bending

The analysis of sections subjected to biaxial bending is performed using compatibility of strains and equilibrium of forces. Defining the neutral axis distance from the origin as x_{NA} with an angle ϕ as shown in Fig. 7.26. The strain at any steel bar equals to

$$\varepsilon_{si} = 0.003 \times \frac{c - d_i}{c} \quad \text{.. (7.41)}$$

where $d_i = x_i \sin(\phi) + y_i \cos(\phi)$ and the neutral axis distance c equals

$$c = x_{NA} \sin(\phi) \quad \text{.. (7.42)}$$

It should be clear that a positive strain indicates compression and a negative strain indicates tension. The developed stress in each bar equals to

$$f_{si} = E_s \times \varepsilon_{si} = 200{,}000 \times \varepsilon_{si} = 600\frac{c - d_i}{c} \le \frac{f_y}{\gamma_s} \quad \text{..................(7.43a)}$$

$$f_{si} = 600\frac{c - d_i}{c} \le \frac{f_y}{\gamma_s} \quad \text{.................................. (7.43b)}$$

The total force in each bar is

$$F_{si} = A_{si} \times f_{si} \quad \text{.. (7.44)}$$

where A_{si} is the area of each individual bar

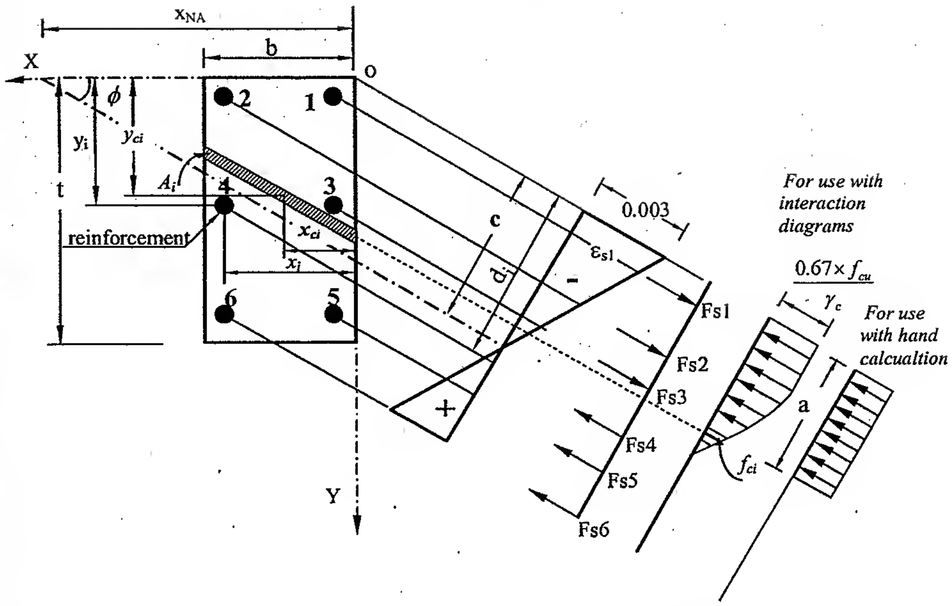


Fig. 7.26 Stress and strain distributions for a column under biaxial bending

The moments of the reinforcement M_{sxi}, M_{syi} is taken about X and Y axis respectively for each bar as

$$M_{sxi} = F_{si} \times y_i \quad \text{.. (7.45)}$$

$$M_{syi} = F_{si} \times x_i \quad \text{.. (7.46)}$$

The ECP-203 does not allow the use of the equivalent stress block in the computation of the capacity of biaxially loaded columns, thus an integration process must be performed. The compression zone will be divided into small areas A_i and multiplied by the corresponding stress f_{ci}. This procedure is used in a computer program developed for the propose of preparing biaxial interaction diagrams. The compressive force developed in each concrete segment is given by

$$F_{ci} = f_{ci}\, A_i \quad \text{.. (7.47)}$$

However, the previous procedure is not suitable in hand calculations. For hand computations, the equivalent stress block is used. Depending on the compression zone shape, it is divided into two or three areas and the developed force in each area equals

$$F_{ci} = \frac{0.67 \times f_{cu}}{\gamma_c} A_i \quad \text{.. (7.48)}$$

The moment of the concrete compression force in each segment equals the individual compressive force multiplied by the distance from the axis X and Y as follows:

$$M_{cxi} = F_{ci} \times y_{ci} \quad \text{...} (7.49)$$

$$M_{cyi} = F_{ci} \times x_{ci} \quad \text{...} (7.50)$$

The total resultant capacity of the section P_u equals to

$$P_u = \sum F_{si} + \sum F_{ci} \quad \text{..} (7.51)$$

The total forces and moments are assumed to be located at the plastic centroid. For sections with symmetrical reinforcement, the plastic centroid coincides with the center of gravity. Thus if one needs to calculate the total moments M_{ux}, M_{uy} about any point rather than the plastic centroid *(point o in Fig. 7.26)*, the moment of the resultant force (P_u) should be taken into consideration as follows

$$M_{ux} = P_u \times y - \left(\sum M_{sxi} + \sum M_{cxi}\right) \quad \text{........................} (7.52)$$

$$M_{uy} = P_u \times x - \left(\sum M_{syi} + \sum M_{cyi}\right) \quad \text{........................} (7.53)$$

where *x* and *y* are the distance from the plastic centroid to the assumed axes. For symmetrical sections x=b/2 and y=t/2.
It should be noted that the reduction safety factors depend on the eccentricity, and the resultant eccentricity (e/t^*) is used for the calculations of these factors as shown in Fig. 7.27.

$$\frac{e}{t^*} = \sqrt{\left(\frac{e_x}{b}\right)^2 + \left(\frac{e_y}{t}\right)^2} \quad \text{..................................} (7.54)$$

where

$$\frac{e_y}{t} = \frac{M_{ux}}{P_u} \qquad \frac{e_x}{b} = \frac{M_{uy}}{P_u}$$

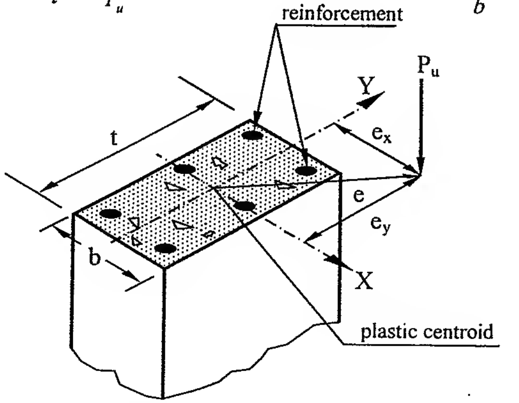
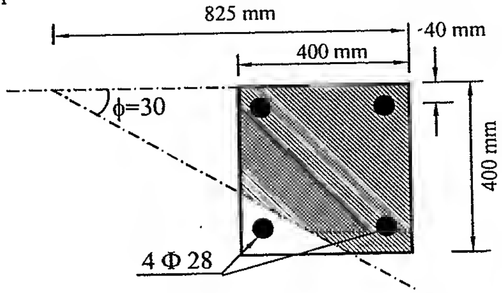


Fig. 7.27 calculation of the eccentricity for a section under biaxial bending

Example 7.16

Calculate the biaxial column capacity for the neutral axis position shown in the figure below knowing that:

f_{cu}= 25 N/mm^2
f_y= 400 N/mm^2



Solution

For the sake of simplicity, the equivalent stress block is assumed to be valid in such a case. The calculation of the capacity can be summarized in the following steps:

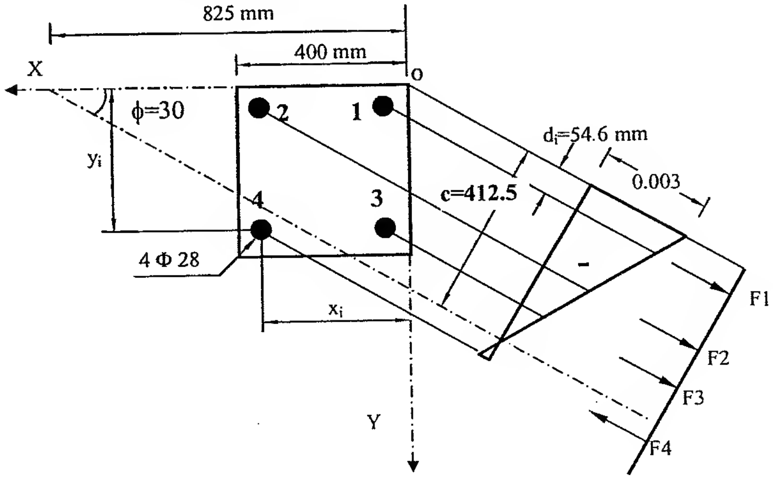
Step 1: Moments due to Forces Developed in the Steel Bars

c= 825 sin (30) = 412.5 mm

The strain in the steel equals

$$\varepsilon_{si} = 0.003 \times \frac{c - d_i}{c} = 0.003 \times \frac{412.5 - d_i}{412.5}$$

where $d_i = x_i \sin(\phi) + y_i \cos(\phi) = x_i \sin(30) + y_i \cos(30)$



Positive strain indicates compression and vice versa. The stress in each bar equals:

$$f_{si} = 200{,}000 \times \varepsilon_{si} \le \frac{f_y}{\gamma_s}$$

Since most of the section is in compression (e/t is small). Assume e/t=0.27.

$$\gamma_c = 1.5 \times \left(\frac{7}{6} - \frac{0.27}{3}\right) \approx 1.6$$

$$\gamma_s = 1.15 \times \left(\frac{7}{6} - \frac{0.27}{3}\right) \approx 1.24$$

Thus concrete and steel safety factors equals γ_s=1.24 and γ_c=1.6. This assumption will be verified later. The area of one bar Φ 28 mm equals=615.75 mm^2
The force in each bar equals

$$F_{si} = A_{si} \times f_{si} = 615.75 \times f_{si}$$

The moment capacity of the section is calculated by taking the moments of the forces about any point (for example point o). The moments of the forces resisted by the steel reinforcement M_{sxi}, M_{syi} are taken about X and Y axes. The moment of the resultant force P_u should be considered since point o does not coincide with the plastic centroid.

$$M_{sxi} = F_{si} \times y_i \qquad M_{syi} = F_{si} \times x_i$$

Calculations of moments due to forces developed in the steel bars

Bar	x_i mm	y_i mm	d_i mm	ε_{si}	f_{si} N/mm^2	F_{si} kN	M_{sxi} kN.m	M_{syi} kN.m
1	40	40	54.6	0.00260	322.6	198.63	7.95	7.95
2	360	40	214.6	0.00144	287.8	177.21	7.09	63.80
3	40	360	331.8	0.00059	117.4	72.31	26.03	2.89
4	360	360	491.8	-0.00058	-115.3	-71.00*	-25.56	-25.56
Total						377.15	15.50	49.07

*(tension force)

Step 2: Moments due to Forces Resisted by Concrete

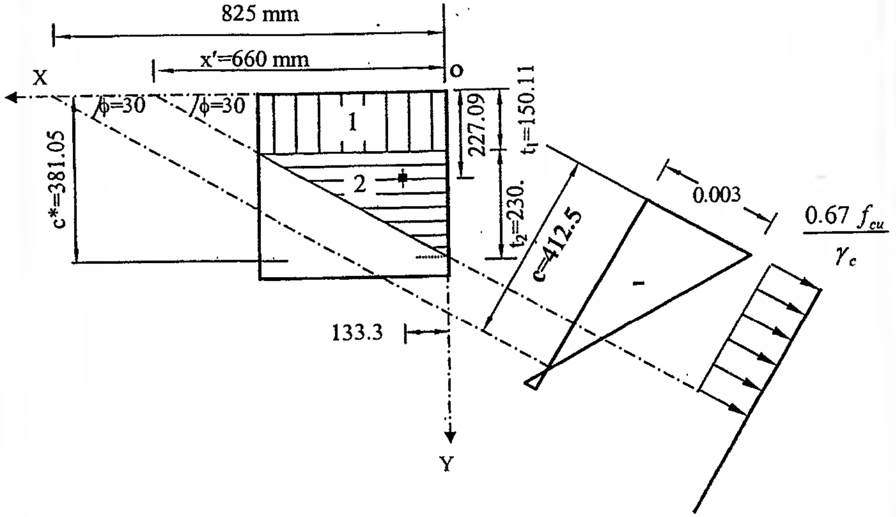
The equivalent stress block is 0.8 of the neutral axis distance

x'= 0.8 (825) =660 mm

Since the compression zone is trapezoidal, it will be divided to rectangular and triangular as shown in figure.

$$t_1 = (x' - 400) \times \tan\phi = (660 - 400) \times \tan 30^\circ = 150.11\ mm$$

$$t_2 = 400 \times \tan\phi = 400 \times \tan 30^\circ = 230.94\ mm$$



$$A_1 = 400 \times t_1 \qquad A_2 = \frac{400 \times t_2}{2}$$

$$y_{c1} = \frac{t_1}{2} \qquad y_{c2} = t_1 + \frac{t_2}{3}$$

$$F_{ci} = \frac{0.67 \times f_{cu}}{\gamma_c} A_i = \frac{0.67 \times 25}{1.60} \frac{A_i}{1000} = 0.01047\ \mathrm{A_i}\ \ \text{(kN)}$$

The moment of the concrete compression force is the sum of the individual compressive force multiplied by the distance from the axis x and y as follows

$$M_{cxi} = F_{ci} \times \frac{y_{ci}}{1000} \qquad \text{(kN.m)}$$

$$M_{cyi} = F_{ci} \times \frac{x_{ci}}{1000} \qquad \text{(kN.m)}$$

Calculations of moments due to forces resisted by concrete

No	shape	b_i mm	t_i mm	Area mm^2	x$_{ci}$ mm	y$_{ci}$ mm	F_{ci} kN	M_{cxi} kN.m	M_{cyi} kN.m
1	▭	400	150.11	60044.43	200.00	75.06	628.59	47.18	125.72
2	◁	400	230.94	46188	133.33	227.09	483.53	109.81	64.47
Total							1112.12	156.98	190.19

Step 3: Total Forces and Moments in Concrete and Steel

$$P_u = \sum F_{si} + \sum F_{ci}$$

$P_u = 377.15 + 1112.12 = 1489.27$ kN.

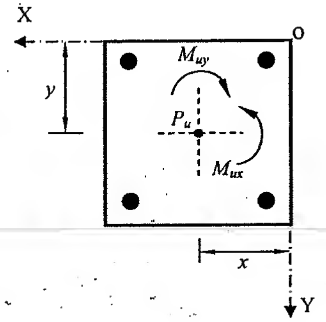
Since the section is symmetrical, the plastic centroid coincides with the c.g. Thus the location of the section compressive force P_u (*the resultant*) is at 200 mm from both X and Y axes. Since the moment is not taken at the plastic centroid of the section, contribution of the P_u must be taken into consideration, thus the total moments about point *o* equals

$$M_{ux} = P_u \times y - \left(\sum M_{sxi} + \sum M_{cxi}\right)$$

$$M_{uy} = P_u \times x - \left(\sum M_{syi} + \sum M_{cyi}\right)$$

$$M_{ux} = 1489.27 \times 200/1000 - (15.50 + 156.98) = 125.36\ kN.m$$

$$M_{uy} = 1489.27 \times 200/1000 - (49.07 + 190.188) = 58.6\ kN.m$$



Step 4: Check reduction safety factors

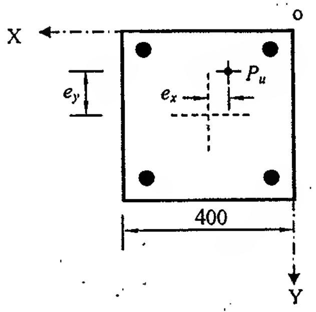
$$\frac{e_y}{t} = \frac{M_{ux}}{P_u \times t} = \frac{125.36}{1489.27 \times 0.40} = 0.21\,m$$

$$\frac{e_x}{b} = \frac{M_{uy}}{P_u \times b} = \frac{58.6}{1489.27 \times 0.40} = 0.098\,m$$

$$\frac{e}{t^{*}} = \sqrt{\left(\frac{e_x}{b}\right)^2 + \left(\frac{e_y}{t}\right)^2} = 0.24$$

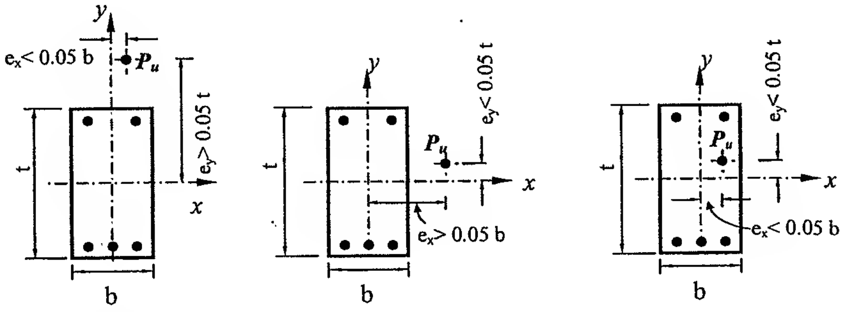
$$\gamma_c = 1.5 \times \left(\frac{7}{6} - \frac{0.24}{3}\right) = 1.63 \approx 1.6 \ldots\ldots o.k$$

$$\gamma_s = 1.15 \times \left(\frac{7}{6} - \frac{0.24}{3}\right) = 1.25 \approx 1.24 \ldots\ldots o.k$$



7.9.3 Minimum Eccentricity for Biaxially Loaded Columns

The Egyptian code states that for columns subjected to biaxial bending, the moment applied on either direction can be neglected if the eccentricity caused by this moment is less than code minimum eccentricity of 0.05 t or 20 mm. The column in such a case will be designed as if it is subjected to a uniaxial bending as shown in Fig. 7.28.



Design for P_u, & M_{ux}
Neglect M_{uy}

Design for P_u & M_{uy}
Neglect M_{ux}

Design for P_u only
Neglect M_{ux} and M_{uy}

Fig. 7.28 Minimum eccentricity requirements for biaxially loaded columns

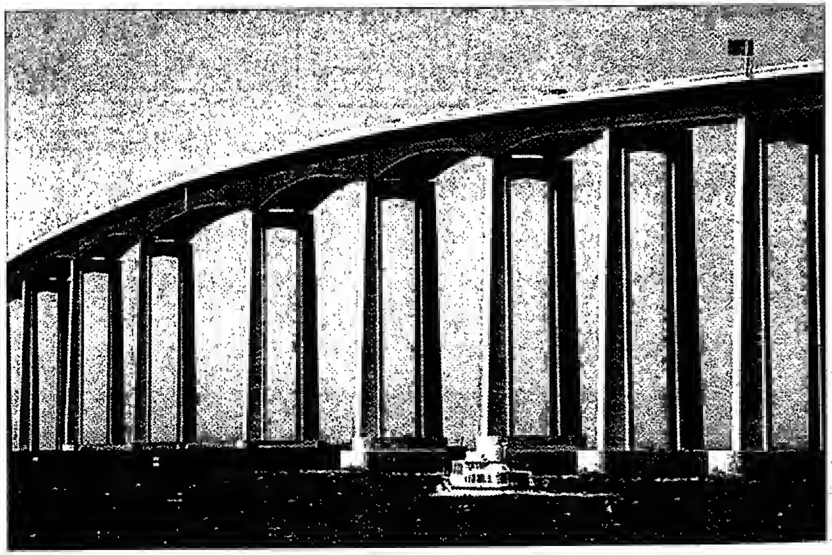
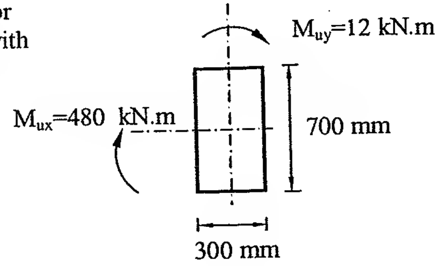


Photo 7.7 Eccentrically loaded columns in a multi-span concrete bridge

Example 7.17

Determine the reinforcement required for a column subjected to biaxial bending with the following data:

P_u = 960 kN
f_{cu} = 25 N/mm^2
f_y = 360 N/mm^2



Solution

The minimum eccentricity in each direction is the bigger of 0.05 of the column dimension and 20 mm

$$e_{x,\min} = 0.05 \times \frac{300}{1000} = 0.015\,m \;\; (or\, 0.02\,m) \qquad e_{y,\min} = 0.05 \times \frac{700}{1000} = 0.035\,m$$

$$e_x = \frac{M_{uy}}{P_u} = \frac{12}{960} = 0.0125\,m < e_{x,\min}\text{, neglect } M_{uy}$$

$$e_y = \frac{M_{ux}}{P_u} = \frac{480}{960} = 0.5\,m > e_{y,\min} \text{ Design for } M_{ux}$$

Thus design as if the section is subjected to **uniaxial bending** P_u, M_{ux}

$$\frac{P_u}{f_{cu}\, b\, t} = \frac{960 \, x \, 1000}{25 \, x \, 300 \, x \, 700} = 0.183$$

$$\frac{M_{ux}}{f_{cu}\, b\, t^2} = \frac{480 \times 10^6}{25 \, x \, 300 \, x \, 700^2} = 0.1306$$

Using interaction diagram with ζ=0.9 f_y=360 N/mm^2, α=1 **(top and bottom)**

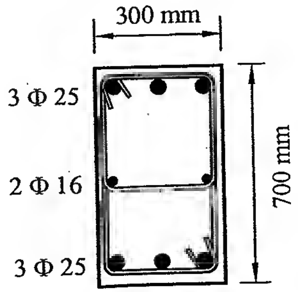
$\rho = 2.7$

$\mu = \rho \times f_{cu} \times 10^{-4} = 2.7 \times 25 \times 10^{-4} = 0.00675$

$A_s = \mu\, b\, t = 0.00675 \times 300 \times 700 = 1418$ mm^2 (3 Φ25)

$A_s' = A_s = 1473\, mm^2$

Please note that the two Φ 16 is added to satisfy code spacing requirement.



7.9.4 Biaxial Interaction Diagrams

The design of sections subjected to biaxial bending can be greatly simplified by using interaction diagrams. The horizontal plane *(constant load level)* is the chosen representation of the failure surface. A computer program was prepared to carry out all the required calculations. The program can be summarized in the following steps

1. The neutral axis is first assumed with certain inclination angel ϕ, the forces and moments in the reinforcement steel are evaluated.
2. The force in the concrete is determined through the integration of the idealized concrete stress strain curve *(not the equivalent stress block)* with the compressed area.
3. A trial and adjustment procedure is performed by changing the neutral axis inclination and/or position until the equilibrium is achieved.
4. Having determined the desired load level, moments are evaluated and a point in the interaction diagram is plotted.
5. For the same reinforcement index (ρ) and load level R_b, several neutral axis positions are assumed and the corresponding bending moments are plotted forming a curve on the interaction diagram.
6. The area of the steel determined from the charts and should be uniformly distributed along the cross section.

7.9.5 The use of Biaxial Interaction Diagrams

The use of biaxially interaction diagrams (refer to Fig. 7.29 and the appendix H) can be summarized in the following steps.

1- Evaluate the load level R_b using the following equation

$$R_b = \frac{P_u}{f_{cu}\,b\,t}$$

2- Calculate the non-dimensional biaxial moments quantities

$$\frac{M_{ux}}{f_{cu}\,b\,t^2} \quad \text{and} \quad \frac{M_{uy}}{f_{cu}\,t\,b^2}$$

3- Locate the reinforcement index ρ from the required load level chart using the previous non-dimensional moments. If the desired load level is not available, use interpolation to find ρ using charts of higher and lower value.

4- Calculate the total area of steel using the following

$$\mu = \rho \times f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\,b\,t$$

This area of steel should be distributed uniformly (uniform in area) along the cross section perimeter. There should be at least three-four bars in each side of the column to ensure uniformity

A computer program was prepared to carryout the computations required to construct the biaxial interaction diagrams. An example of the developed interaction diagrams for biaxially load column is shown in Fig. 7.29. The rest of the design aids is given in Appendix H.

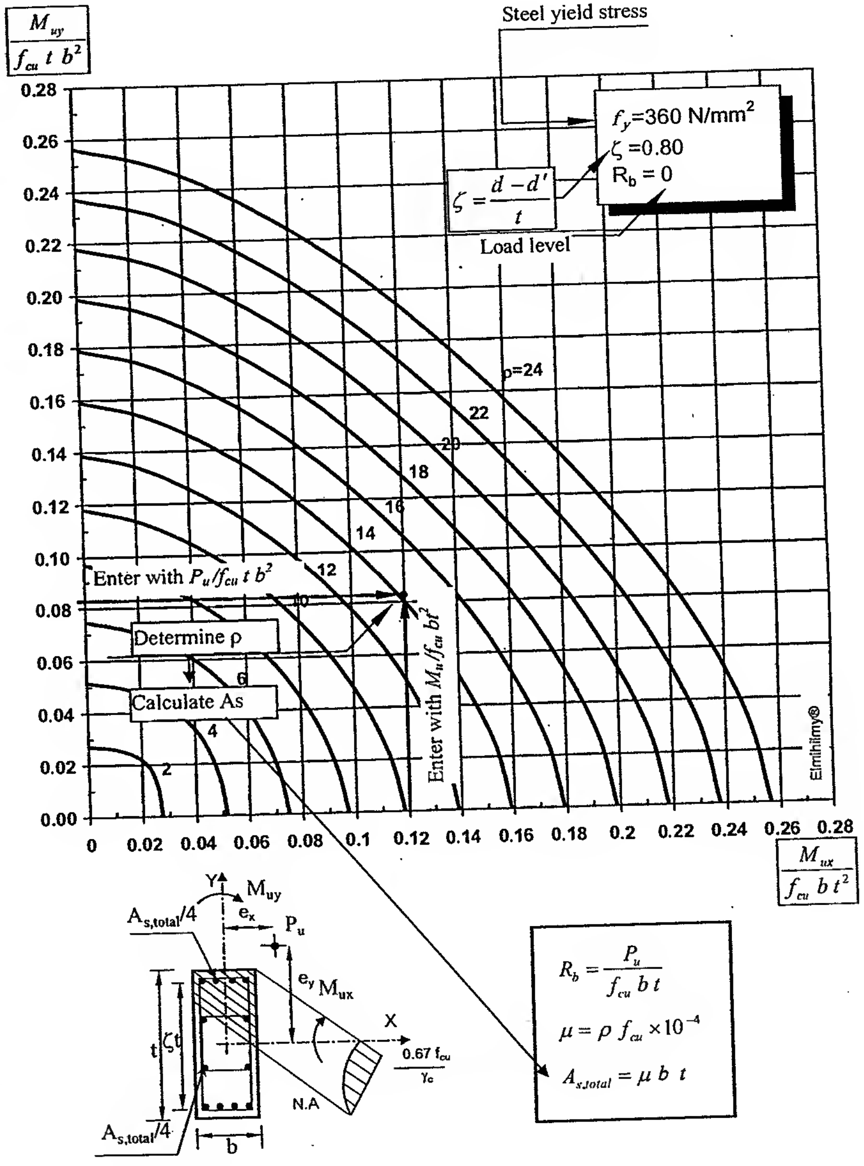
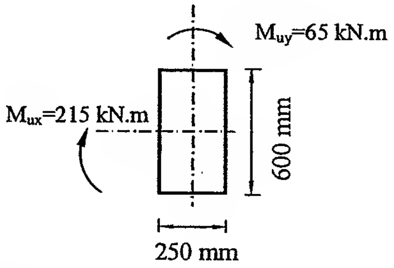


Fig. 7.29 Interaction diagram for biaxially load columns (appendix H)

Example 7.18

Design the reinforcement for a short column subjected to biaxial bending accompanied with compressive force using the following data:

f_{cu} $=35\ N/mm^2$
f_y $=360\ N/mm^2$
P_u $=1200\ kN$
M_{ux} $=215\ kN.m$
M_{uy} $=65\ kN.m$



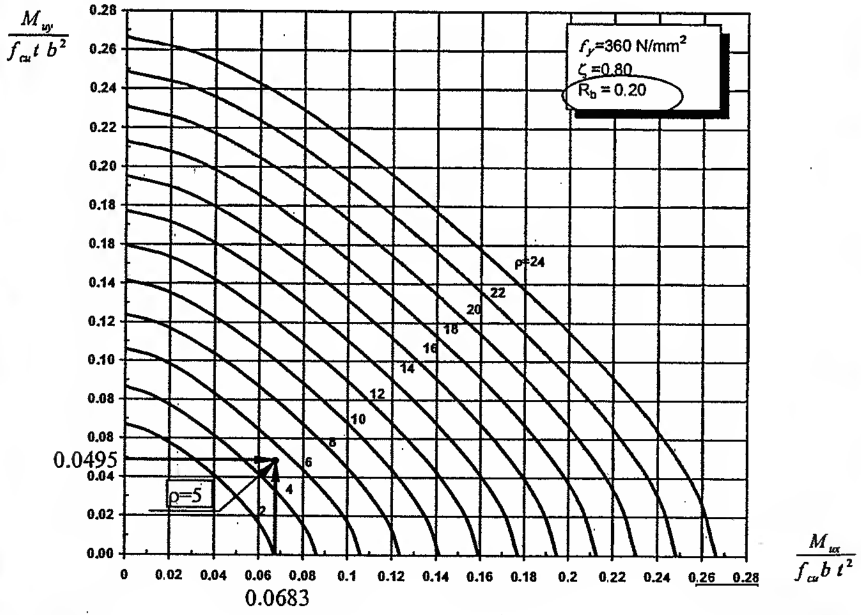
Solution

Step 1: Calculate the following terms

$$R_b=\frac{P_u}{f_{cu}\,b\,t}=\frac{1200\times 1000}{35\times 250\times 600}=0.228$$

$$\frac{M_{ux}}{f_{cu}\,b\,t^2}=\frac{215\times 10^6}{35\times 250\times 600^2}=0.0683$$

$$\frac{M_{uy}}{f_{cu}\,t\,b^2}=\frac{65\times 10^6}{35\times 600\times 250^2}=0.0495$$



Step 2: Calculate reinforcement area

Since the desired load level R_b=0.228 is not available in the biaxial interaction diagrams (see Appendix H; f_y=360 N/mm^2, ζ=0.8), interpolation is performed between R_b=0.2 and R_b=0.3

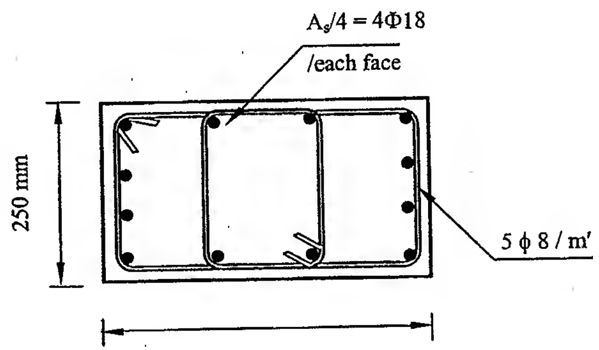
R_b=0.20 → ρ =5.0
R_b=0.30 → ρ =6.8

Interpolating for $R_{b=0.228}$ →→ ρ =5.5

$\mu=\rho\ f_{cu}\ 10^{-4}=5.5\times 35\times 10^{-4}=0.0193>\mu_{min}\ (0.008)$ and $<\mu_{max}(0.04)$

$A_{s,total}=\mu\ b\ t=0.0193\times 250\times 600=2895\ mm^2=28.95\ cm^2$

Choose 12 Φ 18.



7.9.6 ECP-203 Design Procedure for Biaxial Bending

The calculation procedure for sections subjected to biaxial bending is laborious. Many design codes including the Egyptian code adopt the use of the simplified methods for designing members subjected to biaxial bending.

The approximation used in the ECP-203 is to assume that the interaction curve can be represented by two straight lines. Then, transferring the two applied biaxial moments into one magnified (*increased*) uniaxial moment either M'_x or M'_y depending on the ratio of the applied moments and the load level. Fig. 7.30 shows the ratio between the section capacities in each direction.

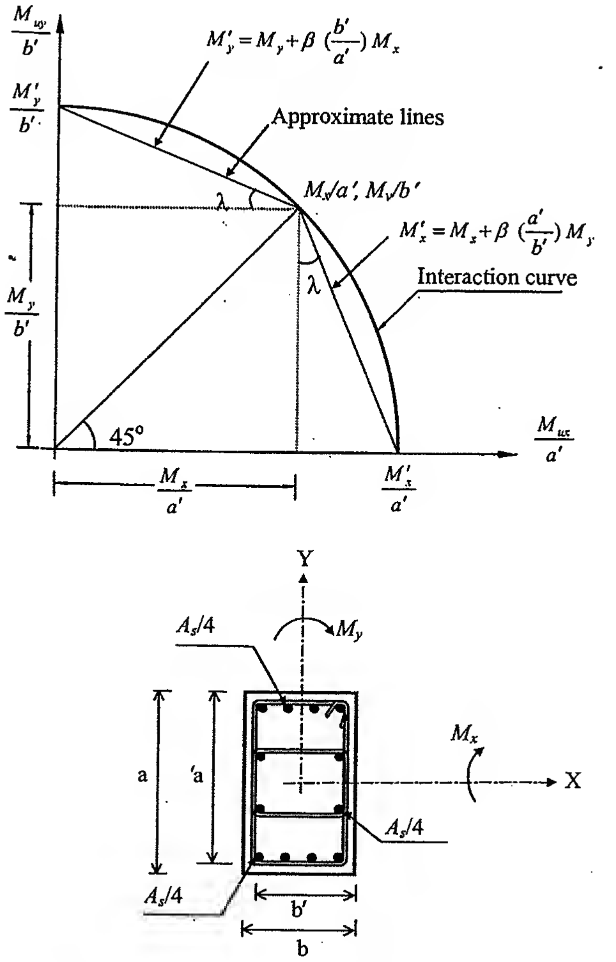


Fig. 7.30 Developemtn of the ECP-203 design method

The angle λ can be calculated as

$$\tan \lambda = \frac{\frac{M'_x}{a'} - \frac{M_x}{a'}}{\frac{M_y}{b'}} \qquad (7.55)$$

where M_x is the design moment, and M'_x is the magnified moment

Defining β as tan λ gives

$$\frac{M'_x}{a'} - \frac{M_x}{a'} = \beta \frac{M_y}{b'} \qquad (7.56)$$

$$\boxed{M'_x = M_x + \beta \left(\frac{a'}{b'}\right) M_y} \qquad (7.57)$$

similarly M'_y

$$\boxed{M'_y = M_y + \beta\,(\frac{b'}{a'})\,M_x} \qquad (7.58)$$

The previous equations are the basis of the code-simplified equations, in which the value of the factor β was determined from the comparisons with the biaxial interaction diagrams.

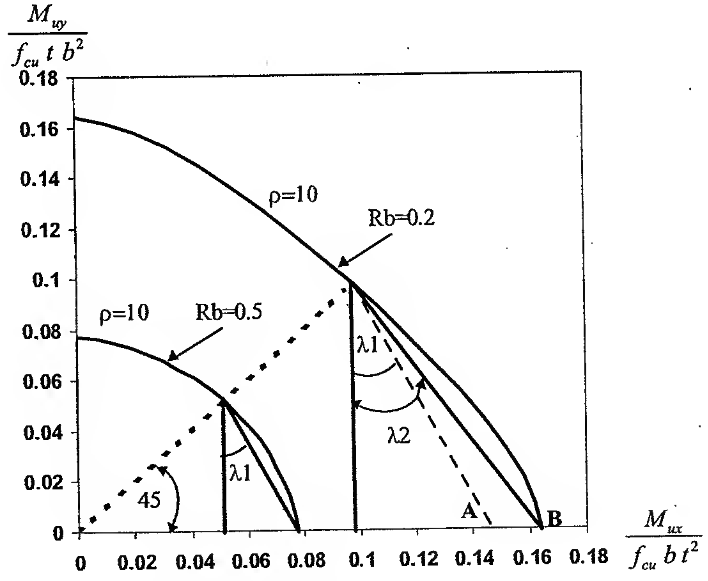


Fig. 7.31 Load contour for different reinforcement ratios and load levels.

Fig. 7.31 shows that for the same reinforcement ratio, the moment capacity for columns with low load level is more than the moment capacity of columns at higher load levels *(above the balanced point)*. Also, the angle λ differs from one load level to another, and a constant value cannot be used. Thus, using λ determined from high load level (R_b=0.5) will lead to unconservative values of M'_x for load levels near the balanced point (R_b =0.2) *(point A instead of point B)*. The actual variation of β (tan λ) with load level is nonlinear. However, the code approximates the relation with a conservative straight line given in Table 7.2 or the following equation.

$$\beta = 0.9 - \frac{R_b}{2} \quad \begin{matrix} \geq 0.6 \\ \leq 0.8 \end{matrix} \qquad \text{.......................} \quad (7.59)$$

Table 7.2 Values of β for different load levels

$R_b = \dfrac{P_u}{f_{cu}\, b\, a}$	≤ 0.2	0.3	0.4	0.5	≥ 0.6
β	0.8	0.75	0.70	0.65	0.60

Figure 7.32 represents the actual values of β determined from the strain compatibility method versus the code approximate values. It can be seen that code value will yield always a conservative design than using the interaction diagrams. The average area of steel using the simplified method is about 10-20 percent higher than the actual vales as shown in Fig. 7.32. However, better accuracy can be hardly obtained from such a simplified design method

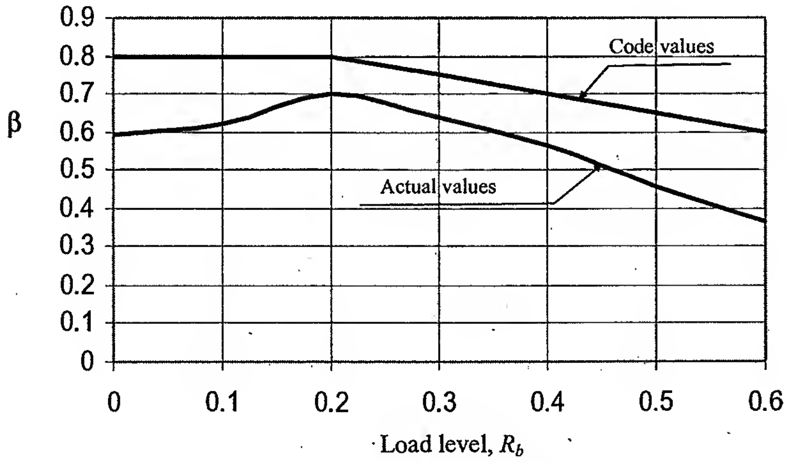


Fig. 7.32 Comparison among actual and code values for β

Design steps for biaxially loaded columns with uniform reinforcement

1- Calculate the applied load level using

$$R_b = \frac{P_u}{f_{cu}\, b\, a}$$

2- From Table 7.2 or Eq. 7.59 determine β factor, use interpolation if required

$$\text{if } \frac{M_x}{a'} > \frac{M_y}{b'} \text{ then } M'_x = M_x + \beta\left(\frac{a'}{b'}\right) M_y$$

$$\text{if } \frac{M_x}{a'} \leq \frac{M_y}{b'} \text{ then } M'_y = M_y + \beta\,\left(\frac{b'}{a'}\right) M_x$$

3- Locate a point on the <u>uniformly distributed</u> <u>steel uniaxial</u> interaction diagrams using

$$\frac{P_u}{f_{cu}\, b\, a} \quad \text{and} \left(\frac{M'_x}{f_{cu}\, b\, a^2} \quad \text{or} \frac{M'_y}{f_{cu}\, a\, b^2}\right)$$

4- Calculate the total area of steel by determining the reinforcement index ρ

$$\mu = \rho \times f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\, b\, t \qquad > A_{s,min}$$

This area of steel should be distributed uniformly (uniform in area) around the cross section perimeter. There should be at least 3→4 bars in each side of the column to ensure uniformity as shown in Fig. 7.33.

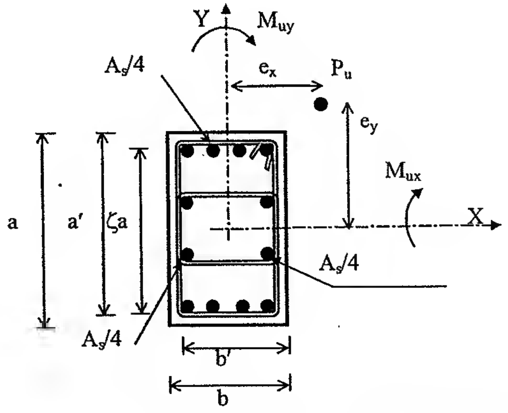


Fig. 7.33 Reinforcement details for biaxially loaded columns

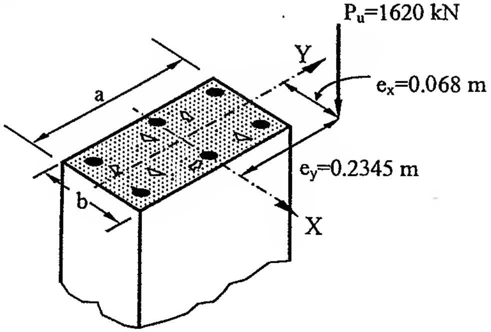
Example 7.19

Design a corner column in a braced building if it is subjected to

P_u =1620 kN
e_x = 0.068 m
e_y = 0.2345 m

The material properties are

f_{cu} =30 N/mm^2
f_y =280 N/mm^2



Solution

Since the column dimensions are not given, assume load level of 0.3 and assume column width of 250 mm. Thus;

$$\frac{P_u}{f_{cu}\,b\,t} = 0.3 = \frac{1620 \times 1000}{30 \times 250 \times t}$$

t=720 mm

Try a column with 250×750 mm

Determine the actual load level R_b using the following equation

$$R_b = \frac{P_u}{f_{cu}\,b\,t} = \frac{1620 \times 1000}{30 \times 250 \times 750} = 0.288$$

From Table 7.2 and using interpolation between R_b=0.2 and R_b=0.3, or directly use Eq. 7.59

$$\beta = 0.90 - \frac{R_b}{2} = 0.90 - \frac{0.288}{2} = 0.756$$

Calculate the applied moments using the given eccentricities

$$M_x = P_u . e_y = 1620 \times 0.2345 = 380\ kN.m$$

$$M_y = P_u . e_x = 1620 \times 0.068 = 110\ kN.m$$

Assume concrete cover = 30 mm, and the distance from the centerline of the reinforcement to the concrete surface 45mm

a' = 750 − 45 = 705 mm

b' = 250- 45= 205 mm

Since M_x/a'= (380/705) > (M_y/b') =(110/205), the design moment will be taken about x. Using Eq. 7.57 gives

$$M'_x = M_x + \beta\left(\frac{a'}{b'}\right)M_y = 380 + 0.756\left(\frac{705}{205}\right)110 = 665.99\ kN.m$$

$$\frac{M'_x}{f_{cu}\,b\,t^2} = \frac{665.99\ x\ 10^6}{30\ x\ 250\ x\ 750^2} = 0.158$$

$$\zeta = \frac{750 - 2 \times 45}{750} = 0.88$$

Using ***uniaxial*** interaction (uniformly distributed steel) f_y= 280 N/mm^2

Since ζ=0.88 use interpolation

For ζ=0.9 ρ =14.0
For ζ=0.8 ρ =16.0

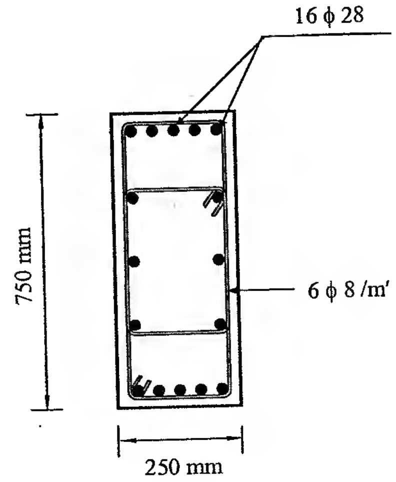
Therefore for ζ= 0.88 →→ ρ=14.4

$\mu = 14.4 \times 30 \times 10^{-4} = 0.0432 > \mu_{min}(0.008)$ and $< \mu_{max}(0.06)$

$A_{s,total} = \mu\ b\ t = 0.0432 \times 250 \times 750 = 8100$ mm^2

The number of bars should be the multiple of 4, thus choose 16 ϕ 28.

The bars are distributed equally on the four sides.



7.9.7 Biaxial Bending in Unsymmetrically Reinforced Sections

Sections with unsymmetrical reinforcement are often encountered in shear walls, beam-columns and columns with high rectangularity ratio. From the cost-effectiveness point of view, placing the reinforcement in the direction of the large moment is more efficient. Thus, the ECP 203 provides a simple method for designing biaxially loaded sections in which the steel is not uniformly distributed.

This simplified method is based on magnifying the applied moments by a moment magnification factor. The magnification factor α_b was introduced to modify the design moments in both directions according to the load level and the ratio of the applied biaxial moments. The magnification factor α_b was evaluated after the examination of the exact solution of the unsymmetrical interaction diagrams[1] (as the one shown in Fig. 7.34). The code permits the use of this simplified method until a fairly high load level of $R_b \leq 0.50$ given by Eq. 7.60a. If Rb>0.5, biaxial interaction diagram with uniform steel or the simplified method (7.9.2) may be used. The magnified moments about X and Y-axes; M'_x and M'_y, are modified by the same factor α_b provided that $(P_u / f_{cu}\, b\ a) \leq 0.50$ as follows:

$$R_b = \frac{P_u}{f_{cu}\, b\ a} \quad \text{...(7.60a)}$$

$$M'_x = \alpha_b \quad M_x \quad \text{...(7.60b)}$$

$$M'_y = \alpha_b \quad M_y \quad \text{...(7.60c)}$$

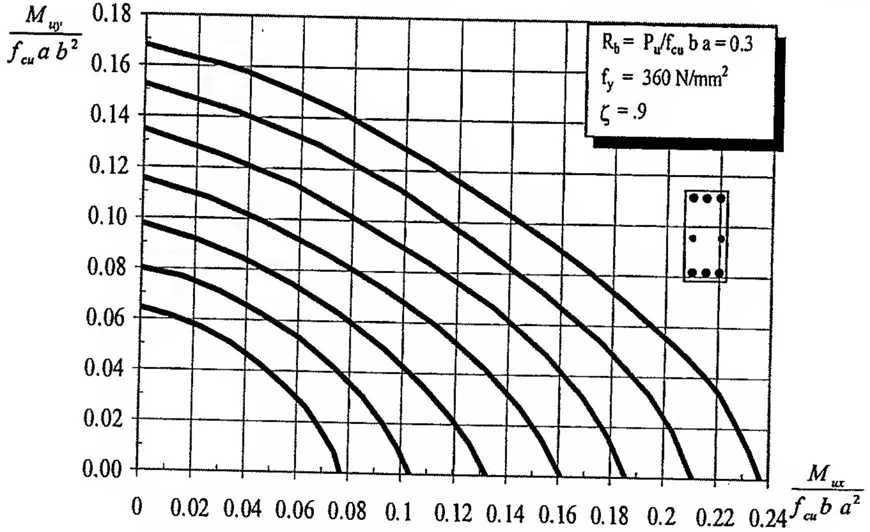


Fig. 7.34 Biaxial interaction diagram for unsymmetrically reinforced sections⁽¹⁾

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[1] Developed by the authors

The uniaxial bending interaction diagrams (top and bottom steel only-Appendix B) is used <u>twice</u> to calculate the area of steel for each direction.

This approach is easy to use in routine calculations and yields approximately the same results obtained through the interaction design curves for biaxially loaded members with unsymmetrical reinforcement. The suggested values of α_b are given in Table 7.3. It can be noticed that the values of the coefficient α_b are symmetrical about $(M_x/a')/(M_y/b') = 1$ (case of uniform steel). It can also be noticed from Fig. 7.35 that the general practice of designing rectangular reinforced concrete beams subjected to pure biaxial bending (R_b=0) twice by using the design moments M_x and M_y without magnification (α_b=1) is valid. However for sections subjected to biaxial bending with normal force this assumption will yield unconservative results, as the design bending moments need to be magnified.

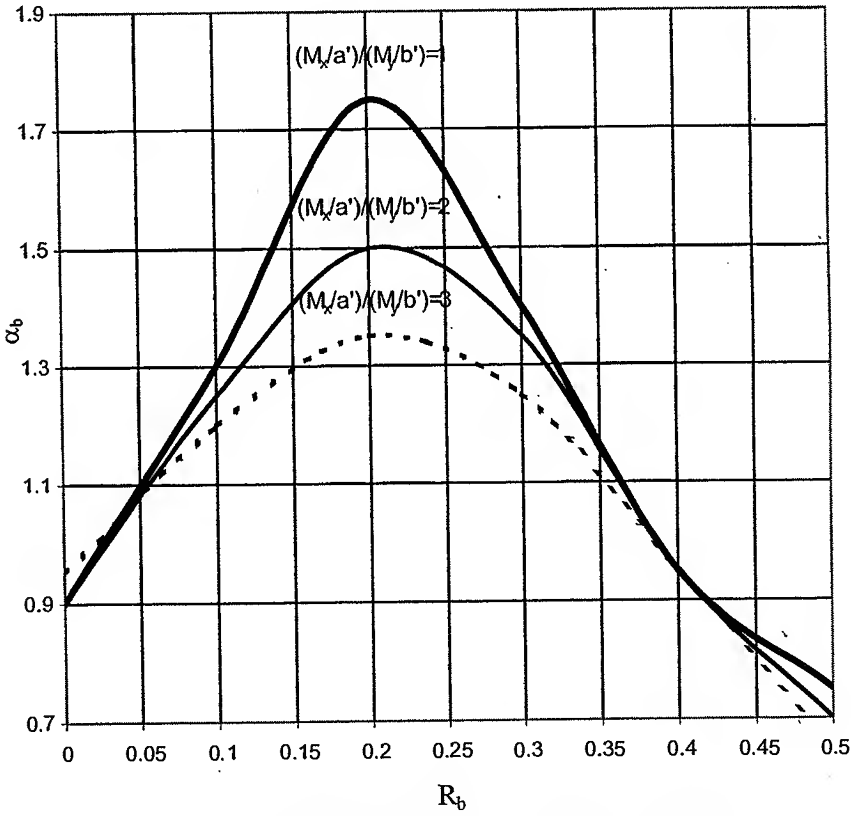


Fig. 7.35 Actual values of α_b for rectangular cross sections

Table 7. 3 values of α_b for rectangular cross sections

$(M_x/a')/(M_y/b')$ $R_b=P_u/\ f_{cu}\ b\ a$	∞*	3	2	1	0.5	0.33	0
$R_b \le 0.1$	1	1.2	1.25	1.30	1.25	1.20	1
R_b=0.2	1	1.35	1.50	1.75	1.50	1.35	1
R_b=0.3	1	1.25	1.35	1.4	1.35	1.25	1
R_b=0.4	1	0.95	0.95	0.95	0.95	0.95	1
R_b=0.5**	1	0.65	0.70	0.75	0.70	0.65	1

* This is a case of M_y=0. For cases in which M_y does not equal to zero, value of 10 is sufficient for interpolation

** If R_b>0.5, biaxial interaction diagram with uniform steel or the simplified method (7.9.2) may be used

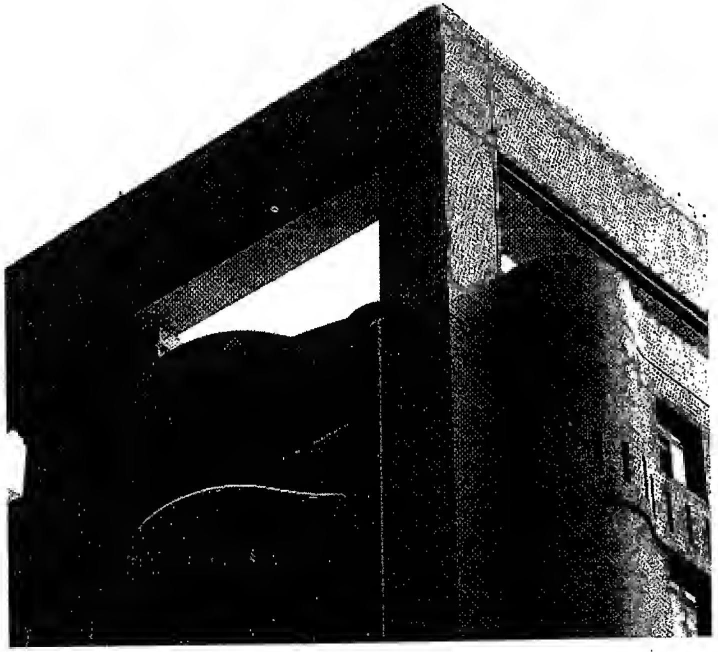


Photo 7.8 Biaxially loaded corner column in court house

Design Steps for biaxially loaded columns with unsymmetrical reinforcement

1- Calculate the applied load level and moment ratio using

$$R_b = \frac{P_u}{f_{cu}\, b\, a} \quad \text{and} \quad \frac{M_x / a'}{M_y / b'}$$

2- From the table determine α_b factor $R_b \le 0.50$, use interpolation if required

$$M'_x = \alpha_b\ M_x$$

$$M'_y = \alpha_b\ M_y$$

3- Use **<u>Top and bottom</u>** steel interaction diagrams **<u>twice</u>** to determine the reinforcement index ρ, calculate the total area of steel.

$$\text{Using } \frac{M'_x}{f_{cu}\, b\, a^2} \text{ and } \frac{P_u}{f_{cu}\ b\ a} \text{ determine } \mu_x = \rho \times f_{cu} \times 10^{-4}$$

$$\text{Using } \frac{M'_y}{f_{cu}\, a\, b^2} \text{ and } \frac{P_u}{f_{cu}\ b\ a} \text{ determine } \mu_y = \rho \times f_{cu} \times 10^{-4}$$

$$\frac{A_{sx}}{2} = \mu_x\ b\ t \qquad \text{and} \qquad \frac{A_{sy}}{2} = \mu_y\ b\ t$$

The area of steel determined for each direction should be duplicated at the opposite face as shown in Fig. 7.36. The total area of steel equals

$$A_{s,total} = A_{sx} + A_{sy} = 2\,(\mu_x + \mu_y)\, b\, a$$

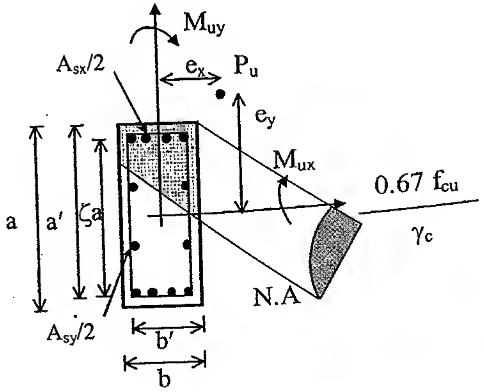
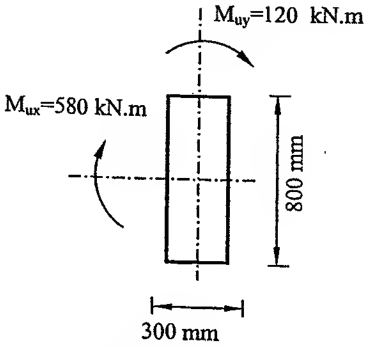


Fig. 7.36 Analysis of sections with unsymmetrical reinforcement

Example 7.20

Determine the **unsymmetrical** reinforcement required for a short interior column subjected to biaxial bending with the following data:
Column size 300 x 800 mm

P_u = 1440 kN
M_{ux} = 580 kN.m
M_{uy} =120 kN.m
f_{cu} = 30 N/mm^2
f_y = 400 N/mm^2



Solution

Step 1: Calculate the magnified moments.

Assuming concrete cover 40 mm, thus

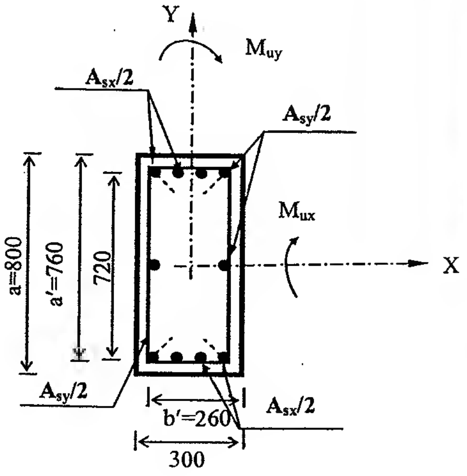
a′=800-40=760 mm

b′=300-40=260 mm

Calculate

$$\frac{M_{ux}/a'}{M_{uy}/b'}=\frac{580/760}{120/260}=1.65$$

$$R_b=\frac{P_u}{f_{cu}\,b\,a}=\frac{1440\ x\ 1000}{30\ x\ 300\ x\ 800}=0.20$$



Note that the corner bar is divided between x and y directions

From Table 7.3 in this text and by using interpolation α_b=1.587

$R_b=P_u/f_{cu}\,b\,a$ \ $(M_x/a')/(M_y/b')$	∞*	3	2	1	0.5	0.33	0
$R_b \le 0.1$	1	1.2	1.25	1.30	1.25	1.20	1
R_b=0.2	1	1.35	1.50	1.75	1.50	1.35	1
R_b=0.3	1	1.25	1.35	1.4	1.35	1.25	1
R_b=0.4	1	0.95	0.95	0.95	0.95	0.95	1
R_b=0.5**	1	0.65	0.70	0.75	0.70	0.65	1

1.587

$M'_x = M_{ux}\ .\ \alpha_b$ = 580 (1.587) = 920.2 kN.m

$M'_y = M_{uy}\ .\ \alpha_b$ = 120 (1.587) = 190.4 kN.m

Calculate the non-dimensional quantities:

$$\frac{M'_x}{f_{cu}\,b\,a^2}=\frac{920.2\ x\ 1000\ x\ 1000}{30\ x\ 300\ x\ 800^2}=0.159$$

$$\frac{M'_y}{f_{cu}\,a\,b^2}=\frac{190.4\ x\ 1000\ x\ 1000}{30\ x\ 800\ x\ 300^2}=0.088$$

$$\zeta_a=\frac{800-2\times40}{800}=0.90$$

$$\zeta_b=\frac{250-2\times40}{250}=0.685\approx0.70$$

Using the regular uniaxial interaction diagrams with symmetrical reinforcement top and bottom (f_y=400 N/mm^2, α=1)

Step 2: Area of steel for M'_x

ζ_a=0.9, $\frac{P_u}{f_{cu}\,b\,a}=0.20$, $\frac{M'_x}{f_{cu}\,b\,a^2}=0.159$, the reinforcement index (ρ_x) equals

ρ_x = 3.4
$\mu_x = \rho \times f_{cu}\ 10^{-4} = 3.4\ x\ 30\ x\ 10^{-4} = 0.0102$
$A_s = A'_s = A_{sx}/2$
$(A_{sx}/2) = \mu_x\ b\ a = 0.0102\ x\ 300\ x\ 800 = 2448\ mm^2$ (@each side)

Step 3: Area of steel for M'_y

$\zeta_b = 0.7$, $\frac{P_u}{f_{cu}\, b\, a} = 0.20$, $\frac{M'_y}{f_{cu}\, a\, b^2} = 0.088$, the reinforcement index (ρ_y) equals

$\rho_y = 1.6$

$\mu_y = \rho_y\ f_{cu}\ 10^{-4} = 1.6 \times 30 \times 10^{-4} = 0.0048$

$(A_{sy}/2) = \mu_y\, b\, a = 0.0048 \times 300 \times 800 = 1152\ \text{mm}^2$ (@each side)

Step 4: Final design

$A_{s,min} = 0.008 \times 300 \times 800 = 1920\ \text{mm}^2$

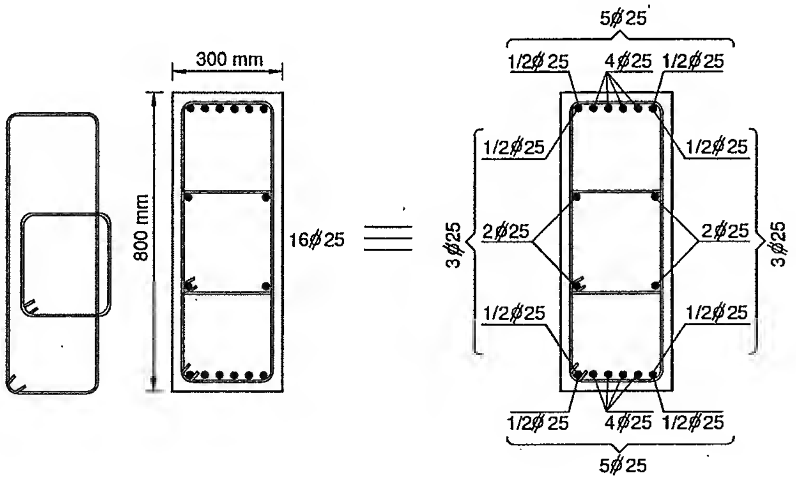
$A_{s,total} = A_{sx} + A_{sy} = 2\ (2448 + 1152) = 7200\ \text{mm}^2 > A_{s,min}$

$\mu = \frac{A_{s,total}}{b \times d} = \frac{7200}{300 \times 800} = 0.03 < (\mu_{max} = 0.04$ for interior column)

(choose 12 Φ 28, 7389 mm^2)

Note : The corner bar is divided between x direction and y direction

[$A_{sx}/2$ = 5 Φ 25 (2454 mm^2), $A_{sy}/2$ = 3 Φ 25 (1473 mm^2)]



7.9.8 Circular Columns under Biaxial Bending

Biaxial moments applied to circular sections can be transformed into equivalent uniaxial bending. This procedure is possible because the bending strength of circular columns is the same in all directions due to the complete symmetry of the section. Thus the design of a circular column subjected to biaxial bending (as the one shown in Fig. 7.37) is the same as the one subjected to uniaxial bending but with a resultant uniaxial moment equals

$$M_u = \sqrt{M_{ux}^2 + M_{uy}^2} \quad \text{.......... (7.61)}$$

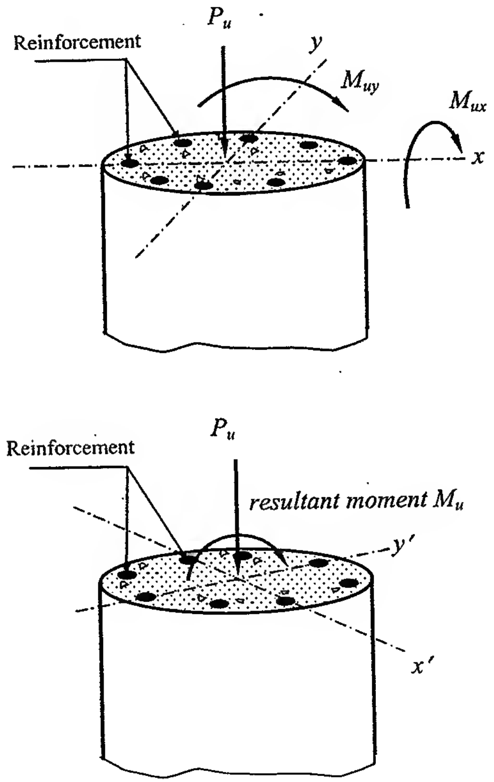


Fig. 7.37 Circular sections under biaxial bending

The previous procedure is valid in case of using almost uniform steel distribution. The use of at least 12 bars is considered sufficient to ensure uniformity.

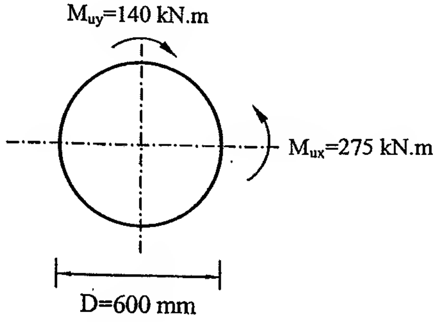
Example 7.21

Design a circular column in a braced building if it is subjected to:

P_u =560 kN
M_{ux} =275 kN.m
M_{uy} =140 kN.m

The material properties are

f_{cu} =25 N/mm^2
f_y =240 N/mm^2



Solution

The column is subjected to biaxial bending moments. It can be designed to withstand a resultant moment M_u

$$M_u = \sqrt{M_{ux}^2 + M_{uy}^2}$$

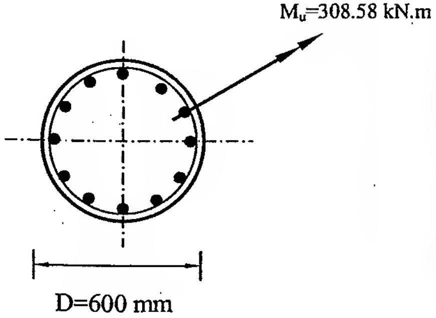
$$M_u = \sqrt{275^2 + 140^2} = 308.58\ kN.m$$

Assume concrete cover of 30 mm

$$\zeta = \frac{r'}{r} = \frac{300-30}{300} = 0.9$$

$$\frac{P_u}{f_{cu}\ r^2} = \frac{560\times 1000}{25\times 300^2} = 0.2489$$

$$\frac{M_u}{f_{cu}\ r^3} = \frac{308.58\times 10^6}{25\times 300^3} = 0.457$$



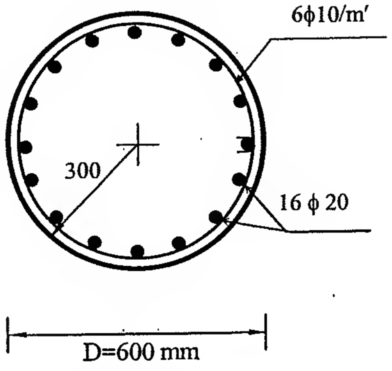
Using interaction diagrams for circular sections (Appendix E) f_y= 240 N/mm^2, ζ=0.9

ρ=6.8

$\mu = \rho\ f_{cu} \times 10^{-4} = 6.8\times 25\times 10^{-4} = 0.017$ $>\mu_{min}$ (0.008) and $< \mu_{max}$ (0.04)

$$A_{s,total} = \mu\ \pi\ r^2 = 0.017\times\pi\times 300^2 = 4806\ mm^2$$

Choose 16 ϕ 20



Reinforcement detail

Assuming that the lateral reinforcement is 6 ϕ10/m′, thus the volume of the stirrups V_s equals ($A_{b(\phi 10)}$=78 mm^2):

$$V_s = n\pi\times D_k A_b = 6\times\pi\times 550\times 78 = 808645\ mm^3$$

$$V_{s,min} = \frac{0.25}{100}\times\frac{\pi}{4}\times D^2\times 1000 = \frac{0.25}{100}\times\frac{\pi}{4}\times 600^2\times 1000 = 706858\ mm^3 < V_s ....o.k$$

7.9.9 Interaction Diagrams for L-Sections

L-section columns are often encountered in the corners of buildings. Most of these columns are subjected to bending in addition to the normal force. Since L-sections are not symmetrical about either axis, evaluating their strength is very complicated and time consuming. In addition, the eccentricity of the applied load with respect to the local axes affects the resistance of the section. Developing interaction diagrams for L-sections is a design tool without any approximations. The construction of the interaction diagram for L-sections is similar to that for rectangular columns subjected to biaxial bending, but the neutral axis has to be assumed in one of four positions as shown in Fig. 7.38.

The developed charts are limited to columns that are symmetrical about a 45° axis as in Fig. 7.38. Furthermore, the width of the column b is defined as a ratio from column height $\lambda = \frac{b}{t}$.

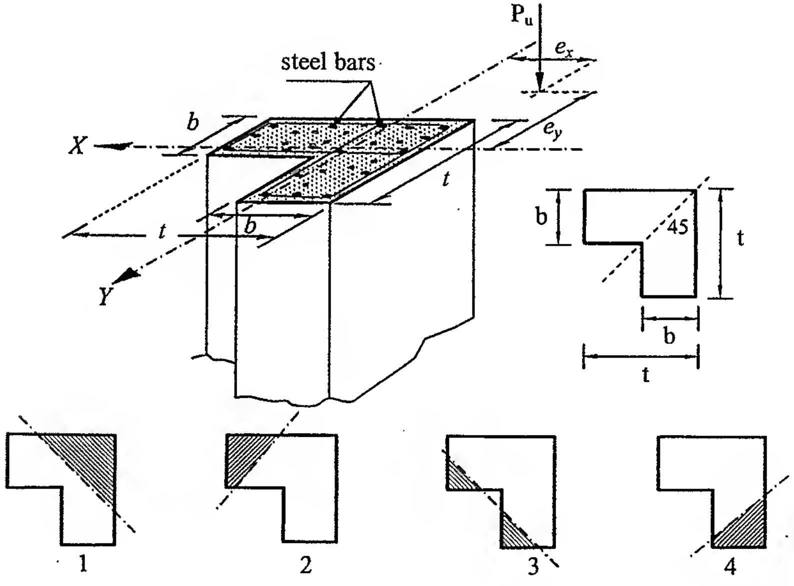


Fig. 7.38 Different positions for the neutral axis in L-sections

The load level is determined by normalizing the applied compressive force with the net concrete area A_c.

$$R_b = \frac{P_u}{f_{cu}\ A_c} \quad \text{..} \quad (7.62)$$

$$A_c = b \times t + b(t - b) = 2 \times t \times b - b^2 \quad \text{..................................} \quad (7.63)$$

The moment is normalized with respect to the net concrete area A_c and the section thickness t.

$$\frac{M_{uy}}{f_{cu}\ A_c\ t} \text{ and } \frac{M_{ux}}{f_{cu} A_c\ t}$$

The area of steel should be at least 16 bars and should be a multiple of 8 to ensure uniformity and equals

$$A_{s,total} = \mu \times A_c \quad \text{..} \quad (7.64)$$

An example of these charts is shown in Fig. 7.39, the rest of the charts is given in the Appendix I.

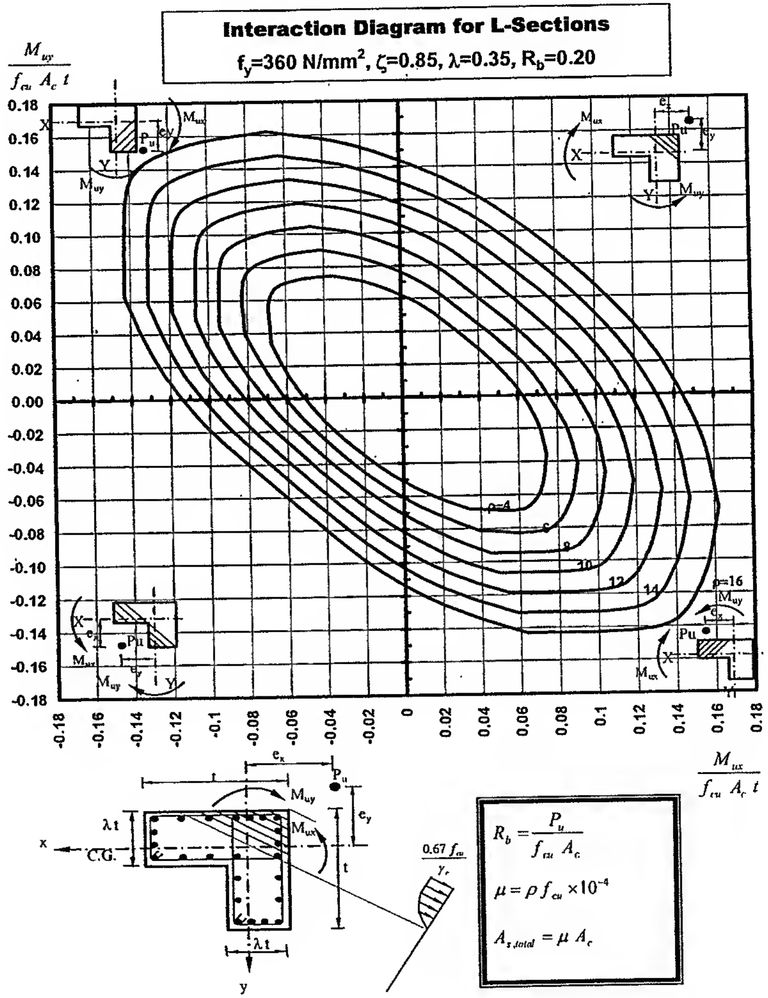


Fig. 7.39 Biaxial Interaction Diagrams for L-sections (appendix I)

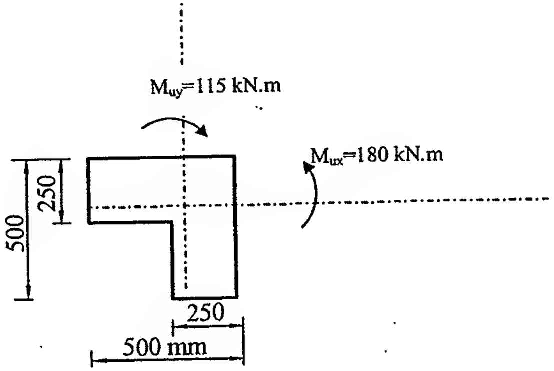
Example 7.22

Determine the area of steel required for an L-shape reinforced concrete column that is subjected to the following straining actions:

P_u =950 kN
M_{ux} =180 kN.m
M_{uy} =115 kN.m

The material properties are as follows:

f_{cu} =25 N/mm^2
f_y =360 N/mm^2



Solution:

Calculate the area of the concrete section A_c

$$A_c = 500\times 250 + 250\times 250 = 187500\ mm^2$$

$$\lambda_t = \frac{b}{t} = \frac{250}{500} = 0.5$$

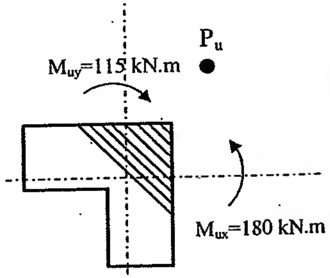
Note that the direction of the given moments will produce a compression in zone 1, thus the normal force will be as shown in figure.

Calculate the following terms

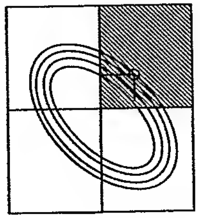
$$R_b = \frac{P_u}{f_{cu}\ A_c} = \frac{950\ x\ 1000}{25\ x\ 187500} = 0.202$$

$$\frac{M_{ux}}{f_{cu} A_c\ t} = \frac{180\ x\ 1000\ x\ 1000}{25\ x\ 187500\times 500} = 0.0768$$

$$\frac{M_{uy}}{f_{cu}\ A_c\ t} = \frac{115\ x\ 1000\ x\ 1000}{25\ x\ 187500\times 500} = 0.049$$



Referring to chart (in Appendix I) with R_b=0.2, λ=0.50, ζ=0.85 the compression zone in the first quarter of the chart, the reinforcement index ρ can be obtained as follows



$\rho = 10.2$

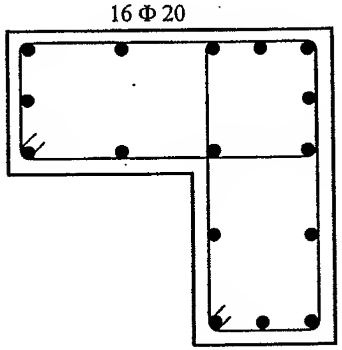
$\mu = \rho\ \ f_{cu}\ 10^{-4} = 10.2 \times 25 \times 10^{-4} = 0.0255$

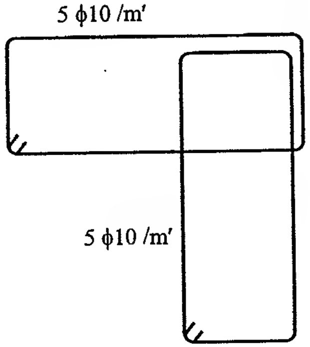
$A_{s,total} = \mu\ \ A_c = 0.0255 \times 187500 = 4781.25\ mm^2$

The area of steel should be at least 8 bars and should be a multiple of 8.

$$Area\ of\ \ one\ bar = \frac{4781}{16} = 299\ mm^2$$

$A_{s,chosen}$= 16 Φ 20, 5026 mm^2





8

SLENDER COLUMNS

Photo 8.1 Unbraced slender columns at Rice University, USA.

8.1 Definition of Slender Columns

Consider a very short column subjected to increasing axial load, eventually the compressive stress is exceeded and compression failure occurs. Now consider a relatively slender column subjected to increasing axial load, when the load reaches a certain value, the column begins to bend about its weaker axis and deflect laterally. The column is said to have buckled.

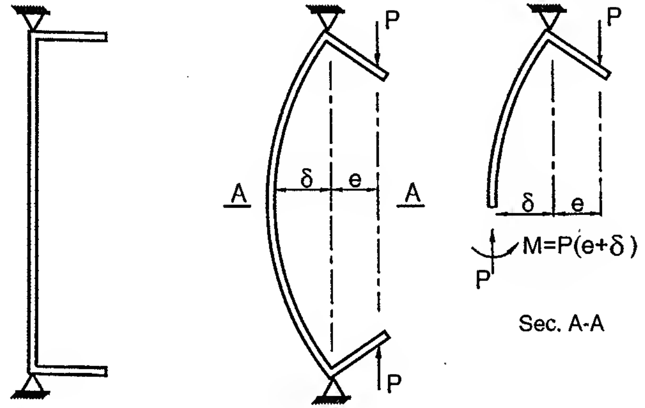
For a pin-pin ideal column (without any imperfections) the buckling load (also called Euler load) is given by

$$P_{cr} = \frac{\pi^2 (EI)}{L^2} \qquad \text{(8.1)}$$

in which *EI* is the flexural rigidity and L is the column length.

Thus, a column is more likely to buckle when either the length *(L)* is increased or the flexural rigidity *(EI)* is reduced. Up to Euler load, for a perfectly straight member, the column is stable without any lateral deformation. However, at Euler load, the column will be at bifurcation "*unstable*" equilibrium, in which it will buckle laterally with indeterminate magnitude. The previous behavior is applied for perfectly straight column, which almost does not exist in reinforced concrete industry. Columns will not be exactly vertical and loads are always slightly eccentric.

Figure 8.1a shows a pin-ended column before loading. Assuming that the column is loaded at an eccentricity e, then it will laterally deflect by an amount δ as shown in Fig. 8.1b. This lateral deflection increases the moments for which the column must be designed. In the symmetrically loaded column shown here, the maximum moment occurs at mid-height where the maximum deflection occurs.
The moments at the ends of the column are:



(a) Before loading (b) During loading

Fig. 8.1 Straining actions for a deflected column

$$M_e = Pe \quad \text{.......................................} (8.2.a)$$

For equilibrium, the internal moment at mid-height must be

$$M = P(e+\delta) \quad \text{.......................................} (8.2.b)$$

A *slender column* is defined as a column that has a significant reduction in its axial load capacity due to moments resulting from lateral deflections of the column.

Figure 8.2 shows an interaction diagram for a cross-section of a reinforced concrete column. This diagram gives the combinations of axial load and moment, which are required to cause failure of a very short column. The dashed radial line O-A is a plot for the end moment on the column in Fig. 8.1b. Since the load is applied at a constant eccentricity, e, the end moment, M_e, is a linear function of P, as given by Eq. (8.2.a). The curved solid line O-B is the moment M_c at mid-height of the column, given by Eq. (8.2.b). At any given load P, the moment at mid-height is the sum of the end moment, *Pe*, and the moment due to the lateral deflection, Pδ. The line O-A is referred to as a load-moment curve for the end moment, while the line O-B is the load-moment curve for the total column moment. If the column is slender, failure occurs when the load-moment curve O-B intersects the interaction diagram for the cross-section at point B as shown in Fig. 8.2. Because of the increase in maximum moment due to the secondary moments, the axial load capacity is reduced from A to B. This reduction in axial load capacity results from what are referred to as *slenderness effect*. Since, failure still occurs at one of the points of the interaction diagram, it is called material failure.
For very slender columns, failure occurs well within the cross-section interaction diagram because of the pronounced second-order effect (slenderness effect). This type of failure is called stability failure. In this type of failure, the collapse load of the column (point C) is less than the actual material given by the interaction diagram.

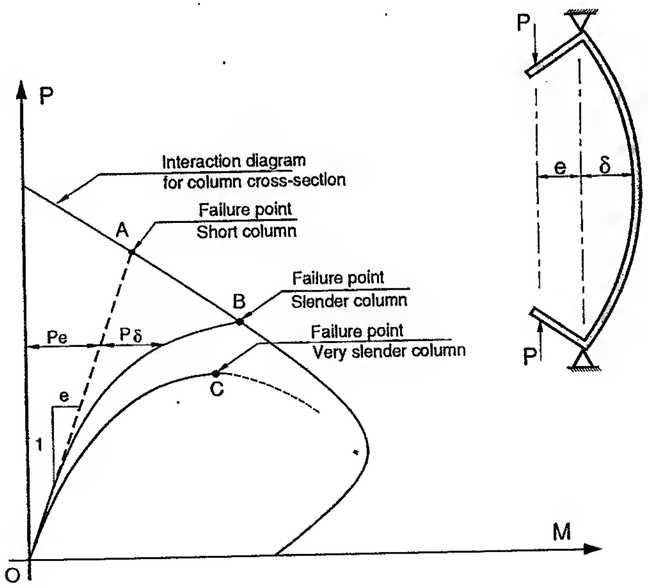


Fig. 8.2 Loads and moments in short and slender columns

8.2 Classification of Buildings

The Egyptian Code ECP classifies concrete structures as being *braced* or *unbraced.*

Many concrete building structures are braced by providing shear walls, cores, or elevator shafts. The stiffness of these elements is considerably higher than the columns themselves and may be assumed to attract all horizontal forces (Fig. 8.4a).

An unbraced building is the one that resists the lateral loads by the framing action of the beams and the columns or that is provided with flexible shear walls (Fig. 8.4b). According to the ECP-203, a building that does not contain shear walls is considered unbraced.

According to the Egyptian Code, a building that contains shear walls or cores that extend the full building height can be considered braced at a certain direction if they were symmetrically distributed and satisfy the following equation:

$$\alpha = H_b \times \sqrt{\frac{\sum N}{\sum EI}} \quad < \ 0.60 \qquad for \quad n \geq 4 \ \text{..................}(8.3a)$$

$$< \ 0.20+0.1n \qquad for \quad n < 4 \ \text{.............}(8.3b)$$

where

H_b = height of the building in meters above foundation

N = sum of all unfactored vertical loads of the building (total working gravity loads)

$\sum EI$ = sum of the flexural rigidities of all the vertical stiffening elements under service conditions.

n = number of stories.

E $= 4400\sqrt{f_{cu}}$

It should be mentioned that a building can be considered braced in one direction and unbraced in the other depending on the distribution of the walls on plan. Fig 8.4 shows examples of braced and unbraced structures. To check the bracing of a building in the two directions, one has to calculate the values α_x and α_y as follows:

$$\alpha_x = H_b \times \sqrt{\frac{\sum N}{\sum EI_y}} \qquad \text{(bracing in x-direction) ....(8.3c)}$$

$$\alpha_y = H_b \times \sqrt{\frac{\sum N}{\sum EI_x}} \qquad \text{(bracing in y-direction) ....(8.3d)}$$

Example 8.1 illustrates the application of the above equations

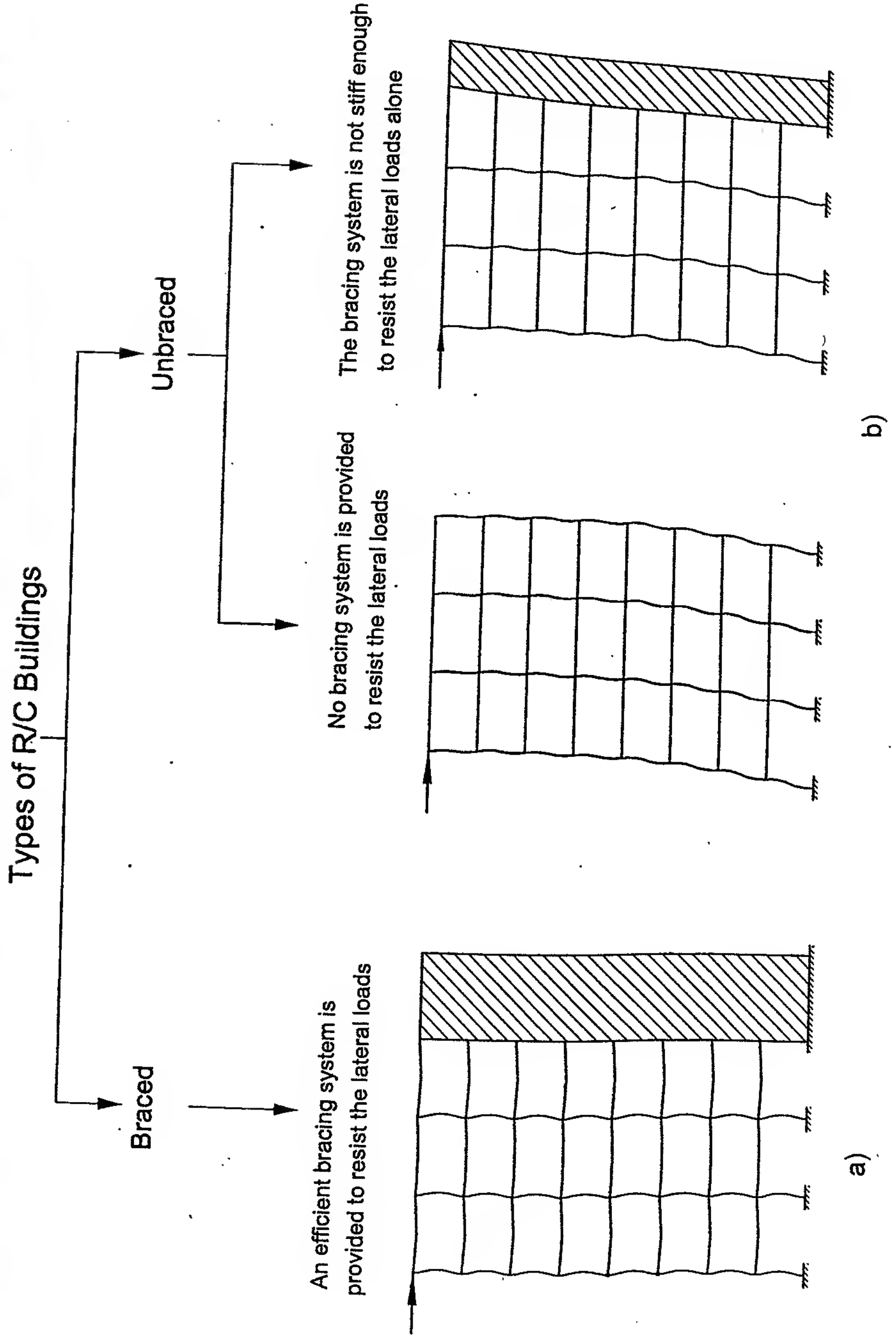
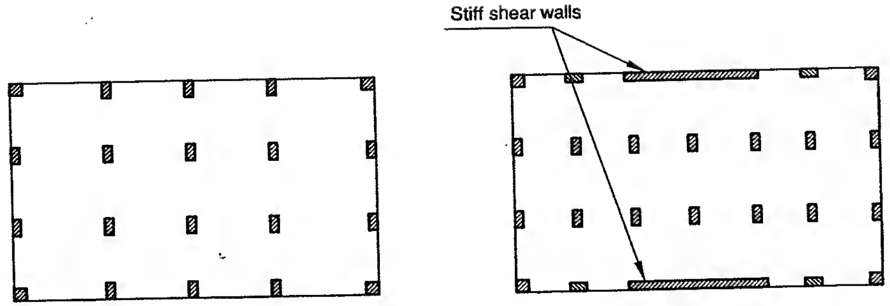


Fig. 8.3 Classification of buildings according to ECCS 203



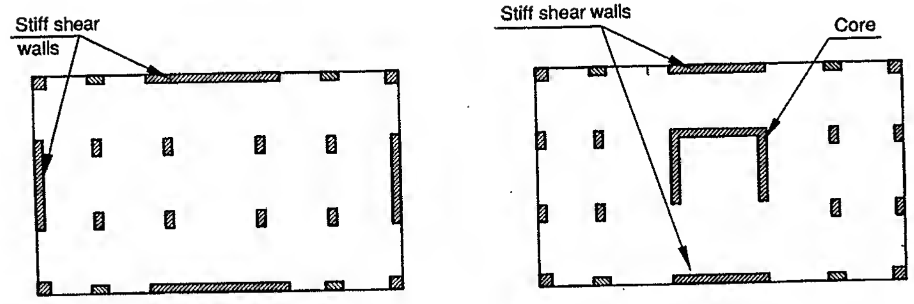
(a) No shear walls or core)

Unbraced structure

(b) (shear walls system)

Unbraced in Y-direction

Braced in X-direction



(c) shear walls system

Braced structure in X&Y directions

(d) shear walls+core system

Braced structure in X&Y directions

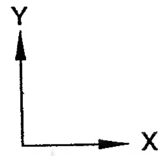


Fig. 8.4 Examples for braced & unbraced structures

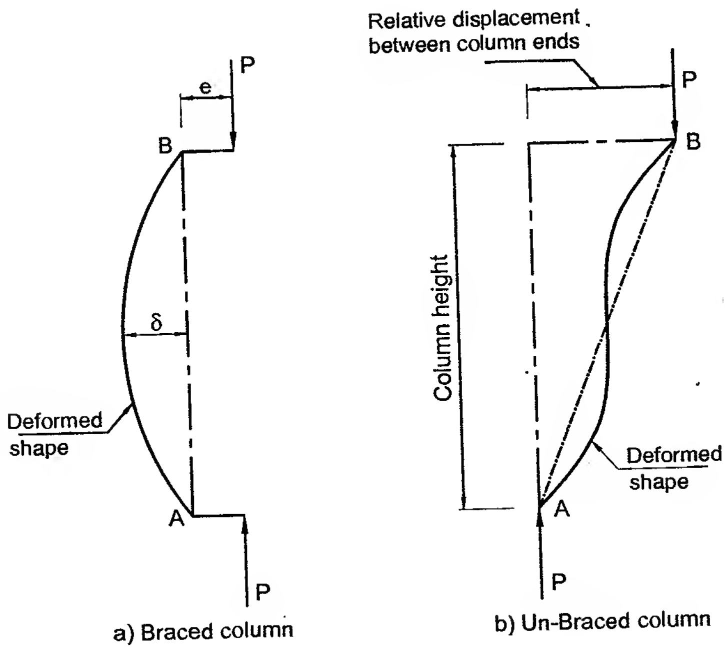
8.3 Braced and Unbraced Columns

Columns located in braced structures are referred to as braced columns. Columns located in unbraced structures are referred to as unbraced columns.

Eccentrically loaded braced slender columns are subjected to additional moments due to the fact that the center-line deviates from the original vertical (un-deformed) shape. This is called additional moments due to *member stability effect* (Fig. 8.5a).

Unbraced columns are subjected to additional moments due mainly to *lateral drift effect,* which occurs due to the fact that each story is laterally shifted with respect to the one below (Fig. 8.5b). Member stability effect still exists but has a minor effect.

It should be noted that member stability effect results in additional moments in braced slender columns. On the other hand, lateral drift effect results in additional moments in unbraced short columns as well as in unbraced slender columns. In unbraced slender columns, however, member stability effect might increase the additional moments.



a) Braced column

b) Un-Braced column

Fig. 8.5 Braced and unbraced columns

8.4 Slenderness Considerations in the Egyptian Code

This section outlines the procedure adopted by the Egyptian Code for slenderness consideration.

8.4.1 Code Definition of Slender Columns

The degree of slenderness is generally expressed in terms of the slenderness ratios

$$\lambda_b = \frac{H_e}{b} \quad \text{.........(8.4.a)}$$

or

$$\lambda_i = \frac{H_e}{i} \quad \text{.........(8.4.b)}$$

in which

$$i = \sqrt{\frac{I_g}{A_g}} \quad \text{.........(8.5)}$$

and

$$H_e = k\, H_o \quad \text{.........(8.6)}$$

where

H_e = the buckling length or the effective height of the compression member.
b = the column dimension perpendicular to the axis of bending.
k = length factor which depends on the end conditions of the compression member as well as the bracing conditions.
H_o = unsupported (clear) height of the compression member.
i = the radius of gyration of the cross section and can be taken as 0.3 b for rectangular section and 0.25 D for circular section. For other shapes, I may be computed for the gross concrete section.

According to the Egyptian Code, slender columns are defined as those that have slenderness ratios greater than those mentioned in Table (8.1) but not more than those mentioned in Table (8.2). The minimum area of steel for slender column is given by

$$A_{s,\min} = \mu_{\min} \times b \times t \quad \text{.........(8.7a)}$$

$$\mu_{\min} = 0.25 + 0.052\, \lambda_b \qquad \text{(for rectangular columns) ....(8.7b)}$$

$$\mu_{\min} = 0.25 + 0.015\, \lambda_i \qquad \text{(for other columns) ....(8.7c)}$$

Table (8.1) Limits of Slenderness Ratio for Short Columns

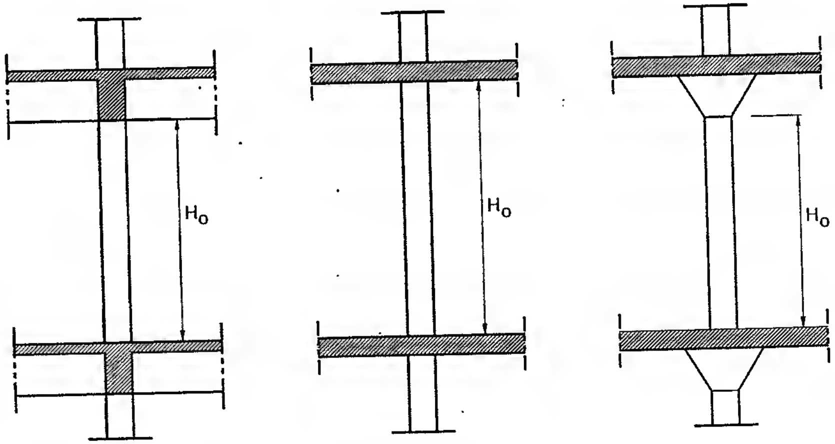
Column Status	λ_t or λ_b	λ_D	λ_i
Braced	15	12	50
Unbraced	10	8	35

Table (8.2) Limits of Slenderness Ratio for Slender Columns

Column Status	λ_t or λ_b	λ_D	λ_i
Braced	30	25	100
Unbraced	23	18	70

8.4.2 Unsupported Height of a Compression Member (H_o)

The unsupported height H_o of a compression member shall be taken as the clear distance between floor slabs, floor beams, girders or other members capable of providing lateral support for the compression member. Where capitals or haunches are present, the unsupported length shall be measured to the lower extremity of the capital or haunch in the plane considered (see Fig. 8.6). It should be noted that the buckling length may be different in X-direction than Y-direction as shown in Fig. xx



a) Slab-beam type

b) Flat plates

c) Flat slab with columns heads

Fig. 8.6 Unsupported length of columns (H_o)

8.4.3 Effective Height of a Compression Member (H_e)

Columns supported by hinges and rollers do not exist in real structures. The ends of a real column are retrained against rotation by their supports, and moments always develop. The effective length concept can be established by examining the deformation of the buckled column with that of a pin-pin column.

The buckling length or the effective height of a compression member (H_e) is the distance between the points of inflection of the diagram representing the buckled shape of the member. It depends on the end conditions of the column and whether it is braced against side-sway or not.

The definition of the effective height is illustrated in Figs. 8.7a and 8.7b for braced and unbraced columns, respectively.

For braced columns, k is the smaller of:

$$k = \left[0.7 + 0.05\ (\alpha_1 + \alpha_2)\right] \le 1.0 \quad \text{..............................(8.8a)}$$

$$k = \left[0.85 + 0.05\ (\alpha_{\min})\right] \le 1.0 \quad \text{................................(8.8b)}$$

For unbraced columns, k is the smaller of:

$$k = \left[1.0 + 0.15\ (\alpha_1 + \alpha_2)\right] \ge 1.0 \quad \text{............................(8.9a)}$$

$$k = \left[2.0 + 0.30\ (\alpha_{\min})\right] \ge 1.0 \quad \text{............................(8.9b)}$$

where

α_1, α_2 = Ratio of the sum of the column stiffness to the sum of the beam stiffness at column lower and upper ends respectively

$\alpha_{\min}$ = Smaller of α_1 & α_2

The coefficient α is given by

$$\alpha = \frac{\sum (E_c\ I_c / H_o)}{\sum (E_b\ I_b / L_b)} \quad \text{.......................................(8.10)}$$

Where

E_c = modulus of elasticity for columns

E_b = modulus of elasticity for beams

I_c = gross moment of inertia of the column cross-section without considering the steel reinforcement

I_b = gross moment of inertia of the beam cross-section

L_b = clear span of the beam

H_o = clear height of the column

Special cases

- α=1.0	for a column connected to a base designed to resist column moment.
- α=10	for a column connected to a base that is not designed to resist moment.
- α=1.0	for simply supported beams framing into a coluPmn.

***Notes*:**

- When calculating the flexural rigidity of the beam cross-section that has a T shape or L shape, the width of the flange is taken as follows

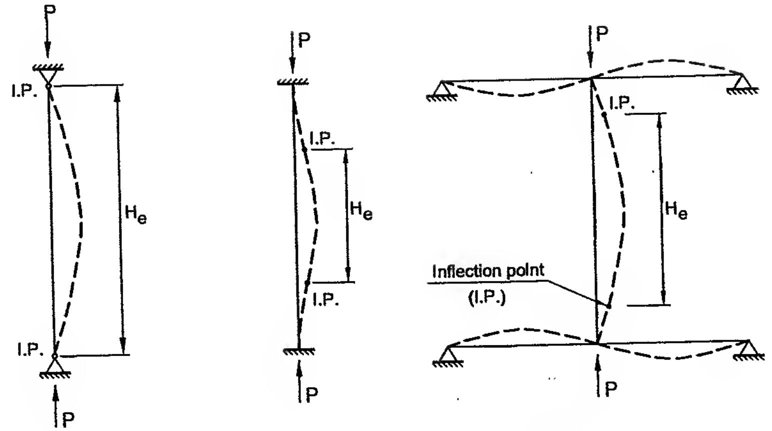
B is the smaller of $(16t_s + b)/2$ or $(L_2/5 + b)/2$ for a T-section

B is the smaller of $(6t_s + b)/2$ or $(L_2/10 + b)/2$ for a L-section

The Egyptian Code permits the use of the gross moment of inertia of the beam section and allows taking the effect of cracking of the beam through using half the value of the gross moment of inertia of the beam section.

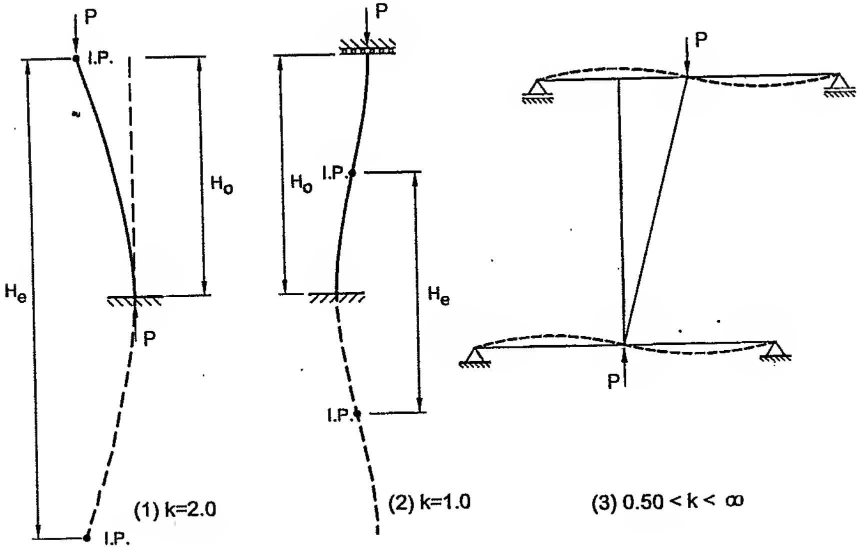
- The ratio (α) may be calculated in flat slab construction on the basis of an equivalent beam having the width and the thickness of the column strip of the slab in the direction of analysis.

As a simplification, the Egyptian Code gives the values of the factor (k) for four cases of end restraint condition. The values of (k) are given in Table (8-3) for braced columns and in Table (8-4) for unbraced columns.



(1) k=1.0 (2) k=0.50 (3) 0.50 < k < 1.0

(a) Braced columns



(1) k=2.0 (2) k=1.0 (3) 0.50 < k < ∞

(b) Unbraced columns

Fig. 8.7 Effective heights of columns

Table (8-3) Values of (k) for Braced Columns

End condition at top	End condition at bottom		
	1	2	3
1	0.75	0.80	0.90
2	0.80	0.85	0.95
3	0.90	0.95	1.00

Table (8-4) Values of (k) for Unbraced Columns

End condition at top	End condition at bottom		
	1	2	3
1	1.20	1.30	1.60
2	1.30	1.50	1.80
3	1.60	1.80	-
4	2.20	-	-

The four conditions are as follows:

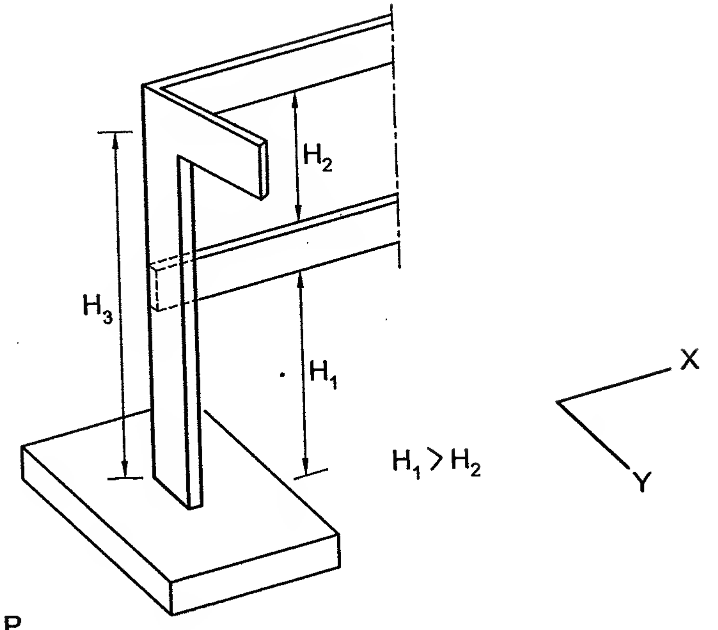
i) **Condition 1**: The end of the column is connected monolithically to beams that are at least as deep as the overall dimension of the column in the plane considered. Foundation designed to withstand moments is considered in this category.

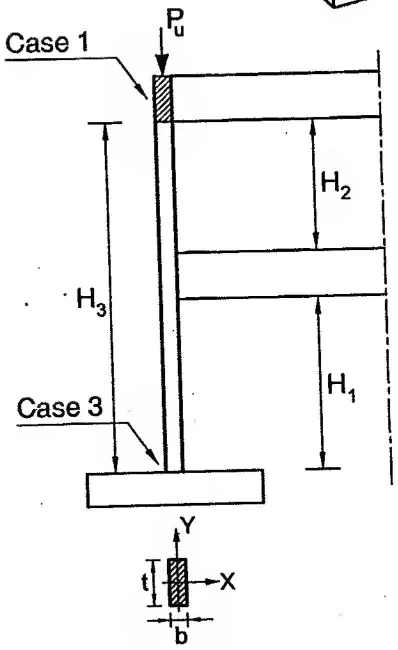
ii) **Condition 2**: The end of the column is connected monolithically to beams or slabs that are shallower than the overall dimensions of the column in the plane considered.

iii) **Condition 3**: The end of the column is connected to members which, while not specifically designed to provide restraint to rotation of the column, nevertheless, provide some nominal restraint (hinged base).

iv) **Condition 4**: The end of the column is unrestrained against both lateral movement and rotation (i.e cantilever column).

Note: The unsupported height of the column might be different in the two orthogonal directions (X- and Y- directions). Figure 8-8 shows an example for such a case.





	X-direction	Y-direction
Case	1-top / 3-bot.	1-top / 3-bot.
H_o	H_1	H_3
K	1.60	1.60
H_e	$1.6 \times H_1$	$1.6 \times H_3$
λ	$\lambda_b = \frac{1.6 \times H_1}{b}$	$\lambda_t = \frac{1.6 \times H_3}{t}$
δ	$\delta_b = \frac{\lambda_b^2 \times b}{2000}$	$\delta_t = \frac{\lambda_t^2 \times t}{2000}$
M_{add}	$M_{add(y)} = P_u . \delta_b$	$M_{add(x)} = P_u . \delta_t$

Fig. 8.8 Effective height for an unbraced column

8.5 Design Moments in Slender Braced Columns

The Egyptian Code takes into account the increase in the applied moments in slender columns by adding to the original moment an additional moment. The additional moment is assumed to occur due to interaction of the applied load with the lateral deflection of the column.

8.5.1 Calculation of the Additional Moments

According to the Egyptian Code the additional moment (M_{add}) induced by the deflection (δ) is given by:

$$M_{add} = P\,\delta \qquad \text{.......................................(8.11)}$$

Figure 8.9 illustrates a case of a rectangular column subjected to additional moments in two directions.

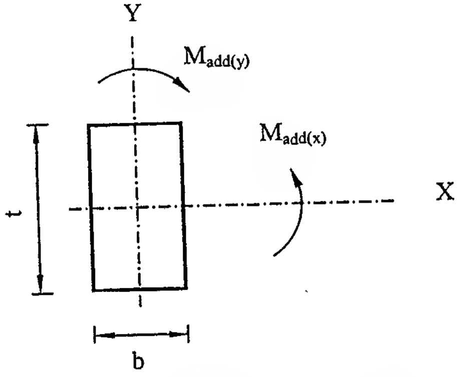


Fig. 8.9 Calculations of additional moments

If the columns is slender in t direction (about X-axis in Fig. (8.7))

$$\delta_t = \frac{\lambda_t^2\, t}{2000} \qquad \text{.......................................(8.12)}$$

$$M_{add(X)} = P_u \cdot \delta_t \qquad \text{.......................................(8.13)}$$

If the columns is slender in b direction (about Y-axis in Fig. (8.7))

$$\delta_b = \frac{\lambda_b^2\, b}{2000} \qquad \text{.......................................(8.14)}$$

$$M_{add(Y)} = P_u \cdot \delta_b \qquad \text{.......................................(8.15)}$$

For circular columns of diameter (D)

$$\delta = \frac{\lambda_D^2\, D}{2000} \quad \text{...(8.16)}$$

For columns with a general shape

$$\delta = \frac{\lambda_i^2\, t'}{30000} \quad \text{...(8.17)}$$

where

t' = column dimension in the direction considered (in mm).

λ_i = slenderness ratio using the column radius of gyration *i*, given by Eq. 8.4b.

8.5.2 Design Moments

Assume a braced slender column subjected to two end moments M_1 and M_2 and the M_2 is the larger of the two end moments. Figure (8.10) shows the distribution of moments assumed over the height of a typical braced column. M_i is the initial moment at a section located near the mid-height and is calculated from:

$$M_i = 0.4M_1 + 0.6M_2 \geq 0.4M_2 \quad \text{..........................(8.18)}$$

For columns subjected to double curvature, the sign of the moment is taken negative.

The design moment for braced columns is taken as the largest value of:

$$M_2$$

$$M_1 + M_{add}/2$$

$$M_i + M_{add} \quad \text{...(8.19)}$$

$$P\, e_{min}$$

Note:

The axial force in a column may be calculated based on the assumption that the beams and slabs transmitting force into it are simply supported. For the case of interior columns supporting approximately symmetrical arrangements of beams, the end moments (M_1) and (M_2) may be assumed equal to zero. This assumption does not apply to columns of flat slab construction for which moments transferred to columns are dealt with explicitly by the code. The initial moments in exterior columns may be estimated as given in Table (8.5).

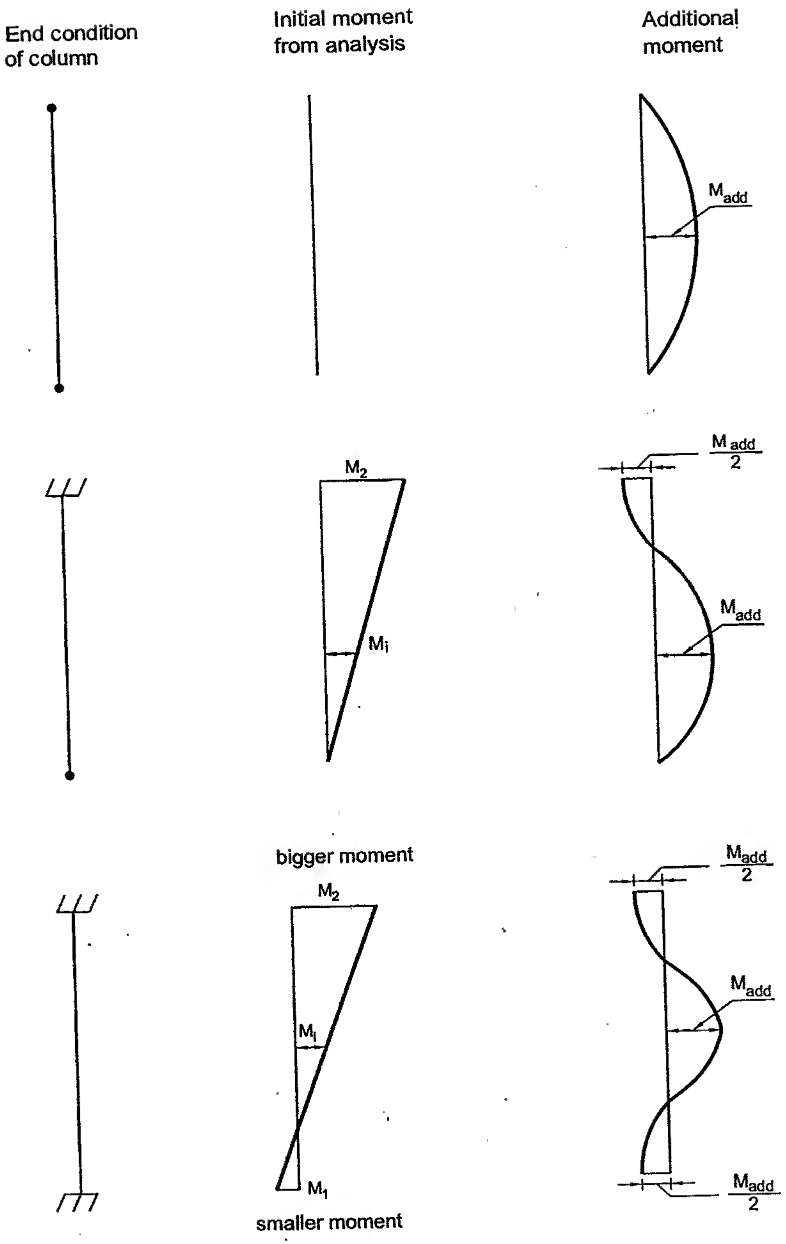


Fig. 8.10 Initial and additional moments in braced slender columns

Table (8.5) Design Moments for Exterior Columns

Position of moment	Moments for frames of one bay	Moments for frames of two bays or more
Moments at foot of upper column	$\frac{K_u \cdot M_f}{K_\ell + K_u + 0.5K_b}$	$\frac{K_u \cdot M_f}{K_\ell + K_u + K_b}$
Moments at head of lower column	$\frac{K_\ell \cdot M_f}{K_\ell + K_u + 0.5K_b}$	$\frac{K_\ell \cdot M_f}{K_\ell + K_u + K_b}$

Where

M_f = bending moment at the end of the beam framing in the column, assuming fixity at both ends of the beam.

K_b = stiffness of the beam

K_ℓ = stiffness of the lower column

K_u = stiffness of the upper column

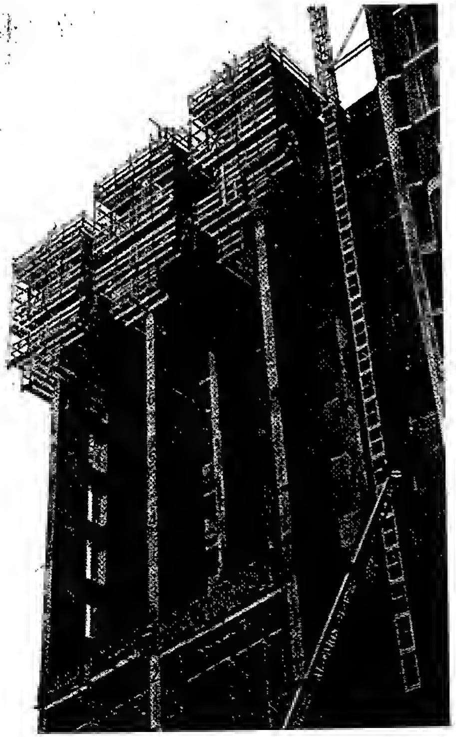


Photo 8.2: Shear walls and slender columns during construction.

8.6 Design Moments in Unbraced Slender Columns

The Egyptian Code takes into account the increase in the applied moments in unbraced slender columns by adding to the original moment an additional moment. The additional moment is assumed to occur due to interaction of the applied load with the lateral deflection of the column. It should be mentioned that unbraced short columns are also subjected to additional moments due the relative displacements of the ends of the columns.

8.6.1 Additional moment

Unbraced columns are generally connected to floors that are rigid enough in their own plane to induce equal deflection (side-sway) to all columns under lateral loads. In such a case an average deflection may be applied to all columns. This deflection can be assessed from the following equation:

$$\delta_{av} = \sum \delta / n \quad \text{..(8.20)}$$

where

δ_{av} = average deflection at ultimate limit state of the floor

δ = deflection at ultimate limit state for each column calculated from Eqs. 8.12

n = the number of the columns in the floor

After the calculation of (δ_{av}), any values of the (δ) more than twice (δ_{av}) should be ignored and the average recalculated. In such a case, (n) in Eq. (8.20) should be reduced appropriately.

$$M_{add} = P \cdot \delta_{av} \quad \text{.......................................(8.21)}$$

8.6.2 Design moments

Assume an unbraced slender column subjected to two end moments M_1 and M_2 and the M_2 is the larger of the two end moments. The distribution of moments assumed over the height of an unbraced column is shown in Fig. (8.11). The design moment for unbraced columns is taken as the largest value of:

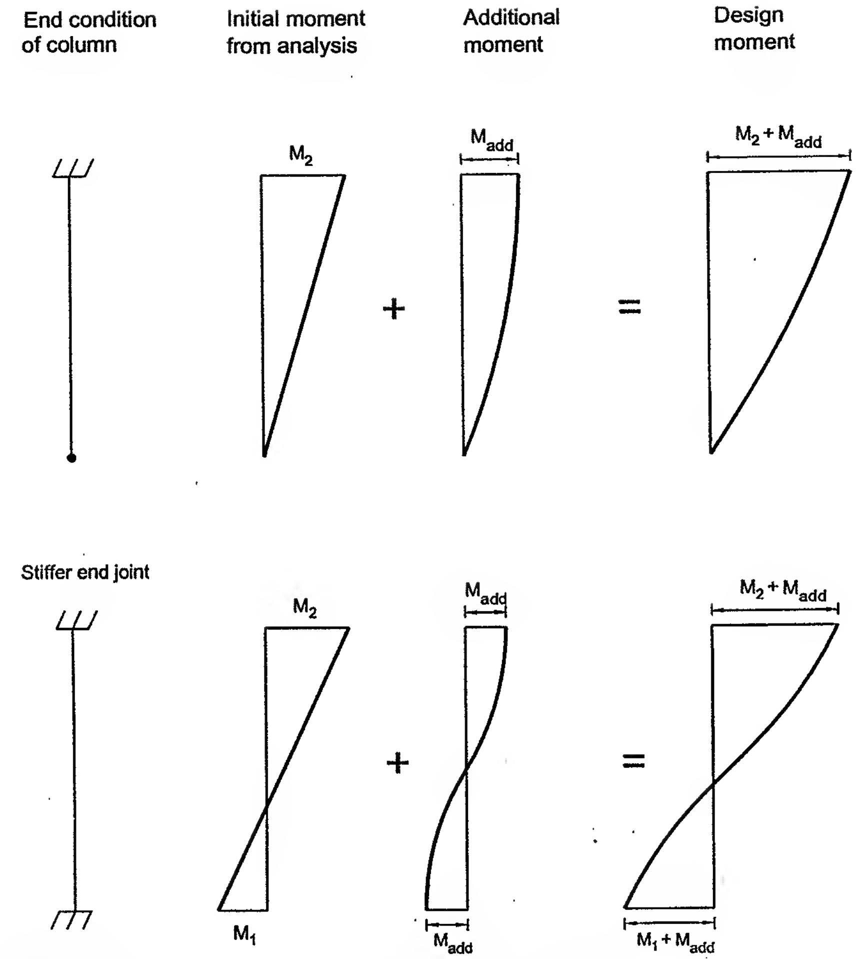
$$M_2 + M_{add}$$

$$P\, e_{min} \quad \text{...(8.22)}$$

where

M_2 is the larger initial end moment due to design ultimate loads.

Fig. 8.12 summarizes the calculations of M_{add} in rectangular slender columns according ECP-203



may be reduced in proportion to the ratio of the stiffness of the less stiff to the stiffer joint

Fig. 8.11 Design moments in unbraced slender columns

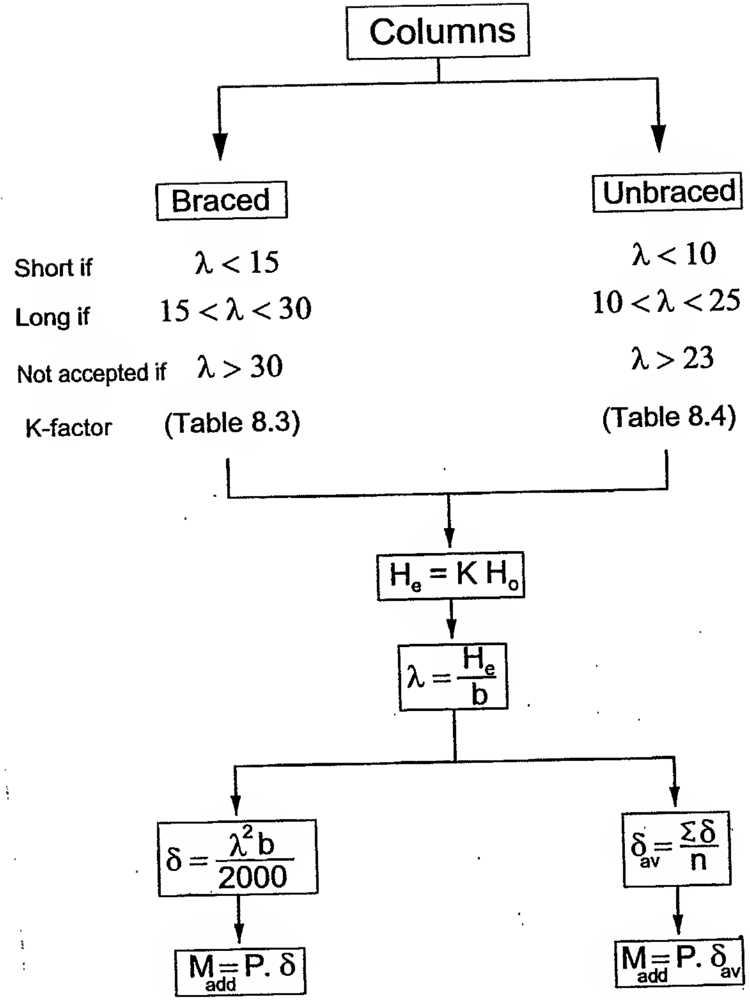
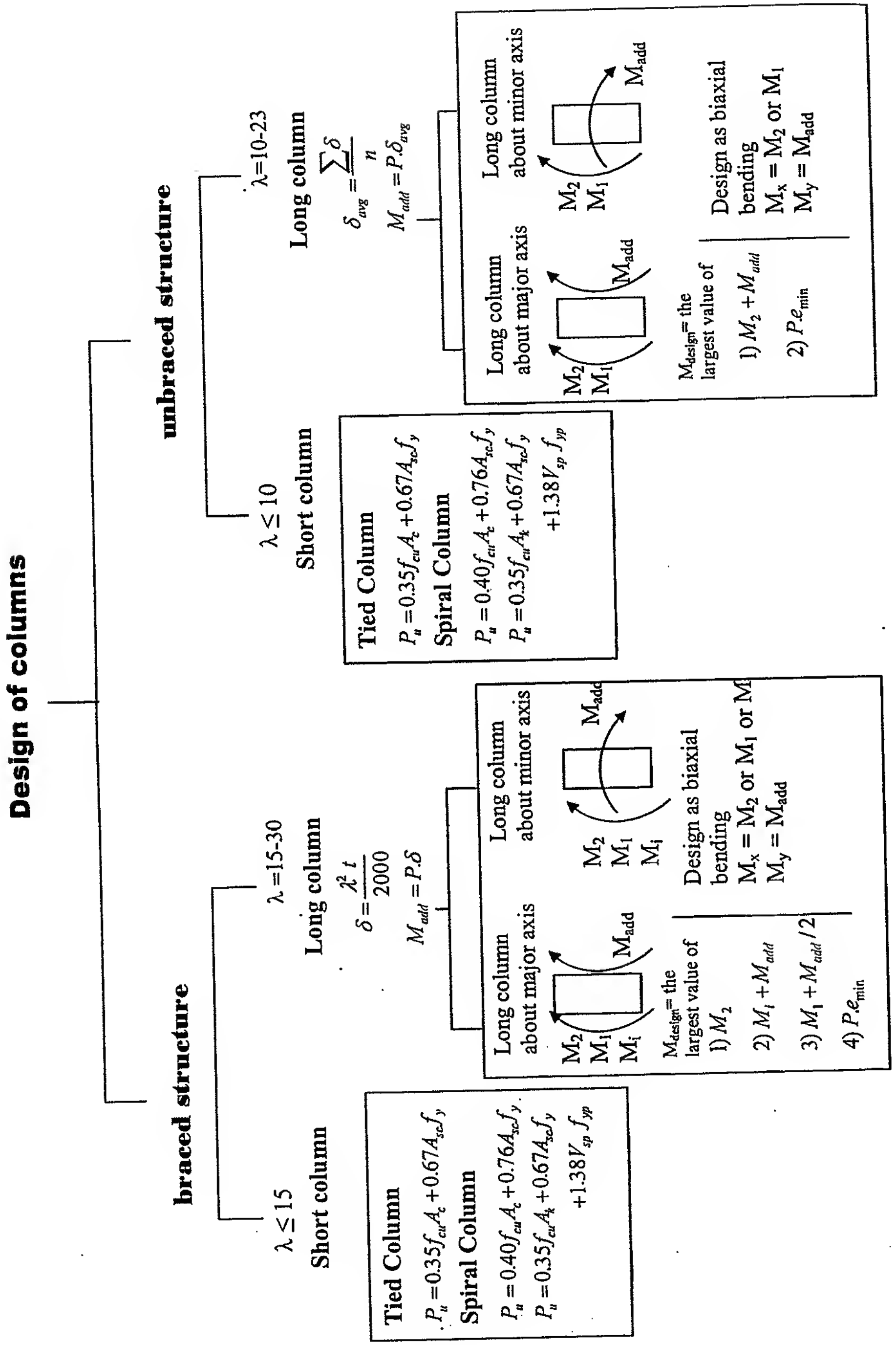


Fig.8.12 Calculations of M_{add} in rectangular slender columns according to ECP-203



Example 8.1

Figure EX-8.1 shows a structural plan of an eleven story residential building. The following data are given:
Thickness of the flat plate at all floors = 220 mm, flooring=1.5 kN/m^2, equivalent wall loads = 2 kN/m^2 and the live load = 3 kN/m^2. The height of the ground floor is 5.0 m and that of the typical floor is 3.0 m. The weight of the core, the walls and the columns can be assumed equal to 20000 kN. The concrete cube strength $f_{cu} = 35\,N/mm^2$

It is required to check the bracing condition of the building in both directions.

Solution

The building is provided with a core and 2 shear walls to increase its capacity to resist the lateral loads. According to the ECP 203, the following equation is to be used to check whether the building is braced or unbraced.

$$\alpha = H_b \sqrt{\frac{N}{\sum EI}} < 0.6$$

where

H_b	The total height of the building above the foundation level
N	Total unfactored vertical loads acting on all vertical elements
$\sum EI$	Summation of flexural rigidity of all vertical walls in the direction under consideration

Step 1: Calculation of N

Weight of typical floor = Own weight + Flooring + Equivalent wall load + Live load

$$= 0.22\times25+1.50 + 2.0 + 3.0$$

$$= 12 \text{ kN/m}^2$$

Total Weight of floor = Area × Unit weight

$$= (26.00\text{x}34.00) \times 12$$

$$= 10608 \text{ kN/floor}$$

Total Weight of Building, N

= No. of floors × Weight of floor + weight of core, walls and columns

$$= 11\times 10608+20000$$

$$= 136688 \text{ kN}$$

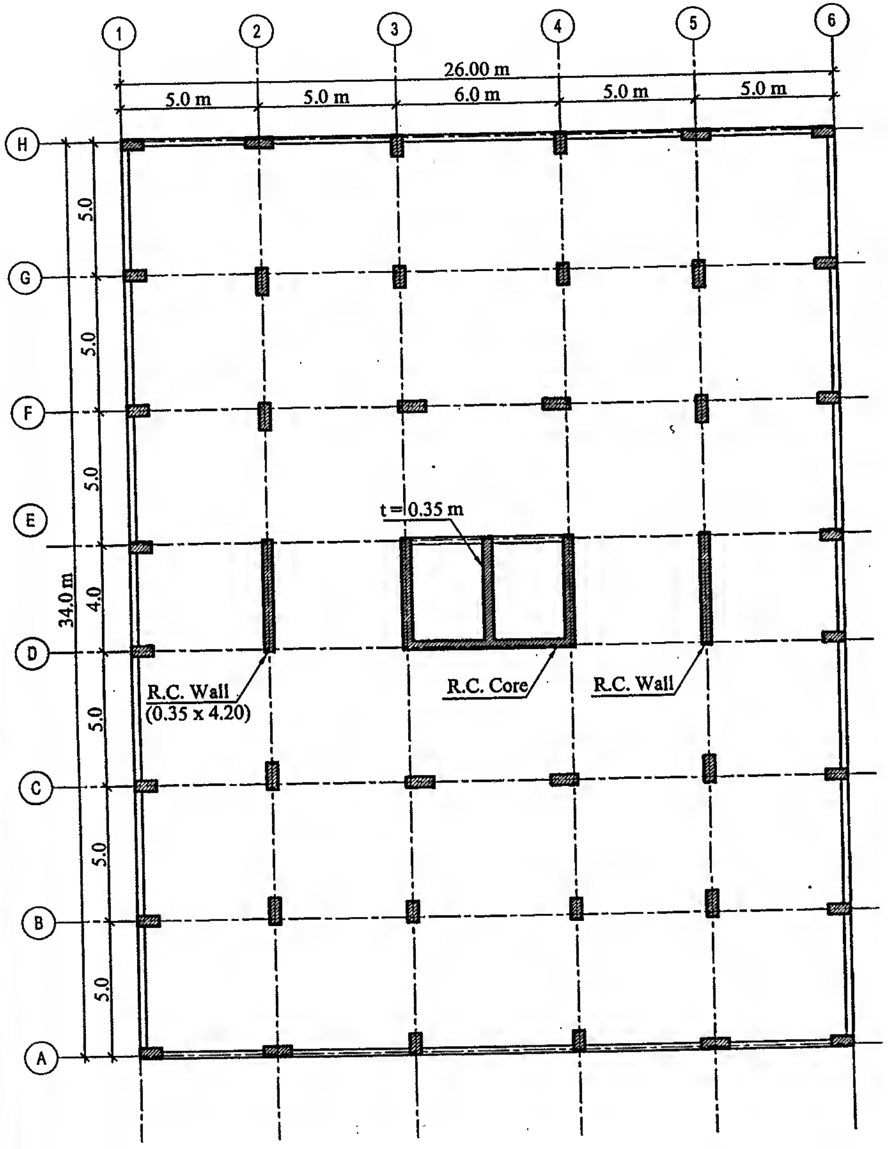


Figure Ex-8.1. Plan of a Typical Story

Step 2: Calculation of the moment of inertia of the core and the walls

The core resists lateral loads in the X- and the Y-directions. The walls resist lateral loads in the Y-direction only.

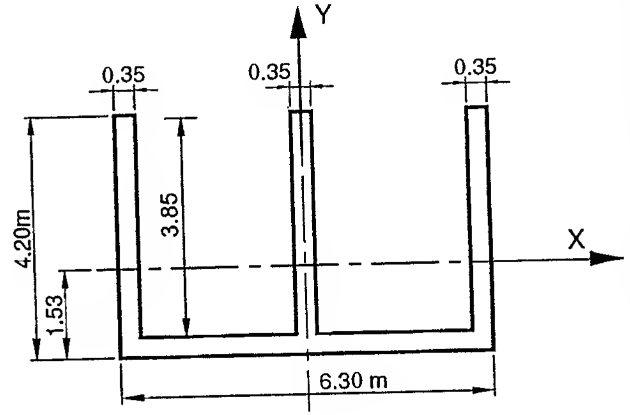
For the Core

$$y = \frac{3\times0.35\times3.85\times2.275 + 6.3\times0.35\times0.175}{3\times0.35\times3.85 + 6.3\times0.35} = 1.53\ m$$

$$I_x = \frac{3\times0.35\times3.85^3}{12} + 3\times0.35\times3.85\times(2.275-1.53)^2 + \frac{6.3\times0.35^3}{12} + 6.3\times0.35\times(1.53-0.175)^2$$

$$I_x = 11.30\,m^4$$

$$I_y = \frac{3\times0.35^3\times3.85}{12} + \frac{0.35\times6.3^3}{12} + 2\times0.35\times3.85\ \left(\frac{6.3}{2} - 0.175\right)^2 = 31.19\,m^4$$

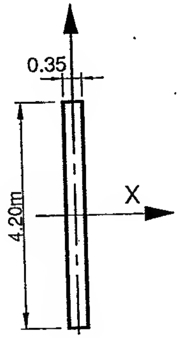


For the Walls

$$I_x = \frac{0.35\times4.2^3}{12} = 2.16\ m^3$$

$$I_x = 2.16\,m^4$$

$$I_y = \frac{4.2\times0.35^3}{12} \cong zero$$



Step 3: Calculation of H_b

Height of the building above the foundation

=10 × 3.0 + 5.0=35.0 m

Step 4: Check bracing condition in Y-direction (Calculation of α_y)

The lateral bracing of the building in Y-direction is achieved by the core, $I_x = 11.3\ m^4$ and the two shear walls.

$$E = 4400\sqrt{f_{cu}}\ \ \mathrm{N/mm^2}$$

$$E = 4400\sqrt{35} = 2.6\times10^4\ N/mm^2 = 2.6\times10^7\ kN/m^2$$

$$\sum EI_x = E\sum(I_{x(core)} + 2\times I_{x(wall)}) = 2.6\times10^7\times(11.30+2\times2.16) = 4.06\times10^8\ kN.m^2$$

$$\alpha_y = H_b\sqrt{\frac{N}{EI_x}}$$

$$\alpha_y = 35.0\times\sqrt{\frac{136688}{4.06\times10^8}}$$

$= 0.64 > 0.60$ *Unbraced Structure in Y-direction*

Step 5: Check bracing condition in X-direction (Calculation of α_x)

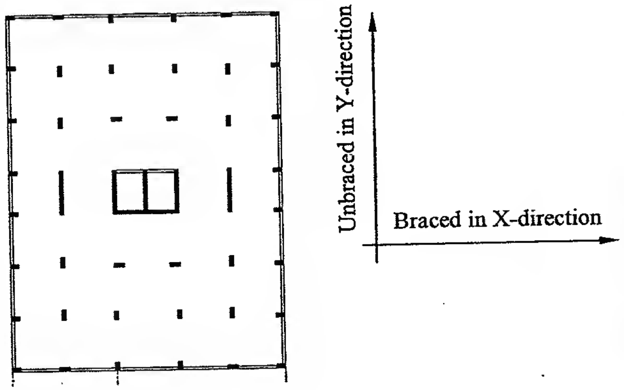
The bracing of the building in X-direction is achieved by the core, $I_y = 31.19\ m^2$.

$$\sum EI_y = E\times I_{core} = 2.6\times10^7\times31.19 = 8.1\times10^8\ kN.m^2$$

$$\alpha_y = H_b\sqrt{\frac{N}{EI_x}}$$

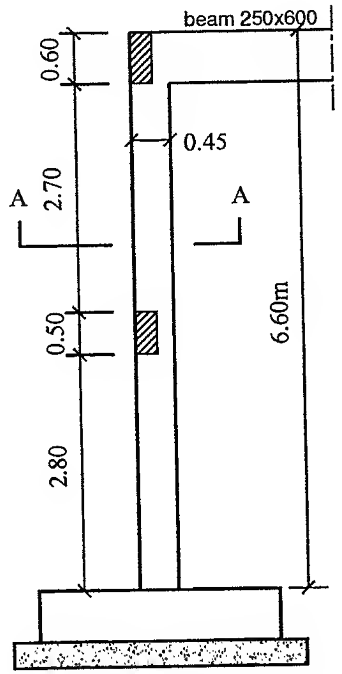
$$\alpha_y = 35.0\times\sqrt{\frac{136688}{8.1\times10^8}}$$

$= 0.45 < 0.60$ *Braced Structure in X-direction*

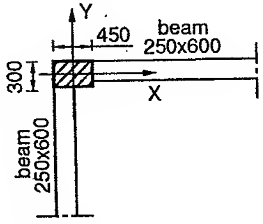


Example 8.2

Design the rectangular column shown in the figure below to support a factored load of 1500 kN. For simplicity the column may be assumed hinged at the foundation level. The column is considered unbraced in x-direction and braced in y-direction. The material properties are $f_{cu} = 30\ N/mm^2$ and $f_y = 360\ N/mm^2$.



Elevation



Sec A-A

Solution:

Step No.1: Considering the moments developed in t-direction (M_y)

The column is considered unbraced in X and Y directions as no lateral resisting system is provided.

Clear height of the column, H_o = 6.6- 0.6=6.0 m

The top end of the column is connected monolithically to beams that are deeper than the dimension of the column (condition1), while the bottom end of the column is given as hinged condition (condition 3).

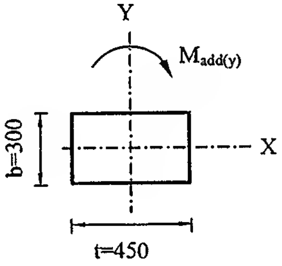
From Table (8-4), the effective length factor →→ k = 1.60.

The effective height, $H_e = k\,H_o$

$$H_e = 1.60 \times 6.0 = 9.60 \text{ m}$$

$$\lambda_{\text{in-plane}} = \frac{H_e}{t} = \frac{9.60}{0.45} = 21.33 > 10 \text{ and not more than } 23$$

The column is classified as slender column in the direction considered, and additional moment is developed.



$$\delta = \frac{\lambda^2 \times t}{2000} = \frac{21.33^2 \times 0.45}{2000} = 0.102\,m$$

$$M_{add} = P_u \times \delta = 1500 \times 0.102 = 153 \text{ kN.m}$$

$$M_{total\,(in\text{-}plane)} = M_u + M_{add}$$

$$\because M_u = 0$$

$$\therefore M_{total\,(in\text{-}plane)} = 0 + 153 = 153 \text{ kN.m}$$

$$\therefore M_y = 153 \text{ kN.m.}$$

Step No.2: Considering the moments developed in b-direction (M_x)

Clear height of the column, H_o = 2.80 m (the largest of the two heights)

The top end of the column is connected monolithically to beams that are deeper than the dimension of the column (condition1), while the bottom end of the column is given as hinged condition(condition 3).

From Table (8-4), the effective length factor→→ k = 0.90.

The effective height, $H_e = k\,H_o$

$$H_e = 0.90 \times 2.80 = 2.52 \text{ m}$$

$$\lambda_{\text{out of plane}} = \frac{H_e}{b} = \frac{2.52}{0.30} = 8.4 < 10$$

The column is classified as short column in the direction considered, and no additional moment is developed.

$$M_{tot,(out\,of\,plane)} = M_u + M_{add}$$

$$\therefore M_{tot,(out\,of\,plane)} = 0$$

$$\therefore M_x = 0$$

Step No.3: Design of the reinforcement:

The column is subjected to uniaxial bending, calculate the following terms:

$$\frac{P_u}{f_{cu}.b.t} = \frac{1500 \times 1000}{30 \times 300 \times 450} = 0.37$$

$$\frac{M_y}{f_{cu}\,b\,t^2} = \frac{153 \times 10^6}{30 \times 300 \times 450^2} = 0.084$$

Assume cover = 40 mm →→ $\zeta = \frac{450 - 2 \times 40}{450} \cong 0.82$ →→Use ζ=0.80(conservative)

Using the **uniaxial** interaction diagram (top & bottom steel) (Appendix B)

From the diagram with f_y = 360 N /mm², ζ = 0.80, α =1→→ ρ = 3.5

$\mu = 3.5 \times 30 \times 10^{-4} = 0.00875 < \mu_{max}$ (0.05) (external column).

$A_s = \mu\, b\, t = 0.00875 \times 300 \times 450 = 1181 \text{ mm}^2$

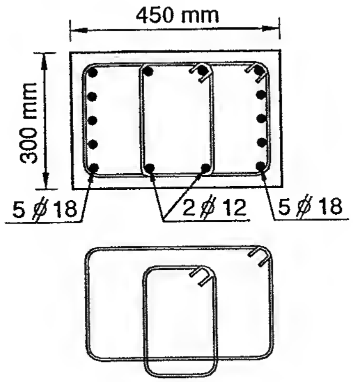
$A_s = A_s' = 1181\,mm^2$

$A_{s,total} = A_s + A_s' = 1181 + 1181 = 2362 \text{ mm}^2$

$\mu_{min} = 0.25 + 0.052 \times 21.33 = 1.359$ %

$A_{s,\,min} = 0.01359 \times 300 \times 450 = 1834.65 \text{ mm}^2 < A_{s,\,total}$

Choose 5 Φ 18 top & 5 Φ 18 bottom ($A_{s,total}$= 2544 mm²)



Example 8.3

Design the rectangular column shown in the figure below to support a factored eccentric load of 500 kN (own weight of column may be neglected). The column is connected to a footing that can resist moment. The material properties are $f_{cu} = 25$ N /mm^2 and $f_y = 400$ N /mm^2.

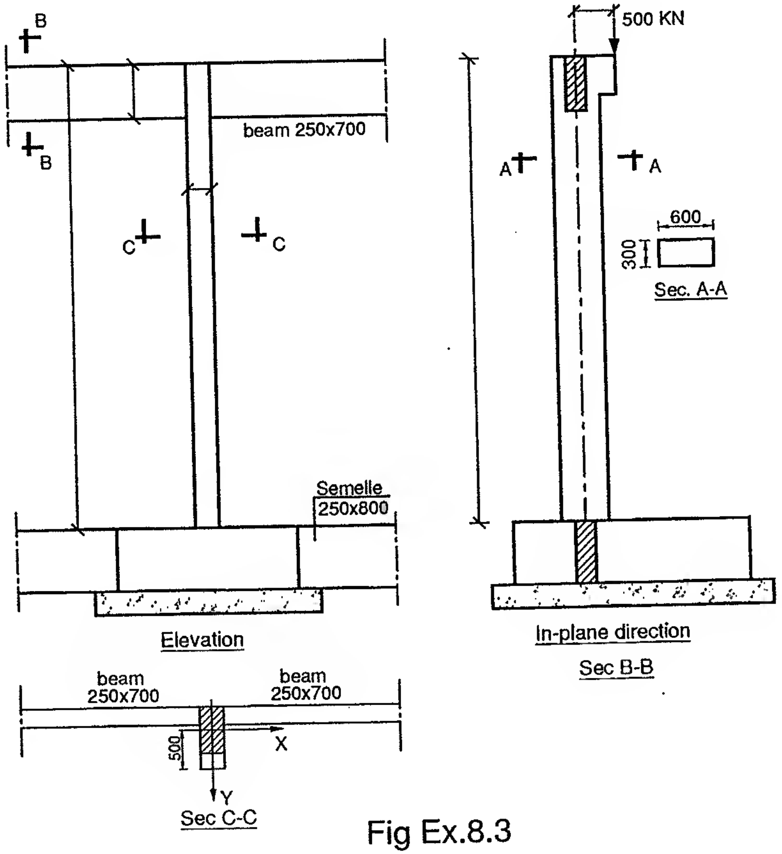


Fig Ex.8.3

Solution:

Step 1: Considering the in-plane direction of the column (M$_x$)

Since the structure is not provided by lateral resisting system, it is considered unbraced. Clear height of the column, $H_o = 6.0$ m

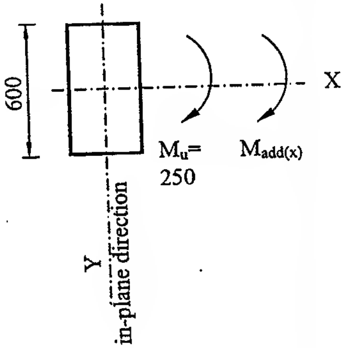
The top end of the column is unrestrained against both lateral movement & rotation (cantilever column) (condition 4) while the bottom end of the column is connected to a footing that can resist moment (condition 1). From Table (8-4), the effective length factor, k = 2.20.

The effective height, $H_e = k\,H_o$

$$H_e = 2.20 \times 6.0 = 13.20 \text{ m}$$

$$\lambda_{\text{in-plane}} = \frac{H_e}{t} = \frac{13.20}{0.60} = 22 > 10 \text{ and not more than } 23$$

The column is classified as a slender column in the direction considered, and an additional moment is developed.



$$\delta = \frac{\lambda^2 \times t}{2000} = \frac{22^2 \times 0.60}{2000} = 0.145$$

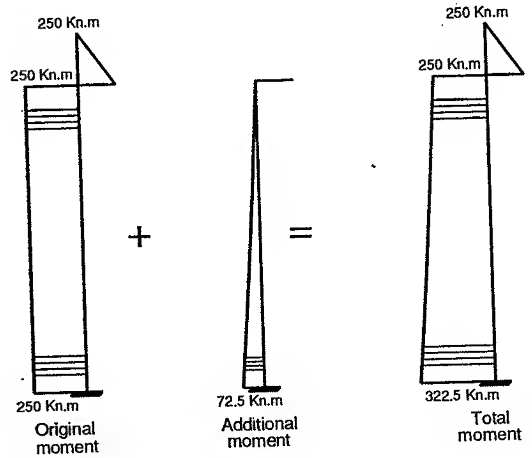
$$M_{add} = P_u \times \delta = 500 \times 0.145 = 72.5 \text{ KN.m}$$

$$M_{\text{total,(in-plane)}} = M_u + M_{add}$$

$$\because M_u = 500 \times 0.50 = 250 \text{ KN.m}$$

$$\therefore M_{\text{total,(in-plane)}} = 250 + 72.5 = 322.5 \text{KN.m}$$

$$\therefore M_x = 322.5 \text{ KN.m.}$$



Step No.2: Considering the out of plane direction of the column(M_y)

Clear height of the column, H_o = 6-0.70 = 5.30 m

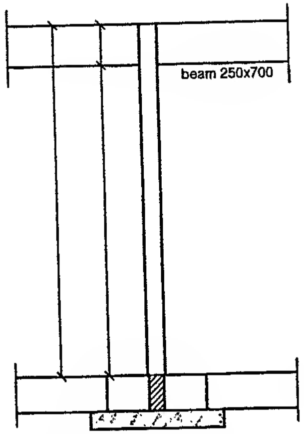
The top end of the column is connected monolithically to beams that are deeper than the dimension of the column (condition1), while the bottom end of the column is connected to a footing that can resist moment (condition 1).

From Table (8-4), the effective length factor →→→ k = 1.20.

The effective height, $H_e = k\,H_o$

$$H_e = 1.20 \times 5.30 = 6.36 \text{ m}$$

$$\lambda_{\text{out of plane}} = \frac{H_e}{b} = \frac{6.36}{0.30} = 21.2 > 10 \text{ and not more than } 23$$



The column is classified as slender column in the direction considered, and additional moment is induced.

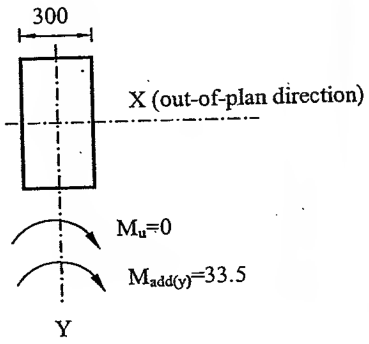
$$\delta = \frac{\lambda^2 \times b}{2000} = \frac{21.2^2 \times 0.30}{2000} = 0.067$$

$$M_{add} = P_u \times \delta = 500 \times 0.067 = 33.5\,kN.m$$

$$M_{\text{total,(out of plane)}} = M_u + M_{add}$$

$$M_{\text{total,(out of plane)}} = 0 + 33.5 = 33.5\,kN.m$$

$$\therefore M_y = 33.5\,KN.m$$



Step No.3: Design of the reinforcement:

The column is subjected to the following straining actions(compression force+ biaxial bending)

P_u = 500 kN

M_x = 322.5 kN.m

M_y =33.5 kN.m

Determine the load level R_b as follows:

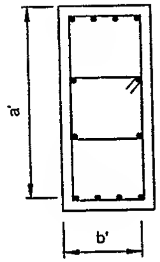
$$R_b = \frac{P_u}{f_{cu}.b.a} = \frac{500 \times 1000}{25 \times 300 \times 600} = 0.111$$

Since R_b<0.2→ β = 0.80

Assume that the concrete cover = 40 mm

$a' = 600 - 40 = 560\,mm$

$b' = 300 - 40 = 260\,mm$

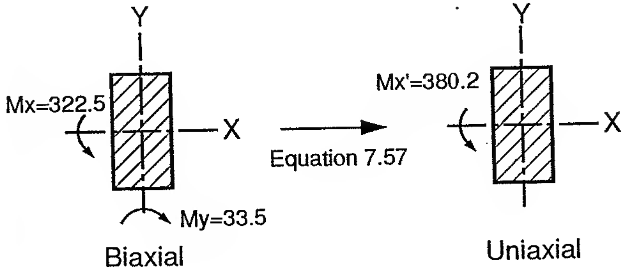


Since $M_x / a' = (322.5/560) > M_y / b' = (33.5/260)$, the design moment will be taken about x.

Using equation 7.57 in this text gives the uniaxial magnified moment.

$$M_x' = M_x + \beta\left(\frac{a'}{b'}\right)M_y$$

$$= 322.5 + 0.8 \times \left(\frac{560}{260}\right) \times 33.5 = 380.2\,kN.m$$



$$\frac{M'_x}{f_{cu}\, b\, t^2}=\frac{380.2\times10^6}{25\times300\times600^2}=0.141$$

$$\zeta=\frac{600-2\times40}{600}\simeq0.9$$

Using uniaxial interaction diagram (uniformly distributed steel)

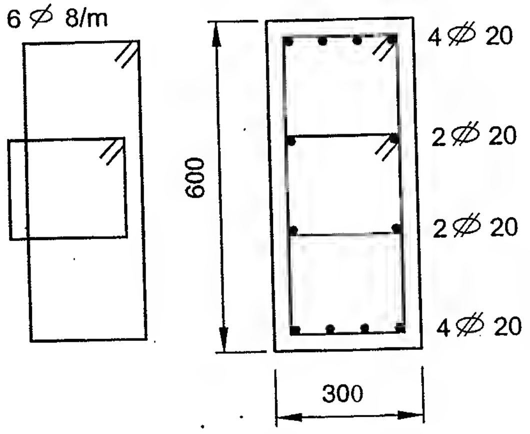
From the diagram with $f_y = 400\ \text{N/mm}^2, \zeta = 0.90 \rightarrow\rightarrow \rho = 8$

$\mu = 8 \times 25 \times 10^{-4} = 0.02 < \mu_{max}$ (0.05) (external column).

$A_{s,\,total} = \mu\, b\, t = 0.02 \times 300 \times 600 = 3600\ \text{mm}^2$

$\mu_{min} = 0.25 + 0.052 \times 22 = 1.394\ \% < \mu$ ......o.k.

Choose 12 Φ 20 (3770 mm^2)



Column Reinforcement

Example 8.4

Fig. Ex-8.4 shows a plan and a sectional elevation of a workshop. It is required to find the straining actions acting on column C1 at the ground floor level knowing that it is subjected to an axial load of a factored value of 2700 kN and an initial moment in X-Z plane (M_y)of a factored value of 400 kN.m. f_{cu}=25 N/mm^2 and f_y=360 N/mm^2

Solution

In order to find the design moments for column C1, it is required to investigate the effects of the slenderness ratios of the column about its principal axes. Since the building contains no shear walls, it is classified according to the Egyptian Code as unbraced. Accordingly, all columns are unbraced columns.

Step 1: Calculation of the ultimate deflection in the X-direction

To calculate the average ultimate deflection in the X-direction, one has to calculate the deflection δ for each column. The following table summarizes the calculations:

Column	Top	Bot.	k	H_o (m)	H_e (m)	b or t (m)	λ	δ_x (m)
C1	1	2	1.3	8.0	10.4	0.65	16.00	0.083
C2	1	2	1.3	8.0	10.4	0.60	17.33	0.090
C3	1	2	1.3	4.0	5.2	0.5	10.40	0.027
C4	1	2	1.3	4.0	5.2	0.5	10.40	0.027

Note: Since the floor beam is thicker than the column dimension under consideration, the top condition is considered case 1. Since the semelle is shallower than the column dimension under consideration, the bottom condition is considered case 2. From Table 8.4 in this text, it can be determined that k=1.3.

Where

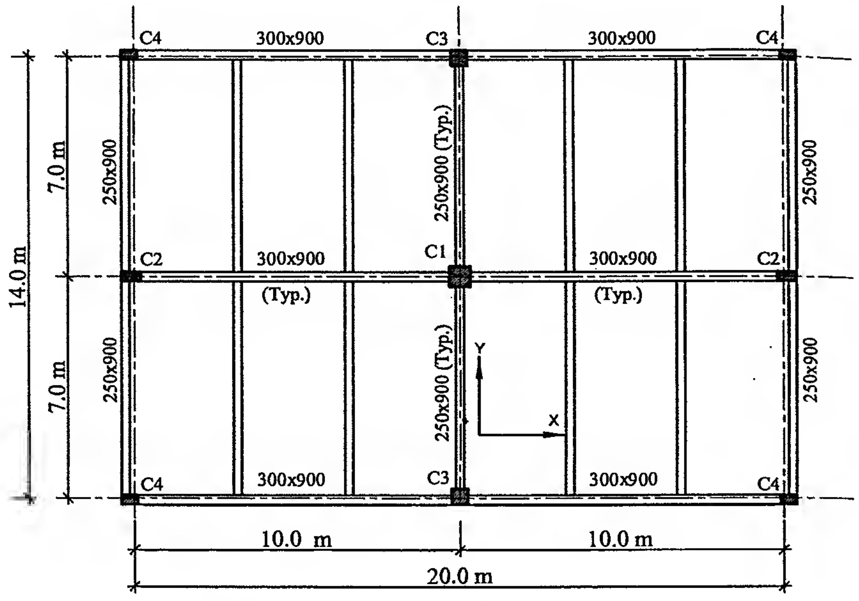
δ_{ci} = deflection at ultimate limit state for C_i in the X-direction

$$= \frac{\lambda_b{}^2 . b}{2000}$$

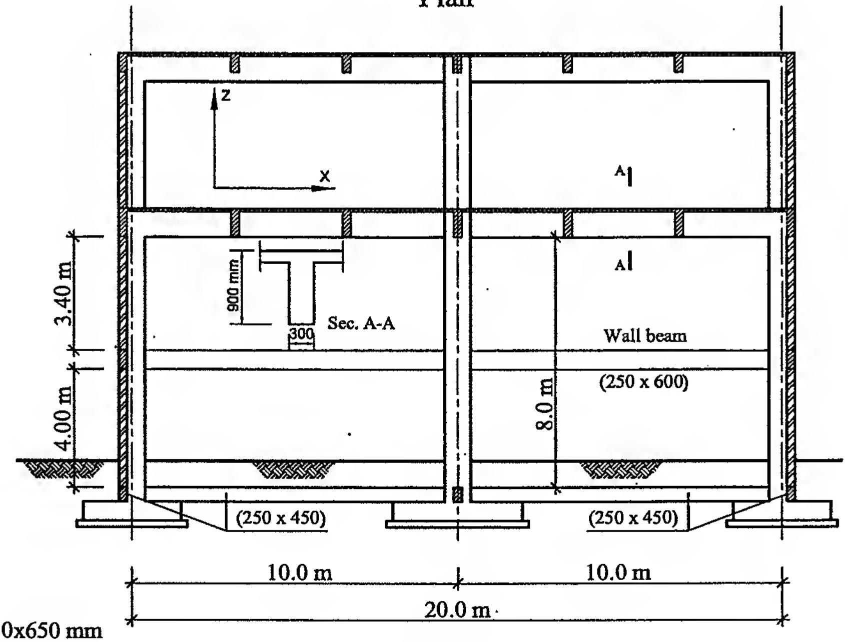
$$\lambda_b = \frac{H_e}{b}$$

$H_e = kH_o$

k = 1.3 {From Table 8.4 of this text with end condition (2) at bottom and (1) at top}



Plan



Elevation

C1 = 650x650 mm
C2 = 300x600 mm
C3 = 500x500 mm
C4 = 300x500 mm

Fig. Ex-8.4

$$\delta_{avx} = \frac{\sum \delta_x}{n}$$, where n = the number of columns

$$\delta_{avx} = \frac{\delta_{c1} + 2\times\delta_{c2} + 2\times\delta_{c3} + 4\times\delta_{c4}}{9}$$

$$\delta_{avx} = \frac{0.083 + 2\times0.090 + 2\times0.027 + 4\times0.027}{9} = 0.0472\,m$$

Note that none of individual δ is larger than 2 δ_{avx}(0.0944)

Step 2: Calculation of the ultimate deflections in the Y-direction

To calculate the average ultimate deflection in the Y-direction, one has to calculate the deflection δ for each column; the following table summarizes the calculations.

Column	Top	Bot.	k	H_o (m)	H_e (m)	b or t (m)	λ	δ_y (m)
C1	1	2	1.3	8.0	10.4	0.65	16	0.083
C2	1	1	1.2	4.0	4.8	0.3	16	0.038
C3	1	2	1.3	8.0	10.4	0.5	20.8	0.108
C4	1	1	1.2	4.0	4.8	0.3	16	0.038

$$\delta_{avy} = \frac{\sum \delta_y}{n}$$, where n = the number of columns

$$\delta_{avy} = \frac{\delta_{c1} + 2\times\delta_{c2} + 2\times\delta_{c3} + 4\times\delta_{c4}}{9}$$

$$\delta_{avy} = \frac{0.083 + 2\times0.038 + 2\times0.108 + 4\times0.038}{9} = 0.0589\,m$$

Note that all the values of δ are less than 2 δ_{avy}

Step 3: Calculation of the additional moments

Additional Moments in the X-Z plane (M_y) = P δ_{avx}
= 2700 x 0.0472
= 127.44 kN.m

Additional Moments in the Y-Z plane (M_x) = P δ_{avy}
= 2700 x 0.0589
= 159.03 kN.m

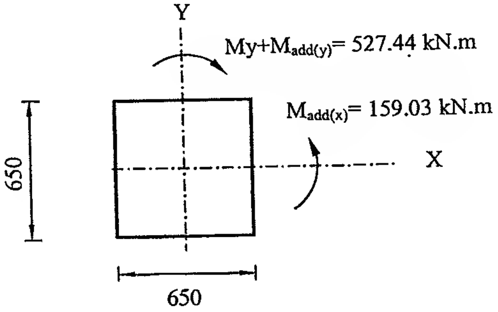
Step 4: Calculation of Final Straining Actions for Column C1

P = 2700 kN

M_y (X-Z plane) = 400+127.44 = 527.44 kN.m

M_x (Y-Z plane) = 159.03 kN.m

It should be noted that in spite of the fact that the column was subjected to an initial uniaxial moments, slenderness effects have resulted in the column being subjected to biaxial moments.



Straining actions for column C1

Step 5: Design of the reinforcement:

The column is subjected to biaxial bending; determine the load level R_b as follows:

$$R_b = \frac{P_u}{f_{cu}.b.a} = \frac{2700\times1000}{25\times650\times650} = 0.256$$

From Table (7.2), using interpolation, get $\beta = 0.772$

Assume concrete cover = 40 mm,

$$a' = 650 - 40 = 610\,mm$$

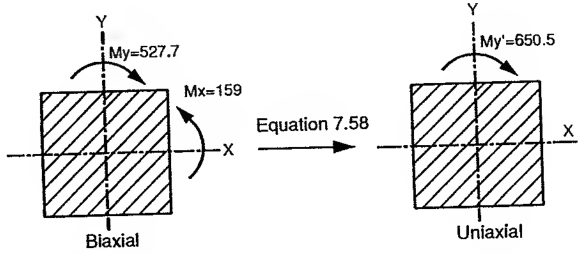
$$b' = 650 - 40 = 610\,mm$$

Since $M_y / b' = (527.7/610) > M_x / a' = (159/610)$, the design moment will be taken about y.

Using equation 7.58 of this text (Chapter 7)

$$M'_y = M_y + \beta(\frac{b'}{a'})M_x$$

$$= 527.7 + 0.772\times(\frac{610}{610})\times159 = 650.50\,kN.m$$



$$\frac{M'_y}{f_{cu}\,b\,t^2} = \frac{650.5\times10^6}{25\times650\times650^2} = 0.095$$

$$\zeta = \frac{650-2\times40}{650} \simeq 0.877$$

Using the **uniaxial** interaction diagram (uniformly distributed steel) $f_y = 360\ N/mm^2$

$\zeta = 0.90 \rightarrow \rho = 4.8$

$\zeta = 0.80 \rightarrow \rho = 5.8$

Using interpolation →→ $\rho = 5.03$

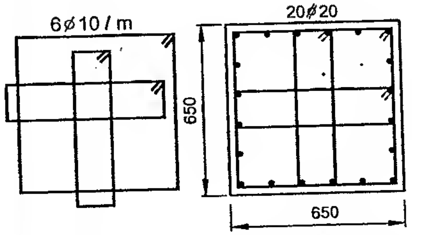
$\mu = 5.03 \times 25 \times 10^{-4} = 0.0126 < \mu_{max}$ (0.04) (internal column).

$A_{s,\,total} = \mu\, b\, t = 0.0126 \times 650 \times 650 = 5313\ mm^2$

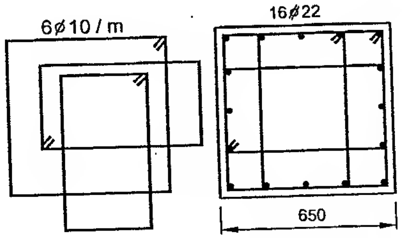
$\mu_{min} = 0.25 + 0.052 \times 16 = 1.082\,\% < \mu$ ......o.k.

Choose 20 Φ 20 (6283 mm^2)

Or 16 Φ 22 (6082 mm^2)



Alternative (1) for column reinforcement



Alternative (2) for column reinforcement

Example 8.5

The frame shown in Fig. Ex-8.5 is a part of the structural system of a braced structure. It is required to find the bending moments in the exterior columns. The unfactored dead and live loads of the exterior beam for all floors may be assumed equal to 32.0 and 12.5 kN/m, respectively. The columns may be assumed fixed to the foundations.

Solution

Table 8.5 gives the values of bending moments in the exterior column.

For the case of frames with two bays or more, the values of bending moments in the exterior column are as follows:

Bending Moment in the lower of upper column is equal to

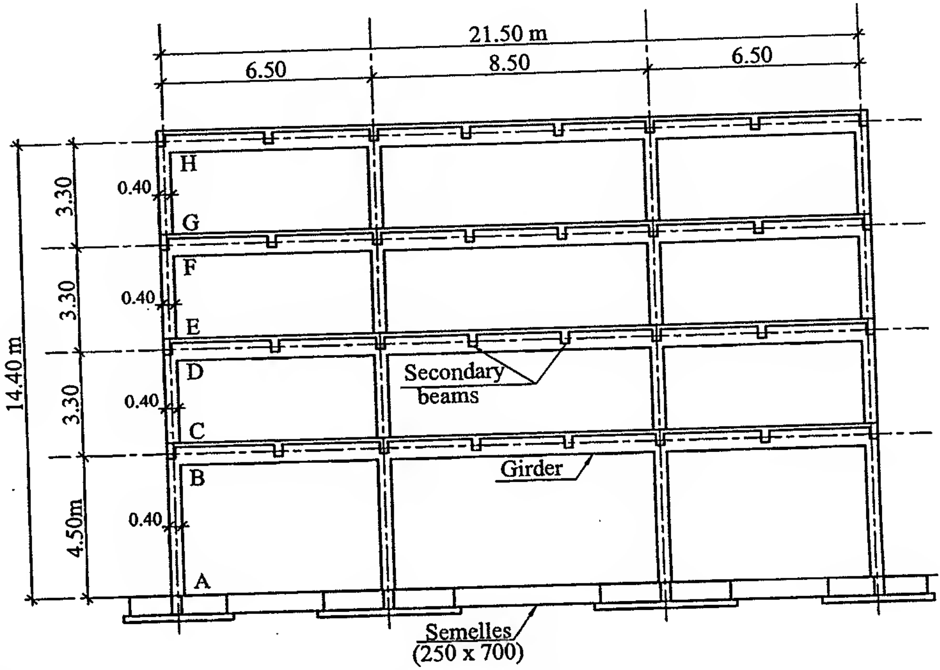
$$= \frac{K_u}{K_l + K_u + K_b} \times M_f$$

and Bending Moment in the upper of lower column

$$= \frac{K_l}{K_l + K_u + K_b} \times M_f$$

where

M_f	is the fixed end moment of the beam
K_u	is the flexural stiffness of upper column with two fixed ends, $K_u = \frac{4EI_u}{h_u}$
K_l	is the flexural stiffness of lower column with two fixed ends, $K_l = \frac{4EI_l}{h_l}$
K_b	is the flexural stiffness of beam, $K_l = \frac{4EI_b}{h_b}$
h_u, h_l	the height of upper and lower column respectively
l_b	the length of beam
I_u, I_l, I_b	the moment of inertia for upper column, lower column and beam respectively.



- All exterior columns are 300x400mm at all floors
- All interior columns are 400x400mm at all floors
- All girders are 250x700mm

Fig. Ex-8.5 Elevation of an industrial building

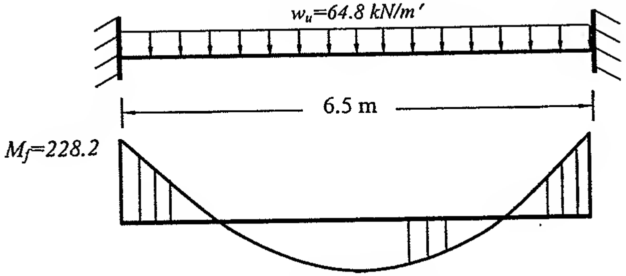
$M_f = w_u l_b^2 / 12$

$w_u = 1.4 \times w_{D.L.} + 1.6 \times w_{L.L.}$

$w_u = 1.4 \times 32.0 + 1.6 \times 12.50 = 64.8$ kN/m'

The beam is assumed fixed from both sides, thus

$M_f = w_u \times L^2/12 = 64.80 \times 6.5^2/12 = 228.2\,kN.m$



Bending moment diagram

For Column AB

$$K_{(AB)} = \frac{4EI_{(AB)}}{h_{(AB)}}$$

$$K_{(AB)} = 4E \times \frac{1}{12}(0.30) \times (0.40)^3 / 4.50$$

$= 1.42E \times 10^{-3}$ kN.m

For Column CD, EF and GH

$$K_{(CD)} = \frac{4EI_{(CD)}}{h_{(CD)}}$$

$$K_{(CD)} = 4E \times \frac{1}{12}(0.30) \times (0.40)^3 / 3.30$$

$= 1.94E \times 10^{-3}$ kN.m

For the Girders

$$K_b = \frac{4EI_b}{l_b}$$

$K_b = 4E \times \frac{1}{12}(0.25) \times (0.70)^3 / 6.50 \quad = 4.4Ex10^{-3}$ kN.m

---

Bending Moment at joint B and C

$$M_{AB} = \frac{K_{(AB)}}{K_{(AB)} + K_{(CD)} + K_b} M_f$$

$$M_{AB} = \frac{1.42E \times 10^{-3}}{1.42E \times 10^{-3} + 1.94E \times 10^{-3} + 4.4E \times 10^{-3}} \times 228.2 = 41.76\,kN.m$$

$$M_{CD} = \frac{K_{(CD)}}{K_{(AB)} + K_{(CD)} + K_b} M_f$$

$$M_{CD} = \frac{1.94E \times 10^{-3}}{1.42E \times 10^{-3} + 1.94E \times 10^{-3} + 4.4E \times 10^{-3}} \times 228.2 = 57.05\,kN.m$$

$$M_{beam} = \frac{K_b}{K_{(AB)} + K_{(CD)} + K_b} M_f$$

$$M_{heam} = \frac{4.4E \times 10^{-3}}{1.42E \times 10^{-3} + 1.94E \times 10^{-3} + 4.4E \times 10^{-3}} \times 228.2 = 129.41\,kN.m$$

Bending Moment at joint D, E, F, and G

$$= \frac{K_{(CD)}}{K_{(CD)} + K_{(EF)} + K_b} M_f$$

$$= \frac{1.94E \times 10^{-3}}{1.94E \times 10^{-3} + 1.94E \times 10^{-3} + 4.4E \times 10^{-3}} \times 228.2$$

$= 53.46\ kN.m$

Bending Moment at joint H

$$= \frac{K_{(GH)}}{K_{(GH)} + 0.0 + K_b} M_f$$

$$= \frac{1.94E \times 10^{-3}}{1.94E \times 10^{-3} + 0.00 + 4.4E \times 10^{-3}} \times 228.2 = 69.8\ kN.m$$

Example 8.6

Figure Ex-8.6 shows a part of an industrial building that is braced in the two orthogonal directions. The cross-sectional dimensions of the columns are 300 x 300 mm. Also shown in this figure is the bending moment diagram acting on columns AB & BC in the X-Z plane. The bending moments acting on these columns in Y-Z plane (out-of plane) can be neglected. Columns AB and BC are also subjected to axial forces of values of 700.0 kN and 450.0 kN, respectively. It is required to find the straining actions acting on columns AB & BC.

Solution

Step 1: Analysis of Columns in the X-Z Plane (in-plane)

Step 1.1: Check slenderness limits of the columns

a) Column AB

Clear height of the column, $H_o = 6.40$ m

The top end of the column is connected monolithically to beams that are at least as deep as the overall dimension of the column in the plane considered (*Condition 1*). The bottom end of the column is connected to a semelle that is deeper than the column (*Condition 1*). From Table 8.3, the effective length factor, k = 0.75

The effective height, $H_e = kH_o$

$$H_e = 0.75 \times 6.40 = 4.80\,m$$

$$\lambda_b = \frac{H_e}{b} = \frac{4.80}{0.30} = 16 > 15 \text{ and not more than } 30$$

Hence, column **AB** is classified as a slender column in the X-Z plane

b) Column BC

Clear height of the column, $H_o = 7.80$ m

The top and bottom ends of the column are connected monolithically to beams that are at least as deep as the overall dimension of the column in the plane considered (*Condition 1*) for top and bottom. From Table 8.3, the effective length factor, k = 0.75.

The effective height, $H_e = kH_o$

$$H_e = 0.75 \times 7.80 = 5.85\,m$$

$$\lambda_b = \frac{H_e}{b} = \frac{5.85}{0.30} = 19.5 > 15 \text{ and not more than } 30$$

Hence, column **BC** is classified as a slender column in the X-Z plane

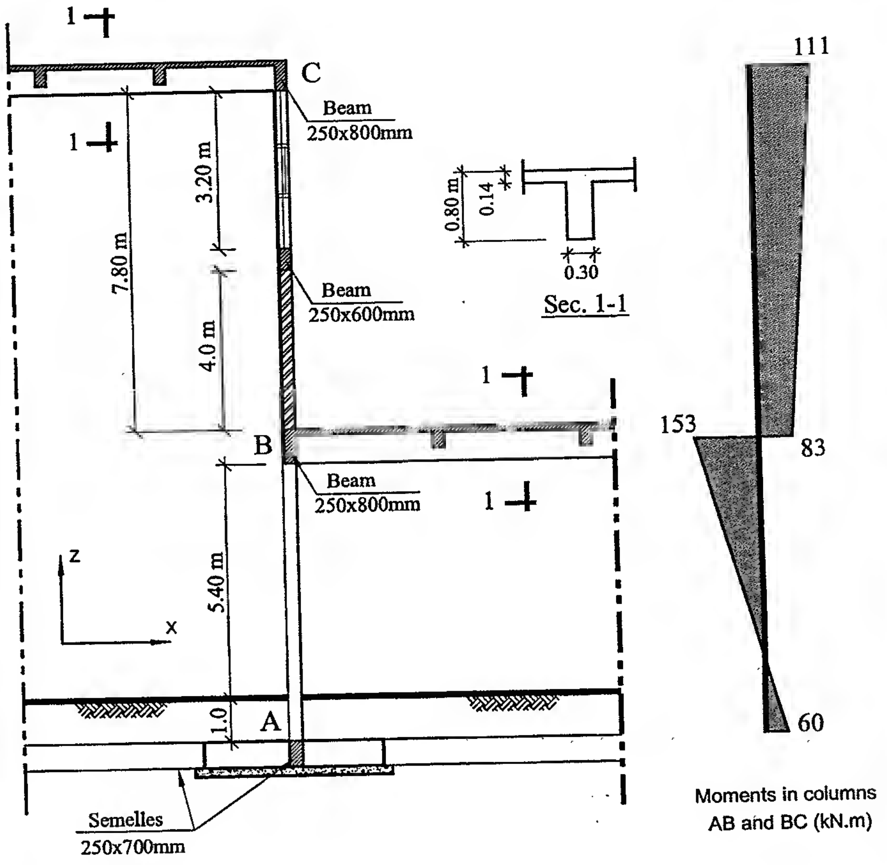


Fig. Ex-8.6 Elevation of an Industrial Building

Step 1.2: Compute the Additional Moment in the X-Z plane(in-plane)

The additional moment, $M_{add} = P.\delta$

a) Column AB

$$\delta = \frac{\lambda_t^2}{2000} t = \frac{16^2}{2000} \times 0.30 = 0.0384\,m$$

$$M_{add} = P.\delta = 700 \times 0.0384 = 26.88\,kN.m$$

b) Column BC

$$\delta = \frac{19.5^2}{2000} \times 0.30 = 0.057\,m$$

$$M_{add} = P.\delta = 450 \times 0.057 = 25.65\,kN.m$$

Step 1.3: Compute the Design Moment in the X-Z Plane (in-plane)

The design moment, M$_{des}$ is the larger of the following values:

1) M_2 2) $M_i + M_{add}$

3) $M_1 + (M_{add}/2)$ 4) $P.e_{min}$

where

$M_i = 0.4M_1 + 0.6M_2 \geq 0.4M_2$

a) Column AB

$M_1 = -60\ kN.m$

$M_2 = 153\,kN.m$ (double curvature)

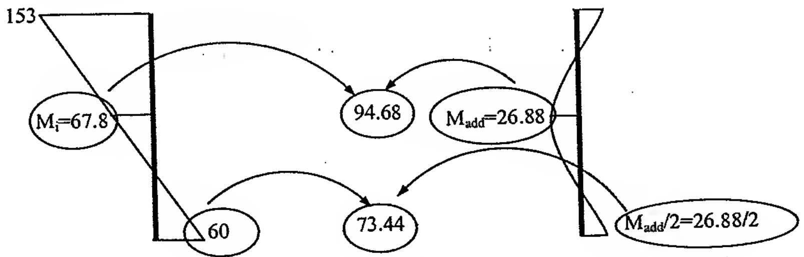
$M_i = -0.4 \times 60 + 0.6 \times 153 = 67.8\,kN.m \geq 0.4M_2$

The design moment is the larger of the following values

1) 153 kN.m 2) 67.8+26.88 = 94.68 kN.m

3) 60+(26.88/2) = 73.44 kN.m 4) 700x(0.05x0.3) = 10.5 kN.m

Accordingly the design bending moment, M$_{des}$ for column **AB** = 153.0 kN.m



b) Column BC

$M_1 = 83\,kN.m$

$M_2 = 111\,kN.m$ (single curvature)

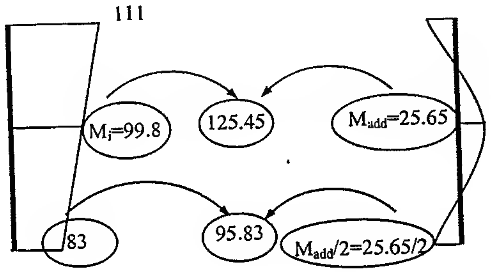
$M_i = 0.4 \times 83 + 0.6 \times 111 = 99.8\,kN.m > 0.4M_2$

The design moment is the larger of the following values

1) 111 kN.m 2) 99.8+25.65 = 125.45 kN.m

3) 83+(25.65/2) = 95.83 kN.m 4) 450x(0.05x0.3) = 6.75 kN.m

Accordingly the design bending moment, M$_{des}$ for column **BC** = 125.45 kN.m



Step 2: Analysis of Columns in the Y-Z Plane (out-of-plane)

Step 2.1: Check slenderness limits of the columns

a) Column AB

Clear height, H$_o$ = 6.40 m

The top end of the column is connected monolithically to beams that are at least as deep as the overall dimension of the column in the plane considered (*Condition 1*). The bottom end of the column is connected to a semelle that is deeper than the column (*Condition 1*). From Table 8.3, the effective length factor, k = 0.75

The effective height, $H_e = kH_o$

$$H_e = 0.75 \times 6.40 = 4.80\,m$$

$$\lambda_b = \frac{H_e}{b} = \frac{4.80}{0.30} = 16 > 15 \text{ and not more than } 30$$

Hence, column **AB** is classified as a slender column in the Y-Z plane

b) Column BC

Clear height, $H_o = 4.0$ m

The top and bottom ends of the column are connected monolithically to beams that are at least as deep as the overall dimension of the column in the plane considered (*Condition 1*) for top and bottom. From Table 8.3, the effective length factor, k = 0.75

The effective height, $H_e = kH_o$

$$H_e = 0.75 \times 4.00 = 3.00m$$

The ratio $\lambda_b = \frac{H_e}{b} = \frac{3.00}{0.30} = 10.0 < 15$

Hence, column **BC** is classified as a short column in the Y-Z plane.

Step 2.2: Compute the additional moment in the Y-Z plane

The additional moment, $M_{add} = P.\delta$

$$\delta = \frac{\lambda_t^2}{2000} t$$

Column AB

$\delta = \frac{16^2}{2000} \times 0.30 = 0.0384m$

$M_{add} = P.\delta = 700 \times 0.0384 = 26.88 kN.m$

Step 2.3: Compute the Design Moment in the Y-Z Plane

Since the initial bending moments in the Y-Z plane were neglected, the design moments in that plane can be considered as the larger of the following two values:

1) M_{add} 2) $P.e_{min}$

a) Column AB

1) 26.88 kN.m 2) 700x(0.05x0.3) = 10.5 kN.m

Accordingly the design bending moment in the Y-Z plane for column **AB** = 26.88 kN.m

b) Column BC

Since the column is classified as a short column the design bending moment in Y-Z plane is equal to zero.

Summary

The following table gives the design straining actions for columns **AB** & **BC**.

Column	Axial Force (kN)	M_{des} (X-Z) plane (kN.m)(M_y)	M_{des} (Y-Z) plane (kN.m) (M_x)
AB	700	153.0	26.88
BC	450	125.45	0.00

Step 3: Design of the reinforcement:

a) Column AB:

The column is subjected to biaxial bending; determine the load level R_b as follows:

$$R_b = \frac{P_u}{f_{cu}.b.a} = \frac{700 \times 1000}{40 \times 300 \times 300} = 0.194$$

From Table (7.2), get β = 0.8

Assume concrete cover = 30 mm,

$a' = 300 - 30 = 270mm$

$b' = 300 - 30 = 270mm$

Since $M_y / a' = (153/270) > M_x / b' = (26.9/270)$, the design moment will be taken about y.

Using 7.58 in this text (chapter 7)

$$M'_y = M_y + \beta(\frac{b'}{a'})M_x$$

$$= 153 + 0.8 \times (\frac{270}{270}) \times 26.9 = 174.5 KN.m$$

$$\frac{M'_y}{f_{cu} b\, t^2} = \frac{174.5 \times 10^6}{40 \times 300 \times 300^2} = 0.162$$

$$\zeta = \frac{300 - 2 \times 30}{300} = 0.8$$

Using **uniaxial** interaction (uniformly distributed steel) $f_y = 360$ N/mm², $\zeta = 0.80$

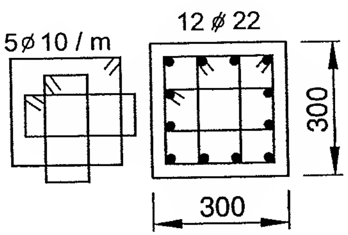
Therefore ρ = 12.2

$\mu = 12.2 \times 40 \times 10^{-4} = 0.0488 < \mu$ max (0.05) (external column).

$A_{s,\,total} = \mu\, b\, t = 0.0488 \times 300 \times 300 = 4392$ mm²

$\mu_{min} = 0.25 + 0.052 \times 16 = 1.082\ \% < \mu$ ......o.k.

Choose 12 Φ 22 (4562 mm²)



Column AB

Note: Since the reinforcement ratio is high (4.8%), therefore it is advisable to increase column dimensions to decrease reinforcement (calculations not shown).

b) Column BC:

The column is subjected to uniaxial bending; determine the following terms:

$$\frac{P_u}{f_{cu}.b.a}=\frac{450\times1000}{40\times300\times300}=0.125$$

$$\frac{M_u}{f_{cu}\,b\,t^2}=\frac{125.45\times10^6}{40\times300\times300^2}=0.116$$

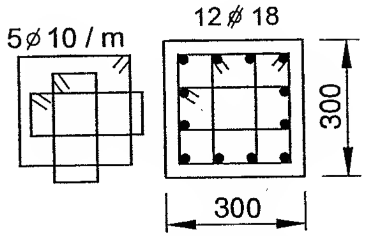
Using **uniaxial** interaction (uniformly distributed steel) $f_y = 360\ \text{N/mm}^2$, $\zeta = 0.80$

Therefore $\rho = 7.5$

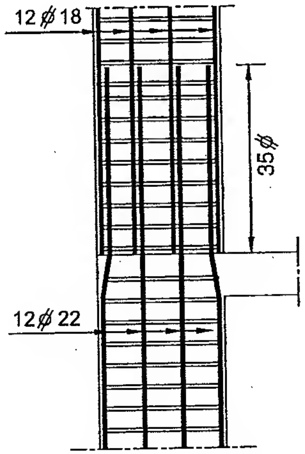
$\mu = 7.5 \times 40 \times 10^{-4} = 0.03 < \mu$ max (0.05) (external column).

$A_{s,\,total} = \mu\, b\, t = 0.03 \times 300 \times 300 = 2700\ \text{mm}^2 \rightarrow\rightarrow$Choose 12 Φ 18 ($3054\ \text{mm}^2$)

$\mu_{min} = 0.25 + 0.052 \times 19.5 = 1.264\ \% < \mu$ ......o.k.



Column BC



Column Lap splice

9

REINFORCED CONCRETE FRAMES

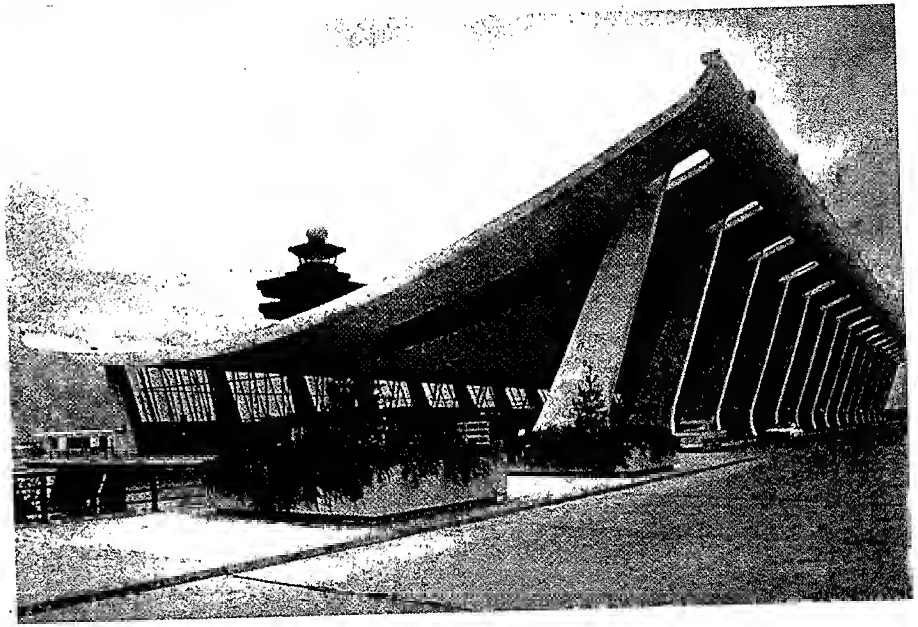


Photo 9.1 Reinforced concrete frames at Dullas airport, USA

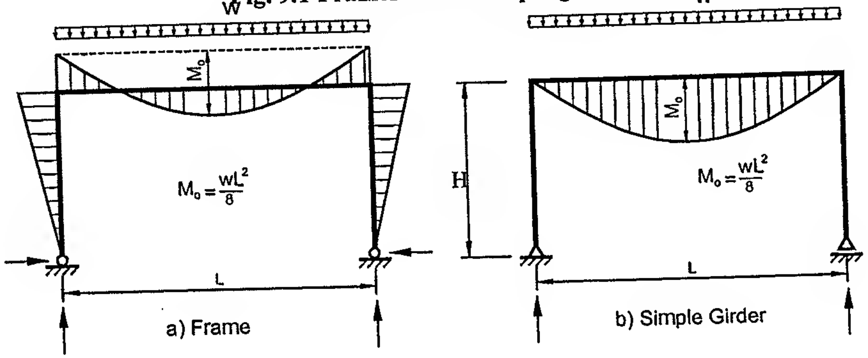
9.1 Introduction

This chapter covers the topic of reinforced concrete frames that are used as supporting elements for halls. The object of a hall is to cover a limited area that has to be utilized for a certain purpose such as meetings, sports, storage, exhibitions and industry.

9.2 Definition of the Frame

A frame is a structure in which the rigid connections between the girders and the supporting columns are utilized so that the internal forces due to the loads are resisted by the combined action of the girder and the columns i.e. the bending moment M_o is distributed among the girder and the columns (Fig. 9.1). In a simple girder, vertical loads are resulted in vertical reactions (Fig. 9.1b), while in a frame; vertical loads give vertical and horizontal reactions (Fig. 9.1a).

Fig. 9.1 Frames versus simple girders



a) Frame

b) Simple Girder

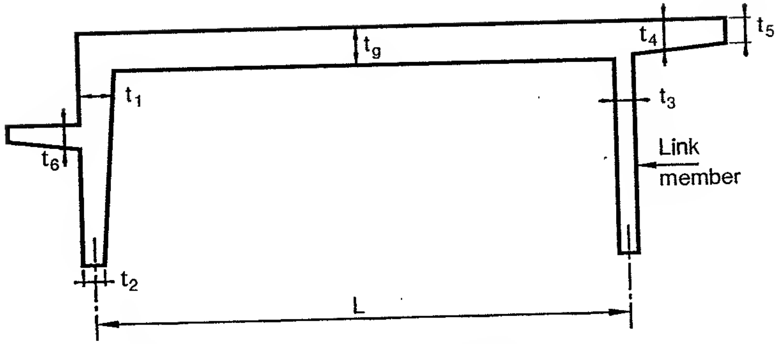
The magnitude of the bending moments resisted by the columns depends on the relative stiffness K_r of the girder with respect to the columns. The bigger the value of K_r, the smaller is the moment resisted by the columns.

$$K_r = \frac{H}{L} \cdot \frac{I_b}{I_c} \qquad (9.1)$$

where

H = height of the column.
L = span of the girder.
I_b = gross moment of inertia of the beam.
I_c = gross inertia moment of of the column.

In order to achieve a good distribution for the bending moments in frame members, it is recommended to choose the concrete dimensions of the frame members as shown in Fig. (9.2)



Member	Recommended value
t_g	L/(12-16)
t_1	(0.8-1.0) t_g
t_2	0.67 t_1
t_3	0.60 t_g
t_4	According to design
t_5	0.67 t_4
t_6	According to design

Fig. 9.2 Dimensioning of the Frame

9.3 The Choice of the Type of the Frame

The choice of the form of a frame is generally governed by the external and internal architectural considerations as well as the purpose at which it is used. The statical system depends on the conditions at the supports.

Two hinged frames are generally used on medium soils as they are not sensitive to displacements of the supports. Figure (9.3) shows several types of two-hinged frames that are extensively used in industrial buildings and workshops. Figure (9.4) shows a two-hinged frame that supports stands.

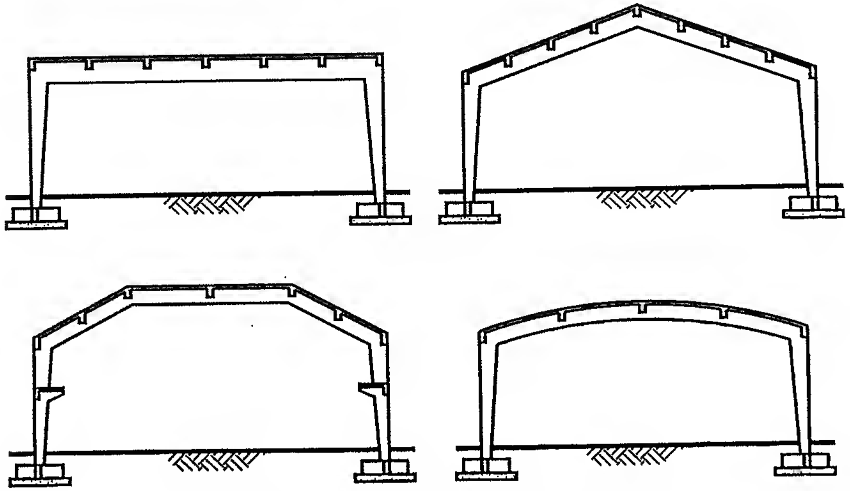


Fig. 9.3 Examples of two hinged frames

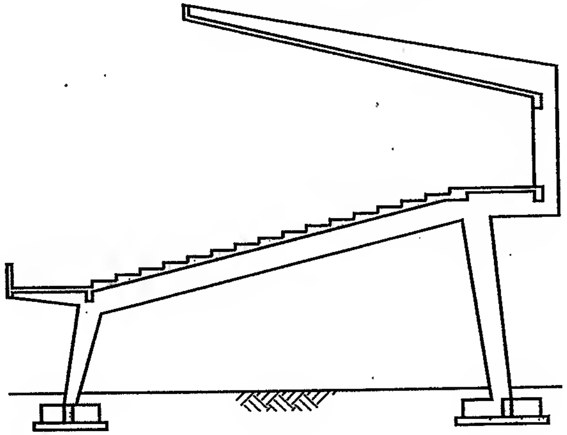


Fig. 9.4 A two hinged frame supporting stands

Statically determined three-hinged frames (Fig. (9.5)) are used on weak soils that may be subjected to small horizontal or vertical movements of the bearing hinges.

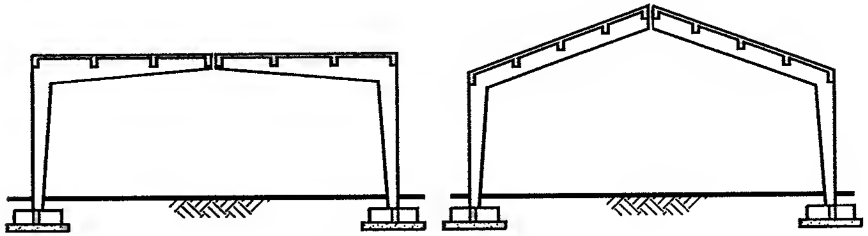
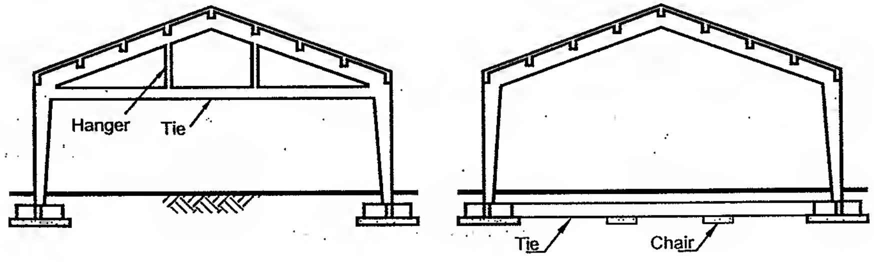


Fig. 9.5 Examples of three-hinged frames

On good firm soils, fixed frames may be used. In this system, the internal stresses due to temperature changes and shrinkage are relatively high and must be considered.

A two-hinged frame with a tie is shown in Fig. (9.6a). Adding a tie to the frame reduces the bending moment in the girder. In order to prevent the sagging of the tie, hangers might be provided at convenient distances. The frame is twice statically indeterminate and hence, the bending moments developed at different sections are less than the case of the two-hinged frame without a tie. The rigid tie at the top of the columns gives a better distribution of the internal forces in the columns and the girder although this does not necessarily more economic solution because of the complicated formwork and the big amount of steel in the tie and the hangers. It gives however a smaller horizontal thrust on the foundations. A frame without a tie is simpler and architecturally more acceptable than a frame with a tie and hangers. If the foundations of a frame cannot resist its horizontal thrust, a tie may be arranged at the bottom hinges to resist the thrust as shown in Fig. (9.6b). In this case, the frame is once statically indeterminate. For elastic ties at foundation level, the horizontal force developed at the foundation level is smaller than that of two hinged frames without ties or with rigid ties. The result is that the corner moment is smaller and the field moment is bigger.



(a) (b)

Fig. 9.6 Examples of two-hinged frames with a tie

Continuous frames can be used to cover areas in which the dimensions are too long to be spanned by single-bay frames (see Fig. (9.7)). Continuous frames are generally high grade statically indeterminate. If the frame is symmetrical in shape and loading, as generally the case in roof structures, the horizontal reaction at the intermediate column will be almost zero. In such a case, the frame may be constructed with a slender intermediate column, which can be assumed as a pendulum. The system in this form is only twice statically indeterminate.

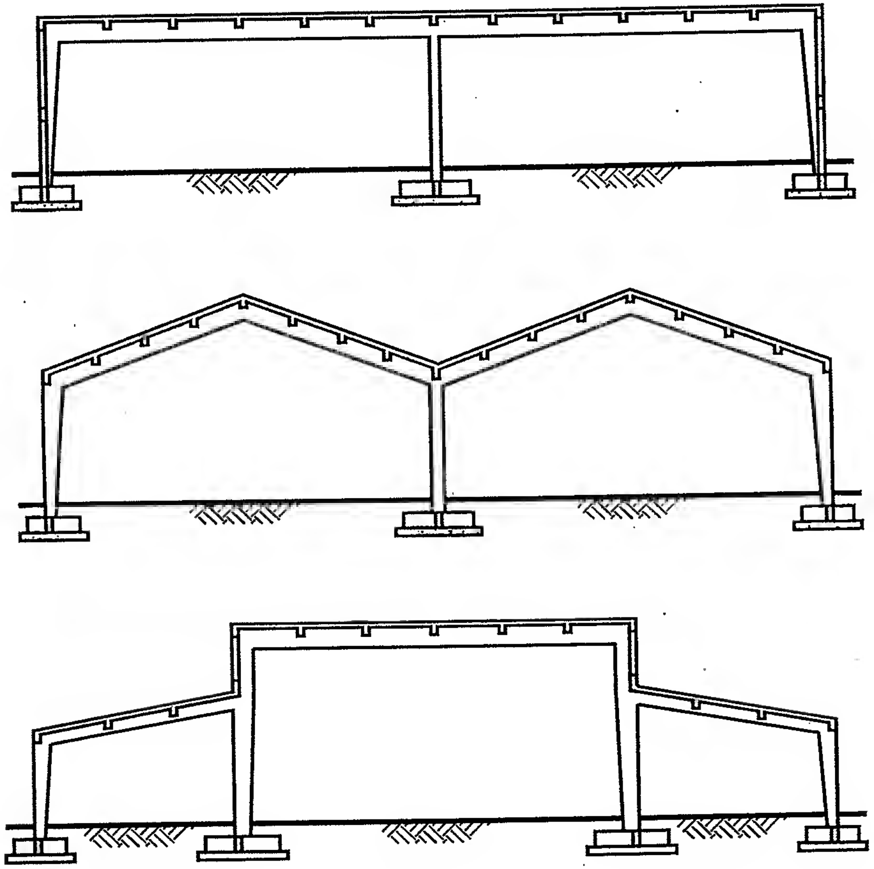


Fig. 9.7 Continuous frames

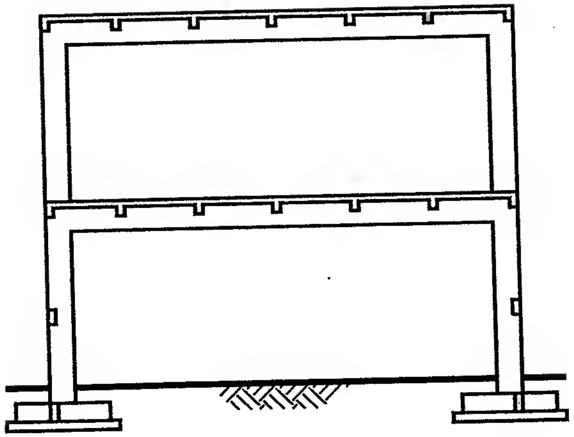


Fig. 9.7 Continuous Frames (Continued)

9.4 Layout of a Hall Supported by R/C Frames

In big covered halls, reinforced concrete frames are usually used as the main supporting element. In order to get relatively reasonable dimensions of the frames, the spacing between frames should be in the range of 5.0 ms to 7.0 ms.

Figure (9.8) shows the layout of the supporting elements of a hall that is 18.0 ms wide, 20.0 ms long and 5.0 ms clear height.

The main supporting element can be chosen as a rectangular frame. The frame are chosen to span in the short direction (18.0 ms) arranged every 5.0 ms.
Continuous secondary beams supported on the frames are used in the roof of the hall in order to get reasonable slab thickness.

In order to get reasonable wall area (15-25 m^2), wall beams are usually provided at the sides of the halls.

In case of a weak soil, semelles are provided at the foundation level in order to connect the footings together to reduce the effect of differential settlement. Otherwise, they could be provided at the bottom end of the frame leg to support the wall above.
It should be mentioned that frames are rigid in their own plane and are very flexible in the out-of-plane direction. Hence, it is essential to provide beams that connect the frames perpendicular to their plane. In many cases, these beams are already existent through the provision of wall and roof beams.

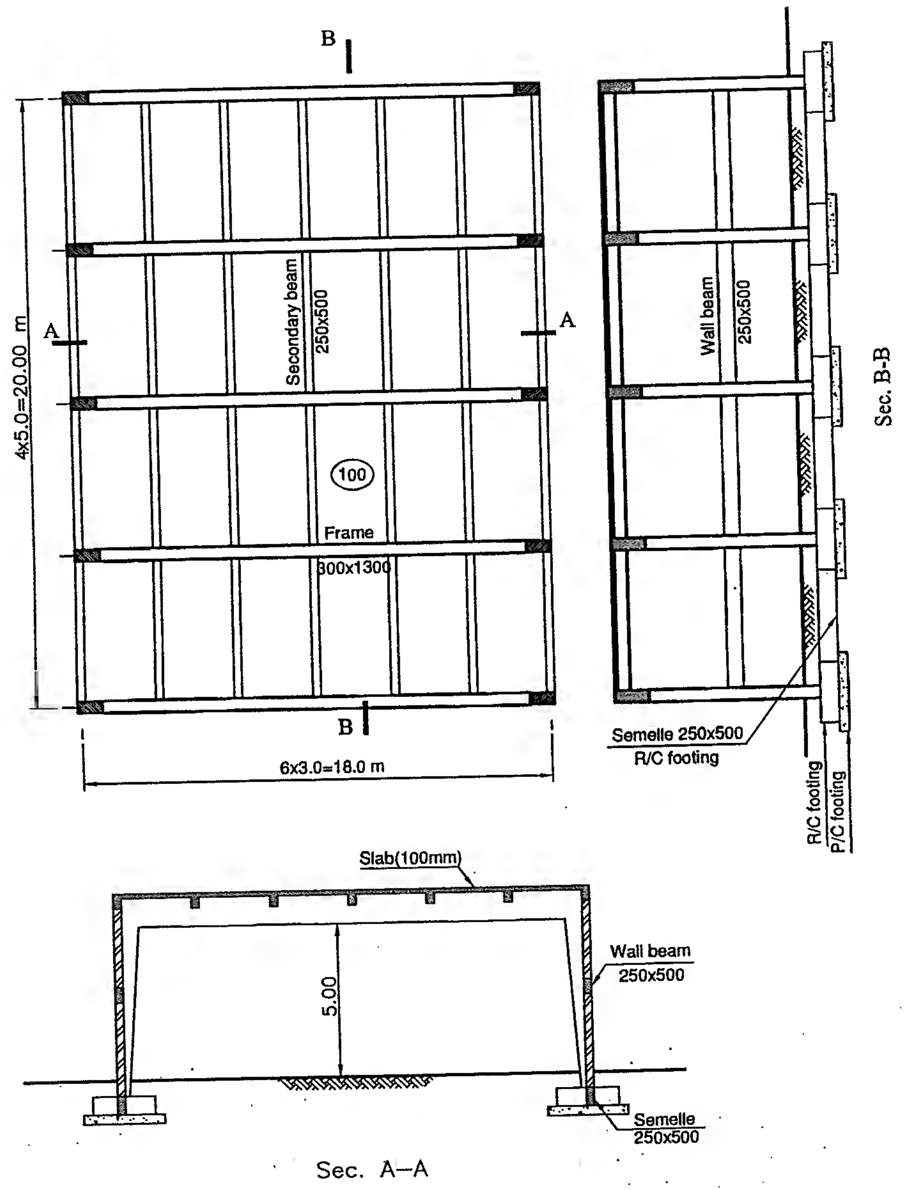


Figure 9.8 Layout of a Hall

9.5 Reinforcement Detailing of Rigid Frames

The art of detailing of reinforcement in rigid frames should follow the general rules of mechanics of reinforced concrete as a composite material. It should also be simple and practical. Figure 9.9 shows some typical details. Other reinforcement details could also be accepted.

1- Detail-1 shows a frame joint that does not resist moment (a joint between a link member and a frame girder)

2- Detail-2 shows a frame joint that does resist moment. The column is a link member and the frame girder has a cantilever part.

3- Detail-3 shows a frame joint designed to resist the moments that developed due to the rigid connection between the girder and the column. The moments applied to the joint produces what is so called "closing joint". The hogging moments of the girder are resisted by the tension column reinforcements. It should be noted that the tensile forces in the reinforcements results in a force $F = T\sqrt{2}$, that produces splitting tensile stresses. Diagonal reinforcement ($A_s = T/(f_y/\gamma_s)$) might be provided to participate in resisting the splitting tensile stresses.

4- Detail-4 shows a frame designed to resist the moments that developed due to the rigid connection between the girder and the column. The moments applied to the joint produces what is so called "opening joint". It can be seen that the reinforcement of the column has not been used to reinforce girder. If the tension reinforcement is continuous over the sharp corner, failure of the cover will take place due to the action of the resultant force F.

5- Detail-5, Detail-6 and Detail-7 show reinforcement detailing of rigid connections in frames in which the girders have cantilever parts.

6- Detail-8 shows the detailing of a double cantilever from the frame leg.

7- Detail-9 shows a broken part of a girder of a frame creating an "opening joint". The bottom reinforcement of the frame girder has to be discontinued at the broken part in order to prevent cover saplling.

8- Detail-10 shows a broken part of a girder of a frame creating a "closing joint". The bottom reinforcement has not been discontinued at the broken part since the resultant of the tension force is inward and has not affect the cover.

Figure 9.10 shows a typical reinforcement detailing of a frame.

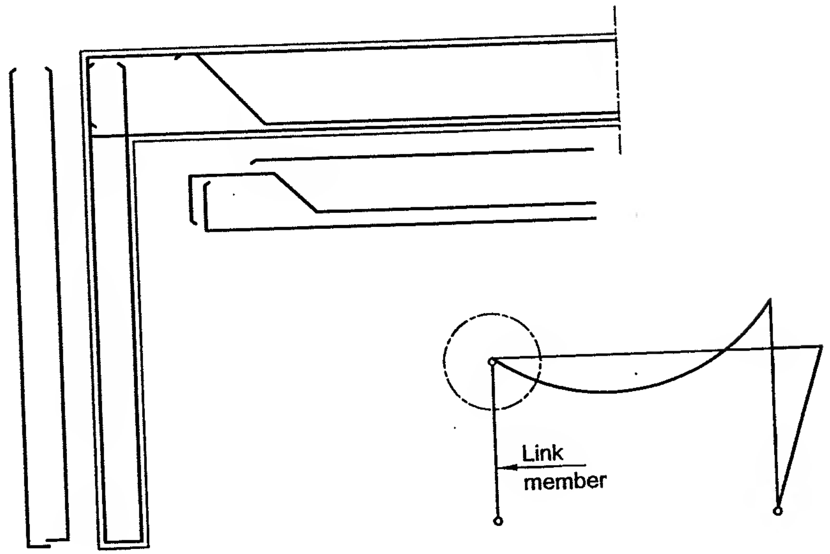


Fig. 9.9 Detailing of Frame Joints (Detail-1)

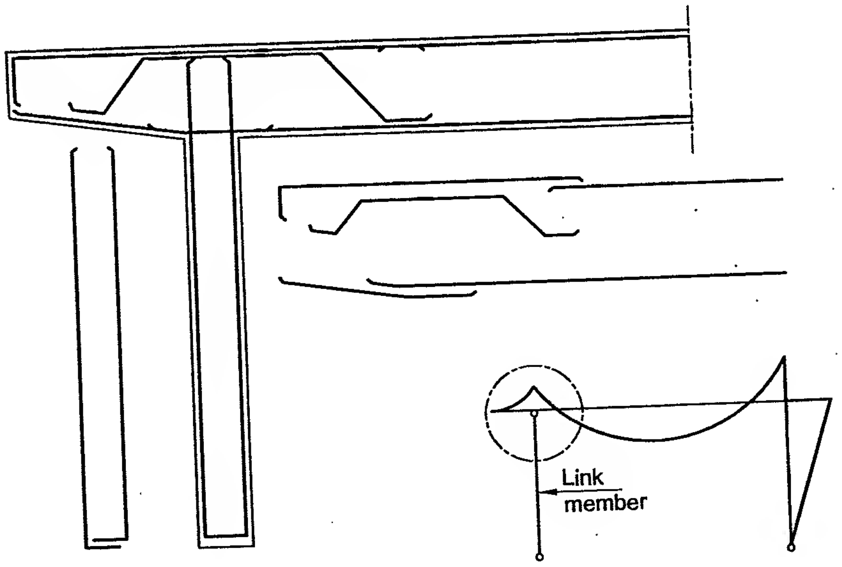


Fig. 9.9 Detailing of Frame Joints (Detail-2)

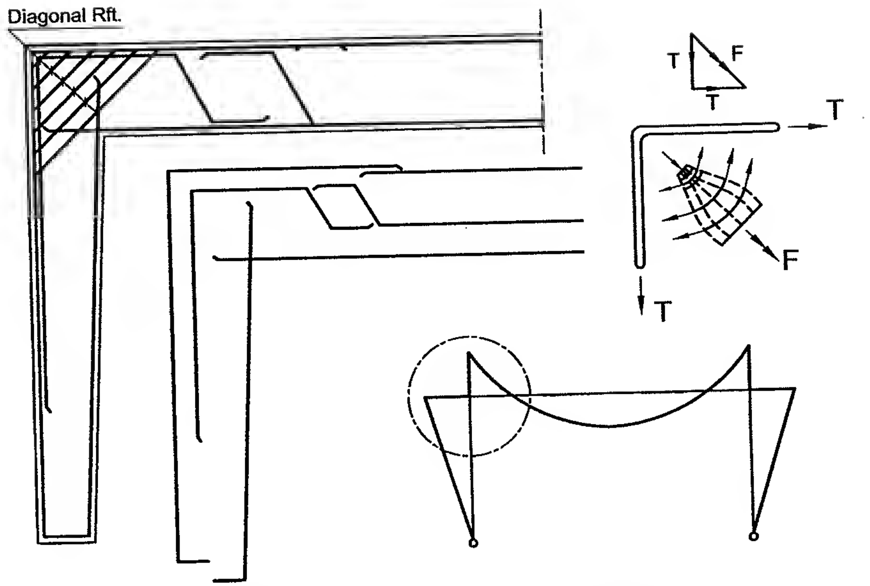


Fig. 9.9 Detailing of Frame Joints (Detail-3)

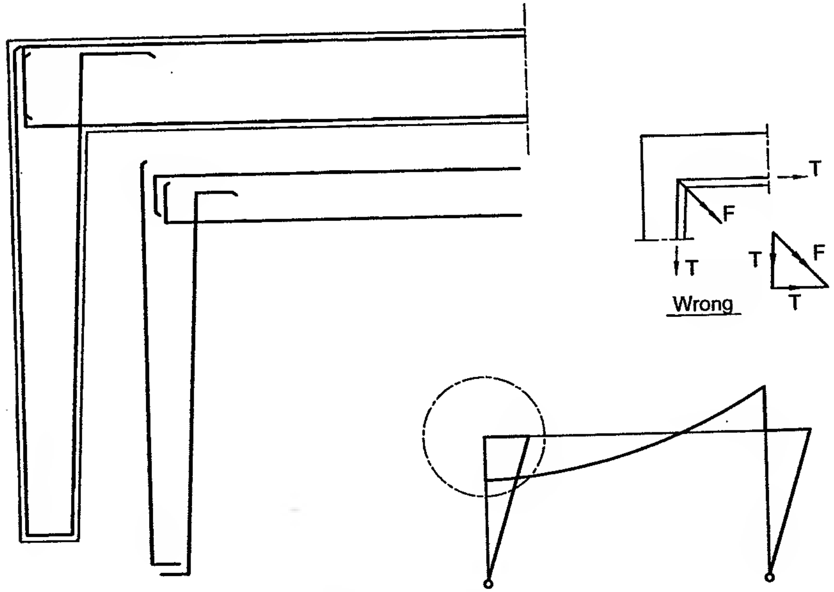


Fig. 9.9 Detailing of Frame Joints (Detail-4)

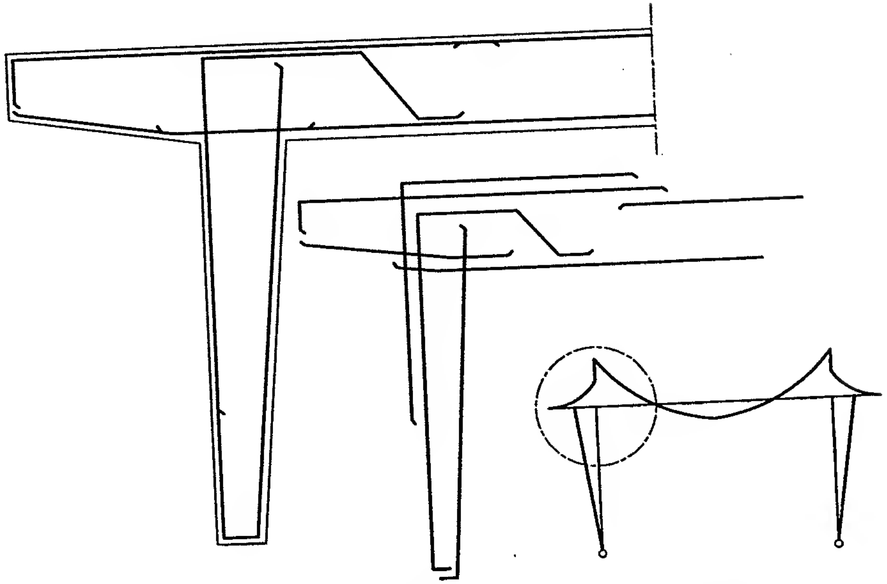


Fig. 9.9 Detailing of Frame Joints (Detail-5)

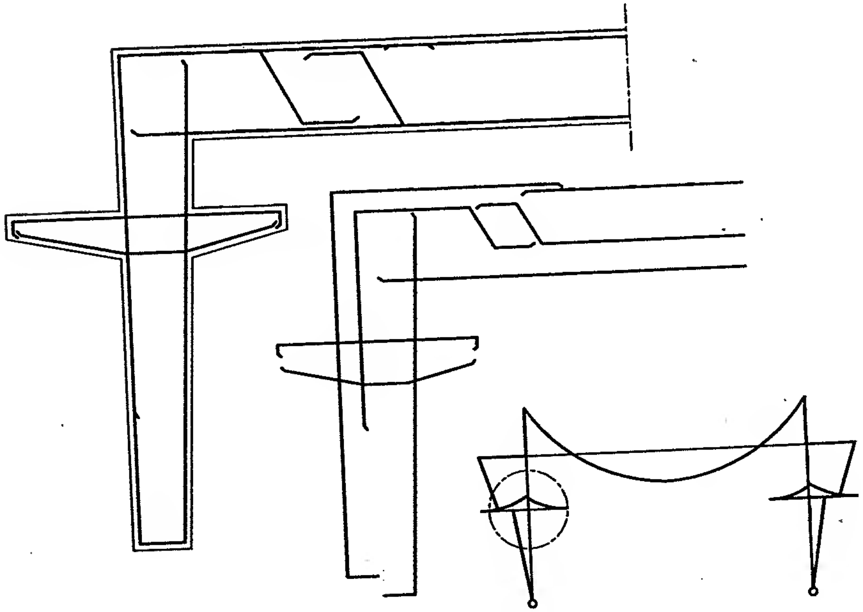


Fig. 9.9 Detailing of Frame Joints (Detail-6)

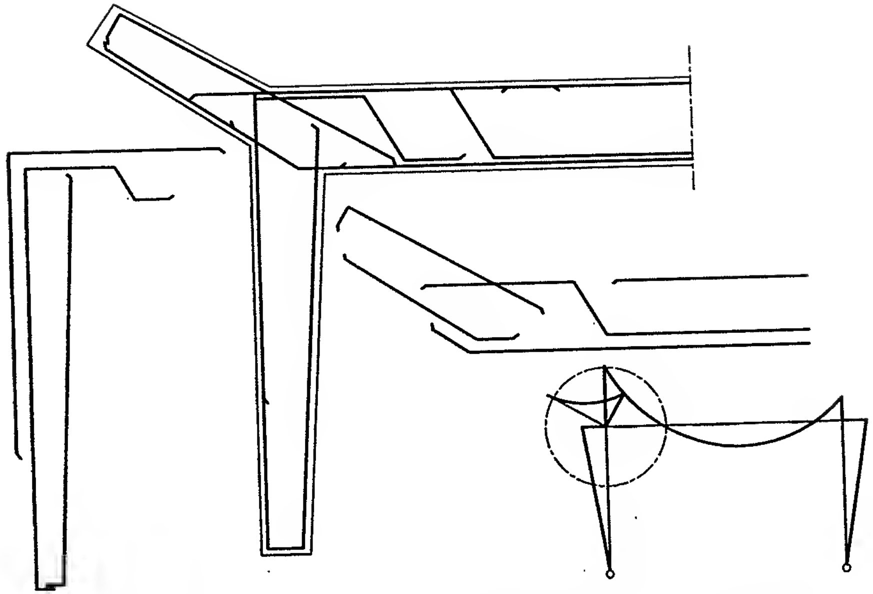


Fig. 9.9 Detailing of Frame Joints (Detail-7)

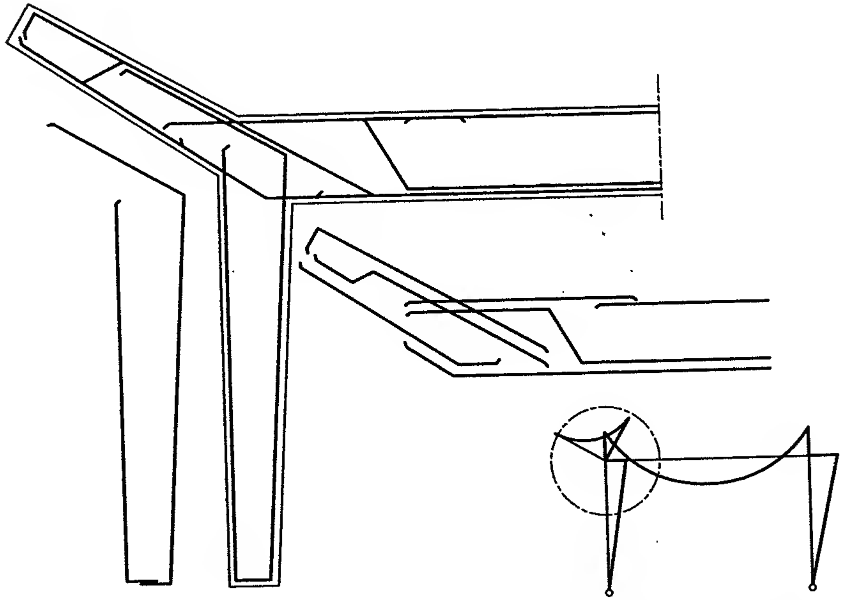


Fig. 9.9 Detailing of Frame Joints (Detail-8)

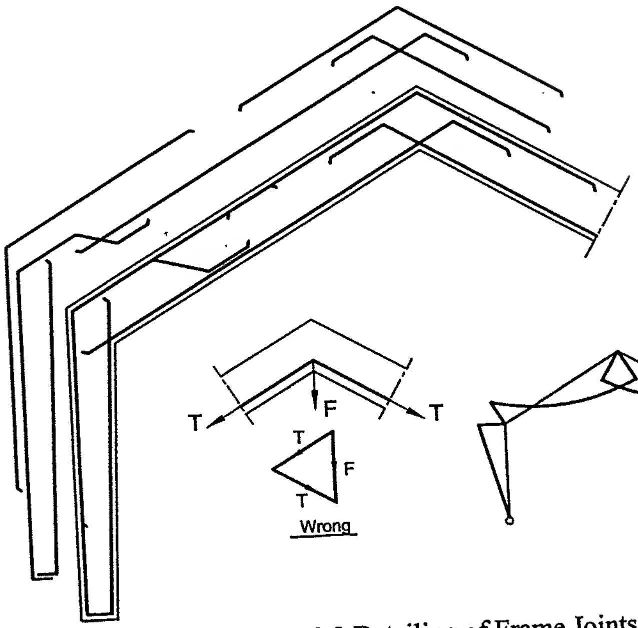


Fig. 9.9 Detailing of Frame Joints (Detail-9)

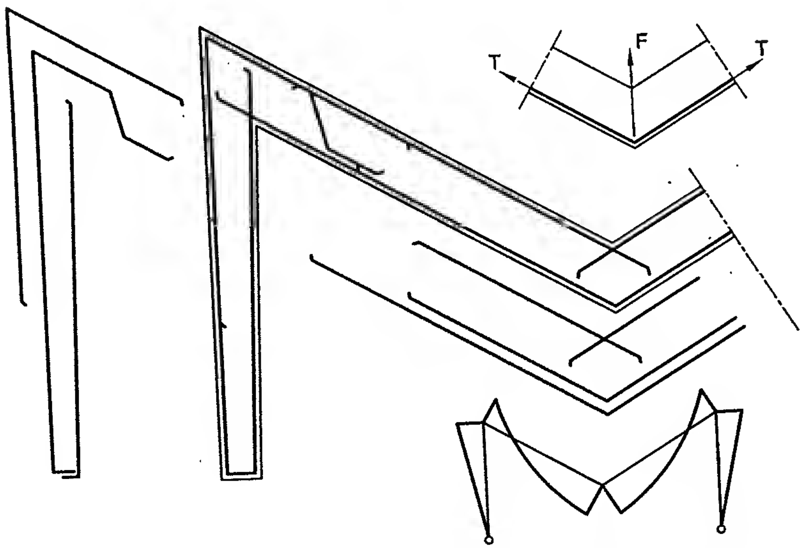


Fig. 9.9 Detailing of Frame Joints (Detail-10)

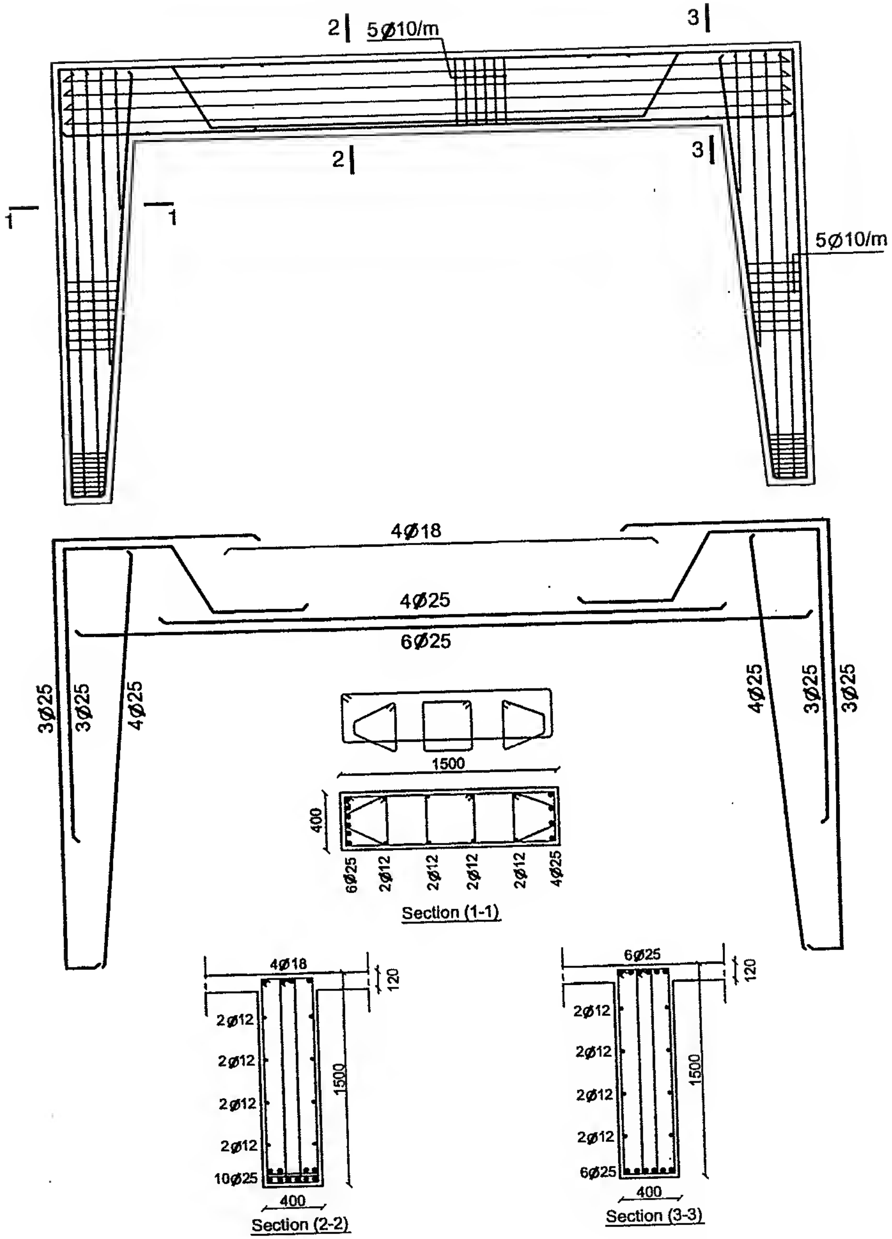


Fig. 9.10 Typical reinforcement detailing of a frame

9.6 Hinged Bearings

The most commonly used type in concrete structures is the lead plate hinge. The design of this type is explained as follows.

In this type of hinges, the normal component of the hinge is transmitted to the foundation by bearing through 20 mm thick lead plate arranged at the middle of the column foot. The horizontal component H is resisted by shear resistance of the connecting bars A_s that are protected from rusting by 20 mm thick bituminous material as shown in Fig. 9.11.

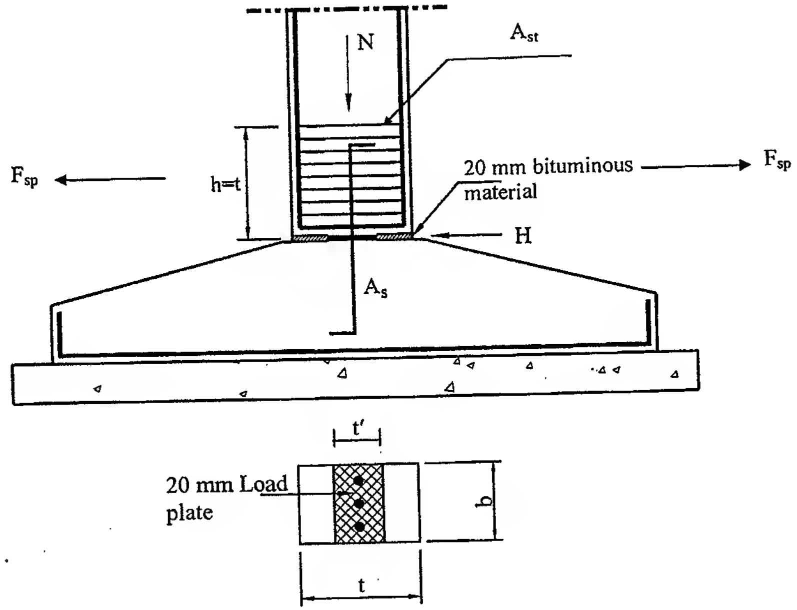


Fig. 9.11 Details of Lead hinge

In order to have acceptable hinge action, the length of the lead plate t' must be smaller than or equal to one third of the depth of the column at the position of the hinge measured in the direction of the required rotation.

The lead hinge can accordingly be calculated in the following manner:

$$\frac{N}{b\,t'} < f_b = 0.67\frac{f_{cu}}{\gamma_c}\sqrt{\frac{A}{A'}} \quad \& \quad \sqrt{\frac{A}{A'}} \le 2 \quad \text{.....................} (9.2)$$

$$\frac{N}{b\,t'} < f_b = 0.67\frac{f_{cu}}{\gamma_c}\sqrt{\frac{b \times t}{b \times t'}}$$

$$\frac{N}{b\,t'} < f_b = 0.67\frac{f_{cu}}{\gamma_c}\sqrt{\frac{t}{t'}}$$

The area of the steel dowels A_s is given by

$$A_s = \frac{H}{0.8\left(f_y/\gamma_s\right)} \quad \text{...} (9.3)$$

The normal compressive stresses are transferred from the breadth of the column t to the breadth of the lead plate t' in a height h approximately equal to the breadth t of the column. Therefore, a splitting tensile force is developed along this length. The transverse splitting force F_{sp} is approximately equal to one forth of the vertical force (N/4). This force must be resisted by horizontal stirrups of area $A_{st} = \dfrac{N/4}{f_y/\gamma_s}$ arranged at the end of the column in a height h=t. Fig. 9.12 shows the graphical representation of finite element model of the hinged base. Far from the lead plate the stress distribution can be considered uniform. Stress concentration can be easily noticed at the connection between the frame leg and the lead plate

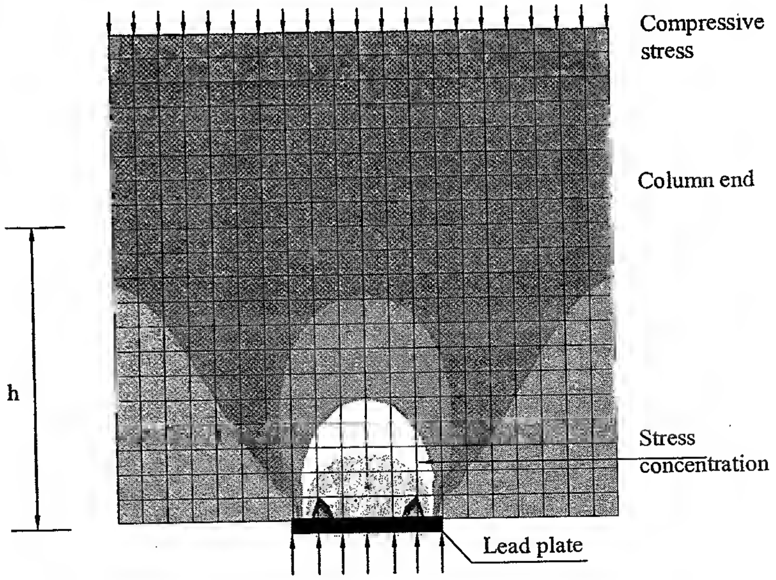


Fig. 9.12 Finite element model for the hinged base

Example 9.1

It is required to design the frame shown in the Figure below and Fig. EX9.1. The ultimate load including dead and live loads is 100 kN/m′. The horizontal reaction at the base equals = 293.7 kN. f_{cu}= 25 N/mm^2 and f_y=360 N/mm^2.

Frame spacing =5.0 m
Slab thickness t_s=120 mm

The frame may be considered unbraced in its plane and braced in the out of plane direction.

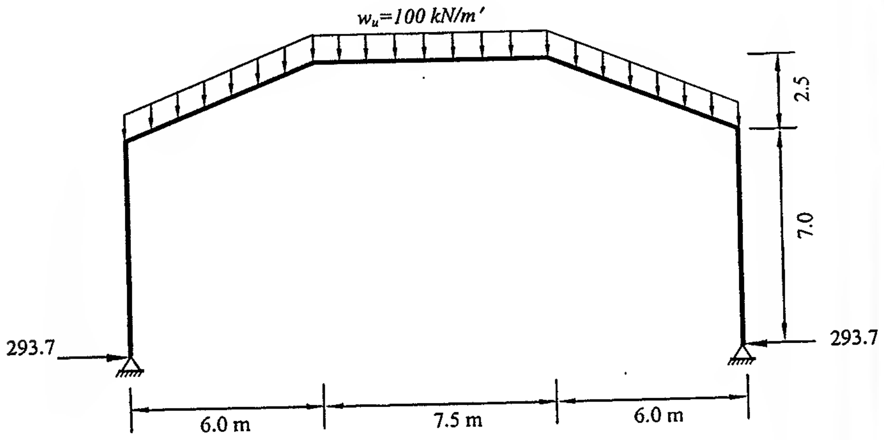
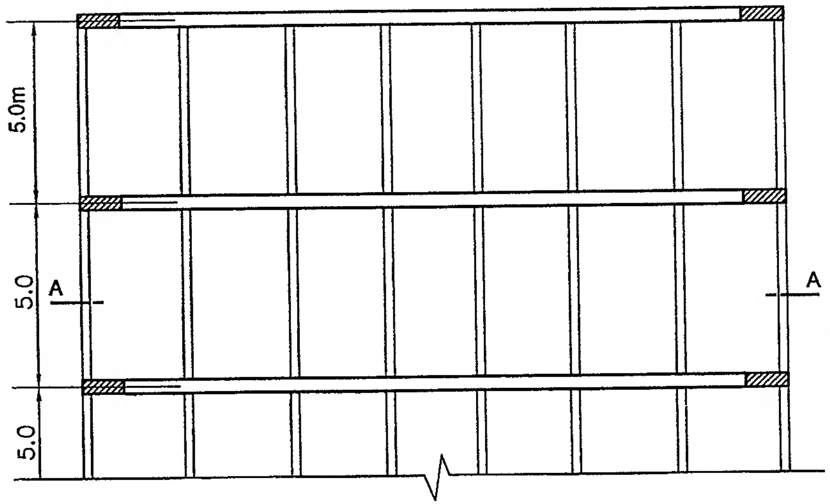
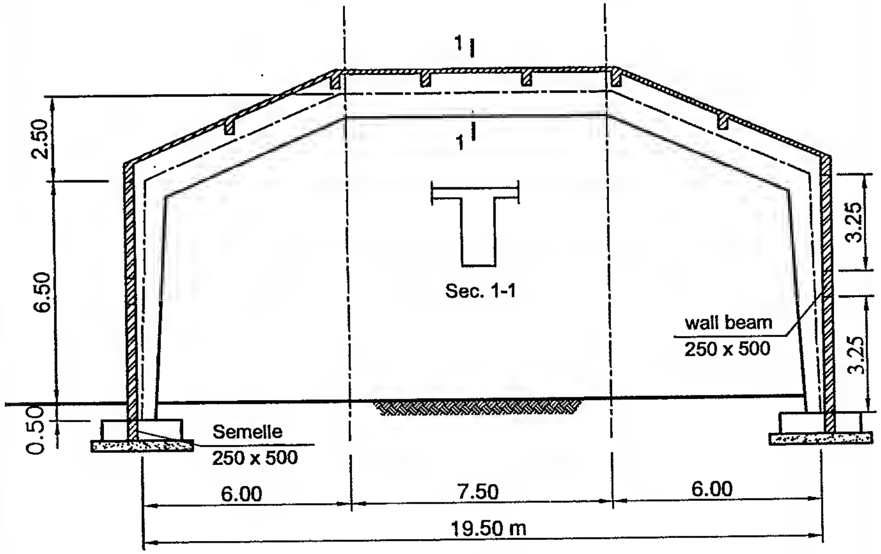


Fig. EX 9.1



Plan



Section A-A

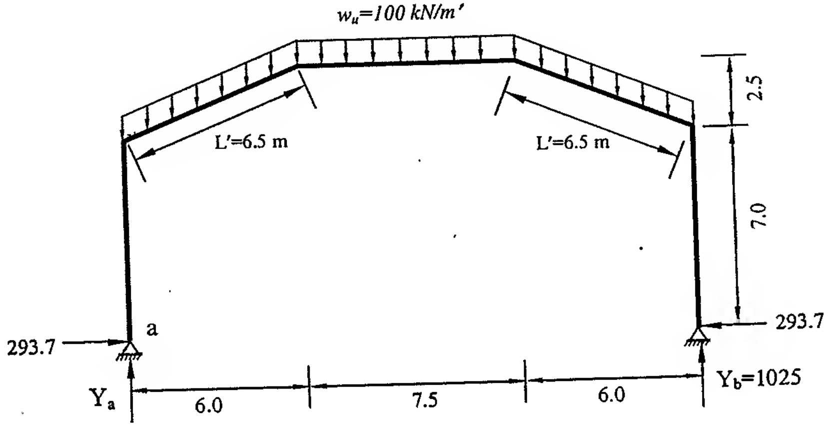
Fig. Ex.-9.1

Solution

Step 1: Calculations of the reactions

The sloped length of the frame L′ equals

$$L' = \sqrt{6.0^2 + 2.5^2} = 6.5\ m$$



Since the frame is symmetrical in loading and in geometry, the vertical reaction equals

$$Y_a = Y_b = \frac{w_u}{2} \times (2 \times L' + 7.5) = \frac{100}{2} \times (2 \times 6.5 + 7.5) = 1025\ kN$$

Step 2: Calculations of Normal, Shear, and Bending Moment

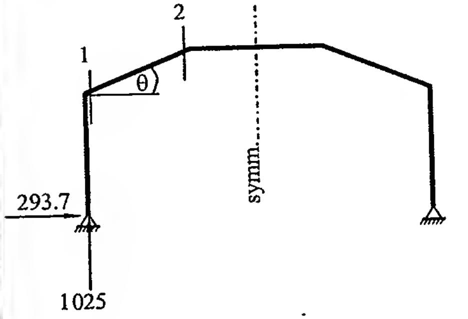
The calculation of shear and normal force for the sloped elements can be obtained using the following formula

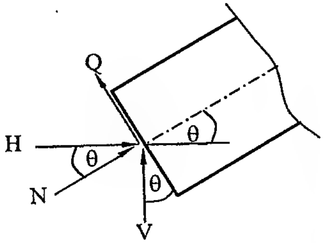
$$\theta = \tan^{-1}\left(\frac{2.5}{6}\right) = 22.6^{\circ}$$

$$N = H_i \times \cos(\theta) + V_i \times \sin(\theta) \qquad \text{(normal force)}$$

$$Q = V_i \times \cos(\theta) - H_i \times \sin(\theta) \qquad \text{(shear force)}$$

Positive direction is given in the figure below





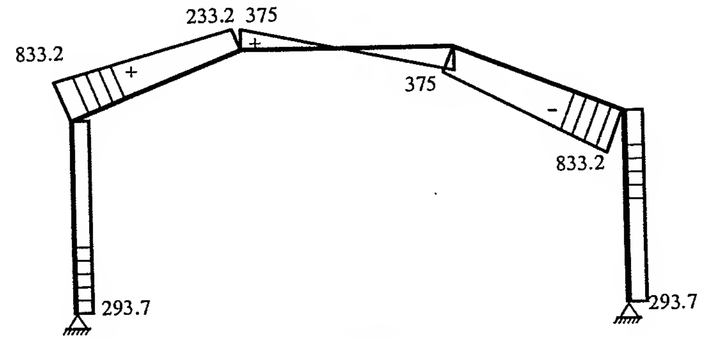
$$V_1 = Y_a = 1025\ kN$$

$$V_2 = Y_a - w_u \times L' = 1025 - 100 \times 6.5 = 375\ kN$$

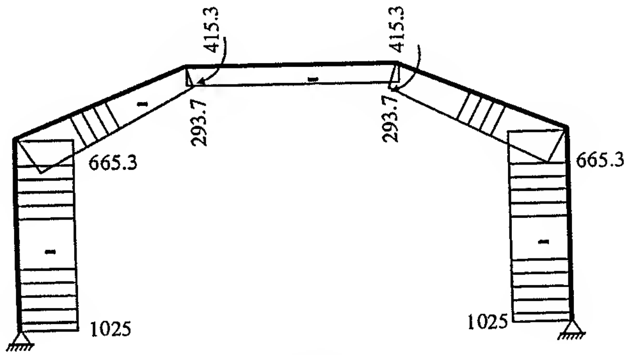
$$H_1 = H_2 = H_a = 293.7\ kN$$

section	H	V	N	Q
1	293.7	1025.0	665.3	833.2
2	293.7	375.0	415.3	233.2

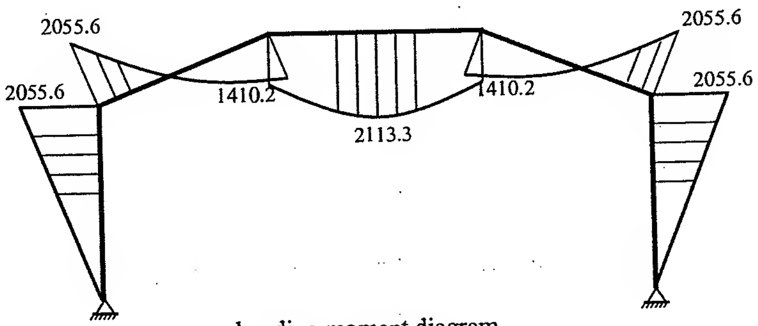
The normal, shear and bending moment are shown in the following figures.



shear force diagram



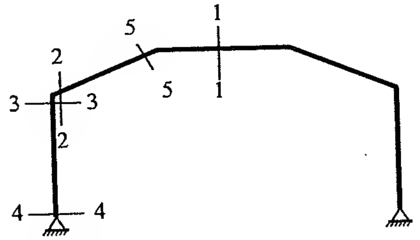
normal force diagram



bending moment diagram

Step 3: Flexural design of critical section

Because of frame symmetry, only five sections need to be designed as shown.



critical sections locations

Step 3.1: Design of Section 1 (400x1400)

Assume $t_g = \frac{span}{12\rightarrow 16} = \frac{19.5}{12\rightarrow 16} = 1.21 \rightarrow 1.625\,m$

Take t_g=1.4 m=1400 mm

Section 1 is subjected to the following straining actions:

M_u= 2113.3 kN.m

P_u=293.7 kN *(compression)*

It acts as a T-section and the width B is taken as:

$$B\ smaller\ of \rightarrow \begin{cases} 16t_s + b \\ \frac{0.7\times L}{5} + b = \\ CL \rightarrow CL \end{cases} \begin{cases} 16\times 120 + 400 = 2320\ mm \\ \frac{0.7\times 19500}{5} + 400 = 3130\ mm \\ 5000\ mm \end{cases}$$

B= 2320 mm

Assuming that the distance from the c.g of the steel to the concrete surface is 80 mm, the effective depth equals

$d = t - 80mm = 1400 - 80 = 1320\ mm$

If P_u/f_{cu} b t is less than 0.04, the normal force can be neglected

$\frac{P_u}{f_{cu}\times b\times t} = \frac{293.7\times 1000}{25\times 400\times 1400} = 0.021 < 0.04$ ....neglect normal force

The design will be carried out as if the section is subjected to moment only. Assuming that the neutral axis is within the flange and using C-J curve gives

$$d = C1\sqrt{\frac{M_u}{f_{cu}\times B}}$$

$$1320 = C1\sqrt{\frac{2113.3\times10^6}{25\times2320}} \qquad \rightarrow \text{C1}=6.91$$

The point is outside the curve(C1>4.8), thus use c/d)$_{min}$=0.125

$$a = 0.8\times c_{min} = 0.1\times d = 0.10\times1320 = 132\ mm$$

Since a(132 mm)>t$_s$(*120 mm*), our assumption is ***not*** valid, because the neutral axis is outside the flange. According to clause 4.2.1.2.f of the ECP 203, one can neglect the compression force in the web and assume that $a=t_s$. Accordingly, A_s equals

$$A_s = \frac{M_u}{f_y/1.15\times(d-t_s/2)} = \frac{2113.3\times10^6}{360/1.15\times(1320-120/2)} = 5357\ mm^2$$

$$A_{s\,min} = smaller\ \ of \begin{cases} \frac{0.225\sqrt{f_{cu}}}{f_y}b\ d = \frac{0.225\sqrt{25}}{360}\times400\times1320 = 1650\ mm^2 \ \lrcorner < A_s \quad o.k \\ 1.3A_s = 1.3\times5357 = 6965\ mm^2 \end{cases}$$

Use 12Φ25 (5890 mm^2) & use 4Φ18 (about 17% of A_s) as stirrups hangers

Step 3.2: Design of Section 2 (400x1400)

Section 2 is a rectangular section with the following straining actions:

M$_u$= 2055.6 kN.m

P$_u$= 665.3 kN *(compression)*

If P$_u$/f$_{cu}$ b t is less than 0.04, the normal force can be neglected

$$\frac{P_u}{f_{cu}\times b\times t} = \frac{665.3\times1000}{25\times400\times1400} = 0.047 > 0.04 \therefore \text{......normal force cannot be neglected}$$

Section 2 is located in the girder of the frame. Hence, it is recommended to limit the amount of the compression steel (if needed) to 40% that of the tension steel. M$_{us}$ approach for the design of this type of sections can be utilized[1] if tension failure occurs. Assuming that tension failure occurs (P$_u$<P$_{ub}$).

$$M_{us} = M_u + P_u\ (\frac{t}{2} - cover) = 2055.6 + 665.3\times\left(\frac{1400}{2} - 80\right)\times\frac{1}{1000} = 2468\ kN.m$$

---

[1] Alternatively, $e_s = e + \frac{t}{2} - \text{cover} = \frac{2055.6}{665.3} + \frac{1.4}{2} - 0.05 = 3.71\,m$

$$M_{us} = P_u\times e_s = 665.3\times3.71 = 2468\ kN.m$$

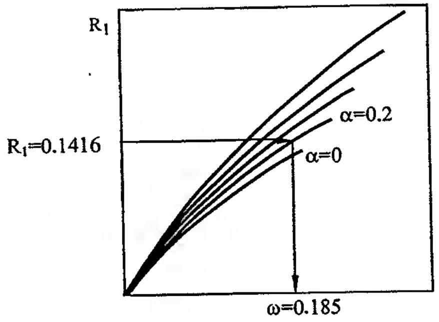
$$R_1 = \frac{M_{us}}{f_{cu}\ b\ d^2} = \frac{2468\ x\ 10^6}{25\ x\ 400\ \ x\ 1320^2} = 0.1416$$

$d'/d = 80/1320 = 0.06$

Since no curves are available for d'/d=0.06, one can use design curves or tables with d'/d = 0.1 (conservative). From the curve, it is clear that the ultimate capacity of the section for α=0 is exceeded, thus α=0.2 is used.

From the curve, ω = 0.185

$$A'_s = \alpha\ \omega\frac{f_{cu}}{f_y}b\times d = 0.2\times0.185\times\frac{25}{360}\times400\times1320 = 1357\ mm^2$$



Check failure condition

$$c_b = \frac{690}{690+f_y}d = \frac{690}{690+360}1320 = 867.42\ mm$$

Applying the equilibrium equation

$$P_{ub} = \frac{0.67\times f_{cu}\ \ b\ \ (0.8\ c_b)}{1.5} + \frac{A'_s\times f_y}{1.15} - \frac{A_s\times f_y}{1.15}$$

$$P_{ub} = \frac{0.67\times25\times400\ \ (0.8\times867.42)}{1.5} + \frac{1357\times360}{1.15} - \frac{4658\times360}{1.15}$$

$P_{ub} = 2066255\,N = 2066.2\ kN > P_u$ (tension failure as assumed)

For practical considerations, the reinforcement for section 2 should be equal to that of section 3. Thus

Use 12Φ25 (5890 mm^2)	Top
Use 4Φ25 (1963 mm^2)	Bottom

Step 3.3: Design of Section 3 (400x1200)

$t_1 \simeq 0.8 t_g \simeq 0.8 \times 1.4 \simeq 1.2\, m$

$t_2 \simeq 0.67 t_1 \simeq 0.67 \times 1.2 \simeq 0.80\, m$

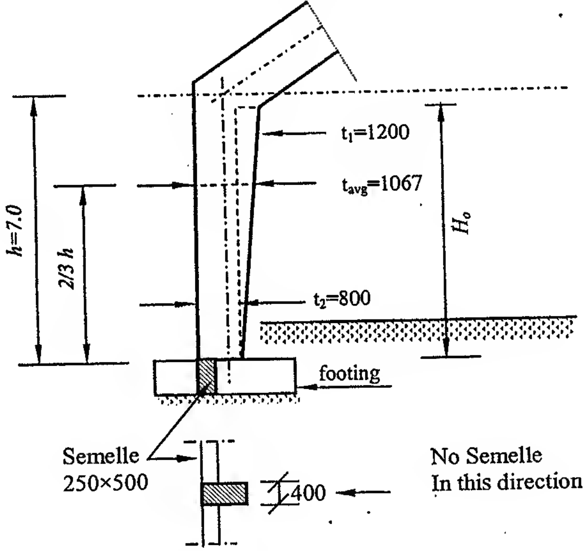
The column is subjected to the following forces

M_u= 2055.6 kN.m

P_u=1025 kN (compression)

The frame column has a variable cross section, for buckling calculations; an average column width at 2/3*h* is used.

$$t_{avg} = t_2 + \frac{2}{3}(t_1 - t_2) = 800 + \frac{2}{3}(1200 - 800) = 1067\ mm$$



Buckling analysis in the out-of-plane direction

From Fig EX9.1, H_o = 3.25 m (from the foundation level to the wall beam)

Since this direction is braced, the effective length factor k can be obtained from Table 8.3 with case 1 at top and case 1 at bottom (the semelle is larger than the column dimension). Thus, k=0.75.

$$H_e = k \times H_o = 0.75 \times 3.25 = 2.44\ m$$

$$\lambda = \frac{H_e}{b} = \frac{2.44}{0.4} = 6.1 < 15 \text{ (case of braced columns)}$$

Thus, no additional moments are induced in the out-of-plane direction

Buckling analysis in the in-plane direction

- The frame column is unbraced and the height of the column is measured from the bottom of the frame girder to the base (H_o) and is equal to

  $H_o \cong h \cong 7.0\ m$

- The effective length factor k can be obtained from Table 8.4. The top part of the column is considered case 1and the bottom part is considered case 3 (isolated footing with no semelle). Thus k=1.6 and the effective length H_e equals

  $H_e = k \times H_o = 1.6 \times 7.0 = 11.2\ m$

The slenderness ratio λ is calculated using the average column thickness not the actual section width, thus λ equals

$$\lambda = \frac{H_e}{t_{avg}} = \frac{11.20}{1.067} = 10.50$$

Since λ is greater than 10, the column is considered long and additional moment calculation is required.

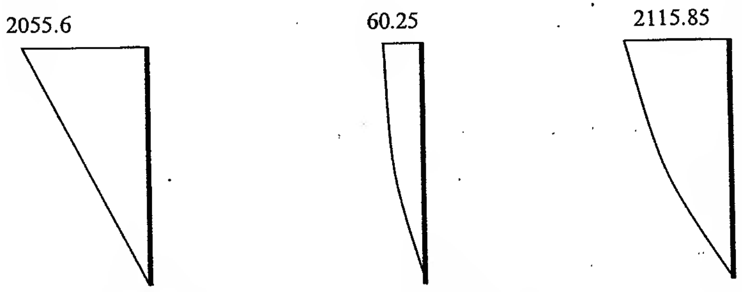
$$\delta = \frac{\lambda^2 \times t_{avg}}{2000} = \frac{10.50^2 \times 1.067}{2000} = 0.0588\ m$$

$$e_{min} = 0.05 \times t_{avg} = 0.05 \times 1.067 = 0.053\ m < \delta$$

Thus the lateral deflection due to buckling is larger than the code minimum eccentricity. The column is subjected to an axial force of P_u=1025 kN and the additional moment equals

$$M_{add} = P_u \times \delta = 1025 \times 0.0588 = 60.25\ kN.m$$

$$M_{tot} = M_u + M_{add} = 2055.6 + 60.25 = 2115.85\ kN.m$$



Original moment (M_u) Additional moment (M_{add}) Total moment (M_{tot})

It is recommended to use compression steel in frame column not less than 40% of the tension steel to allow for moment reversal initiated by lateral loads such as wind or earthquake. Columns in frames is preferred to be designed using the interaction diagram with compression steel ratio (α=0.4→0.80) depend on the magnitude of the moment. It should be noted that the actual member thickness of 1200 mm is used in the design.

$$\frac{P_u}{f_{cu}\times b\times t}=\frac{1025\times 1000}{25\times 400\times 1200}=0.085$$

$$\frac{M_u}{f_{cu}\times b\times t^2}=\frac{2115.85\times 10^6}{25\times 400\times 1200^2}=0.147$$

The factor ζ equals

$$\zeta=\frac{t-2\times cover}{t}=\frac{1200-2\times 80}{1200}=0.867$$

Using interaction diagram with f_y=360 N/mm^2, α=0.4

At ζ=0.8 ρ=5.1

At ζ=0.9 ρ=4.6

Using interpolation for ζ=0.867 gives ρ=4.8

$$\mu=\rho\times f_{cu}\times 10^{-4}=4.8\times 25\times 10^{-4}=0.012$$

$$A_s=\mu\times b\times t=0.012\times 400\times 1200=5760\ mm^2$$ (12Φ25, 5890 mm^2)

$$A'_s=\alpha\cdot A_s=0.4\times 5760=2304\ mm^2$$ (6Φ22, 2280 mm^2)

$$A_{s,total}=A'_s+A_s=5760+2304=8064\ mm^2$$

Since the column is long the minimum reinforcement ratio is

$$\mu_{\min}=0.25+0.052\ \lambda=0.52+0.052\times 10.5=0.8\%$$

$$A_{s,\min}=0.008\times b\times t=0.008\times 400\times 1200=3840\ mm^2<A_{s,tot}\ ....o.k$$

Step 3.4: Design of Section 4 (400x800)

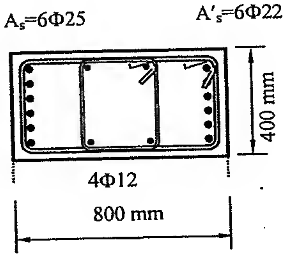
This section is subjected to pure compression (P_u= 1025kN). From the detailing of the frame the section will be reinforced with 6Φ25 and 6Φ22

$$A_{sc}=6\times 491+6\times 380=5226\ mm^2$$

$$A_{s,\min}=\frac{0.6}{100}\times b\times t=\frac{0.6}{100}\times 400\times 800=1920\ mm^2<A_{sc}\ ......o.k$$

$$P_u=0.35\times f_{cu}\times A_c+0.67\times f_y\times A_{sc}$$

$$P_u=\frac{1}{1000}(0.35\times 25\times(400\times 800)+0.67\times 360\times 5226)=4060\ kN>(1025)......o.k$$



Step 3.5: Design of Section 5 (400x1400)

Section 5 is subjected to the following straining actions

M$_u$= 1410.2 kN.m

P$_u$=415.3 kN *(compression)*

$$\frac{P_u}{f_{cu}\times b\times t}=\frac{415.3\times 1000}{25\times 400\times 1400}=0.0296<0.04\ ....\text{neglect normal force}$$

From section 1→→B= 2320 mm

$$1320=C1\sqrt{\frac{1410.2\times 10^6}{25\times 2320}}$$

C1= 8.46

The point is outside the curve, thus c/d)$_{\min}$=0.125

$$a=0.8\times c_{\min}=0.1\times d=0.10\times 1320=132\ mm$$

Since a(132 mm)>t$_s$*(120 mm)*, our assumption is ***not*** valid, because the neutral axis is outside the flange.

$$A_s=\frac{M_u}{f_y/1.15\times(d-t_s/2)}=\frac{1410.2\times 10^6}{360/1.15\times(1320-120/2)}=3575\ mm^2$$

Use 8Φ25 (3927mm^2) & use 4Φ18 as stirrups hangers

Step 4: Design for Shear

Ideally, the critical section for shear should be taken at distance of d/2 from the column leg. However for simplicity the value of the maximum shear from the shear force diagram is used.

Q_u=833.2 kN

$$q_u = \frac{Q_u}{b \times d} = \frac{833.2 \times 1000}{400 \times 1320} = 1.58 \text{ N/mm}^2$$

The applied shear q_u should be less than the maximum shear stress $q_{u,max}$

$$q_{u,\max} = 0.7\sqrt{\frac{f_{cu}}{1.5}} = 0.70\sqrt{\frac{25}{1.5}} = 2.85\ N/mm^2 \ldots > q_u(1.58)\ldots o.k$$

The presence of the compression force increases the concrete shear capacity of the section by the factor δ_c , where

$$\delta_c = 1 + 0.07 \times \frac{P_u}{A_c} = 1 + 0.07 \times \frac{665.3 \times 1000}{400 \times 1400} = 1.083 < 1.5 \ldots\ldots o.k$$

$$q_{cu} = \delta_c \times 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} = 1.083 \times 0.24\sqrt{\frac{25}{1.5}} = 1.06\ N/mm^2$$

Since $q_u > q_{cu}$, the frame has to be provided with special reinforcement for shear..

$$q_{su} = q_u - \frac{q_{cu}}{2} = 1.58 - \frac{1.06}{2} = 1.047\ N/mm^2$$

Assume that steel yield strength of the stirrups is 280 N/mm^2. Since the frame girder equals 400 mm, the code requires *four* branches of stirrups. Try 10 mm (78.5 mm^2)

$$A_{st} = 4 \times 78.5 = 314\ mm^2$$

The required stirrup spacing can be obtained using code equation (4-22)

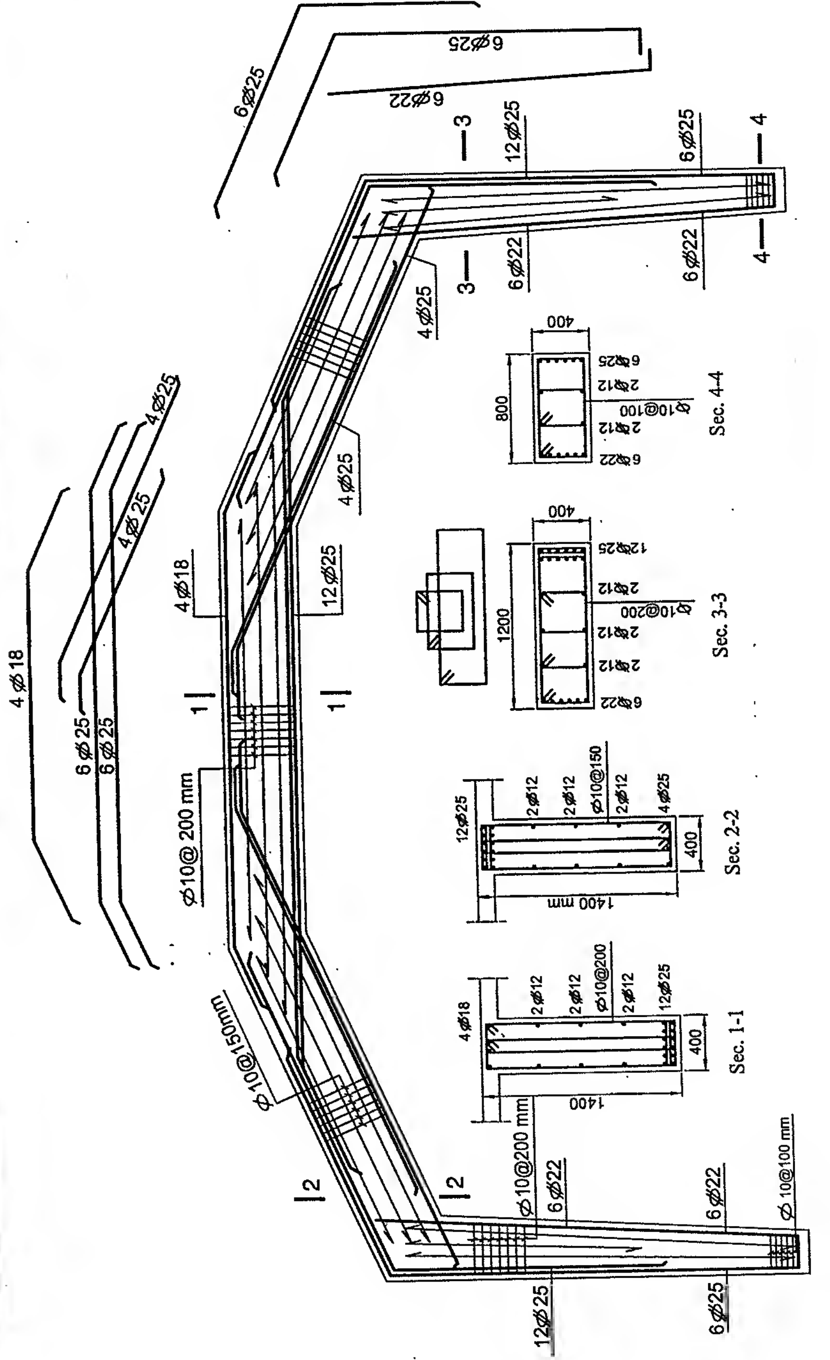
$$s \times q_{su} \times b = A_{st} \times \frac{f_{yst}}{1.15}$$

$$s \times 1.047 \times 400 = 314 \times \frac{280}{1.15}$$

s=182 mm, rounding to the lesser integer number →→ s=150 mm

$$A_{st,\min} = \frac{0.4}{f_y} \times b \times s = \frac{0.4}{280} \times 400 \times 150 = 85\ mm^2 < A_{st}(314)\ldots o.k$$

Use ϕ 10@ 150 mm in the inclined part of the girder and ϕ 10@ 200 mm elsewhere.
Note: use 10 ϕ10 at the foundation level.



Reinforcement Detailing of the Frame

Example 9.2

The frame shown in the Figure below and Fig Ex. 9.2 is to be designed for a new factory in the Six of October City. The ultimate load including dead and live loads is 75 kN/m′. The horizontal reaction at the base equals = 127.56 kN. f_{cu}= 30 N/mm^2 and f_y=360 N/mm^2.

Frame spacing =5.0 m.
Slab thickness =120 mm, b_g=350 mm, t_g=1200 mm.

The frame may be considered unbraced in its plane and braced in the out of plane direction

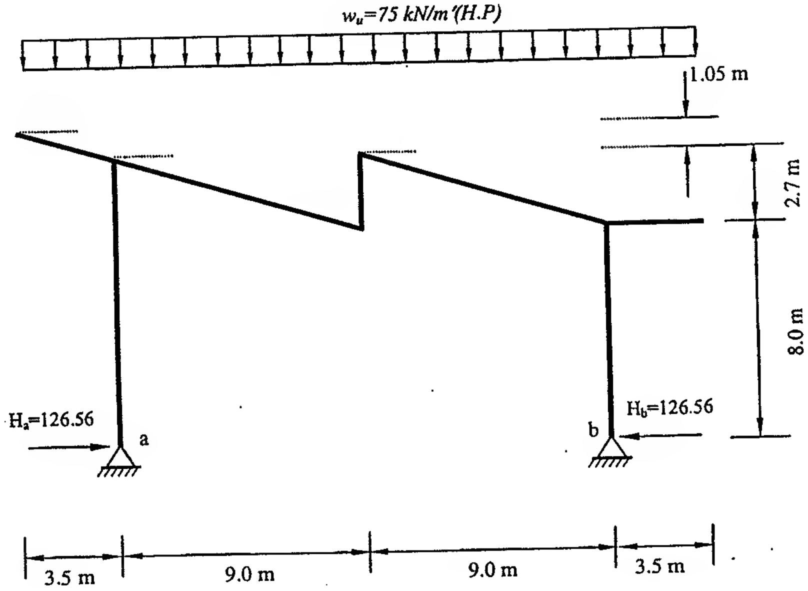


Fig. EX 9.2

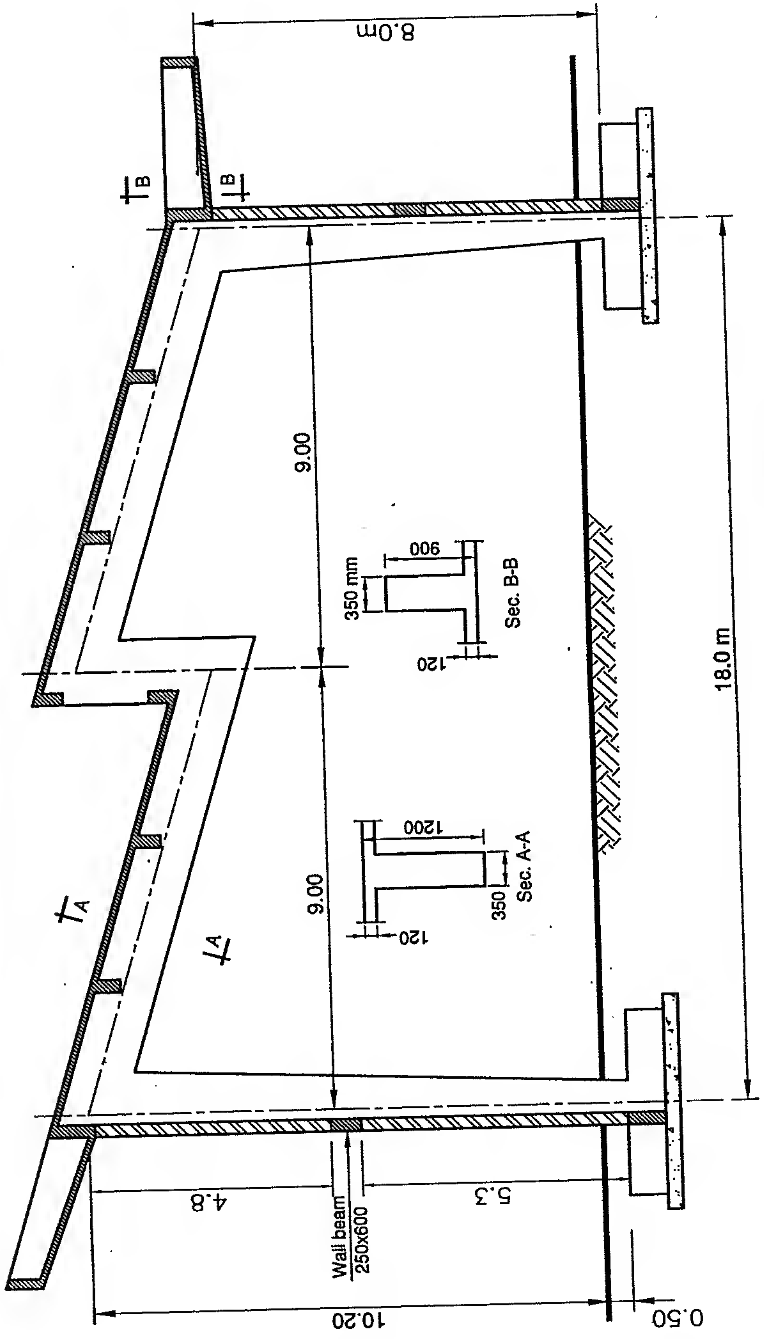


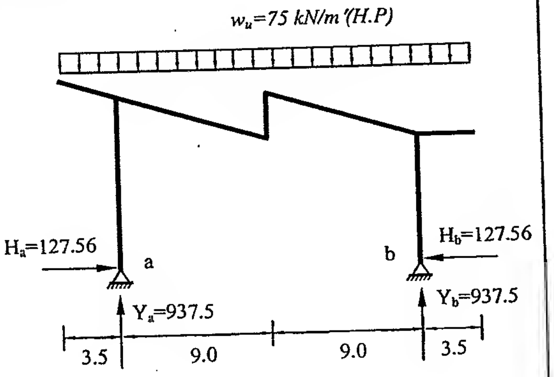
Fig. Ex.-9.2 Frame cross section

Solution

Step 1: Calculations of the reactions

The vertical reaction equals

$$Y_a = Y_b = \frac{75}{2}\times(18+2\times3.5) = 937.5\ kN$$



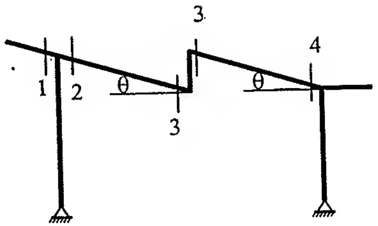
Step 2: Calculations of normal, shear, and bending moments

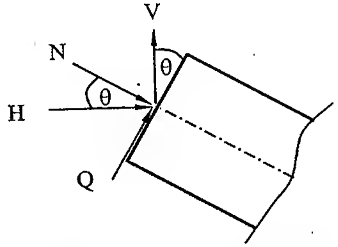
The calculation of shear and normal force for the sloped elements can be obtained using the following formula

$N = H_i\times\cos(\theta) - V_i\times\sin(\theta)$ (normal force)

$Q = V_i\times\cos(\theta) + H_i\times\sin(\theta)$ (shear force)

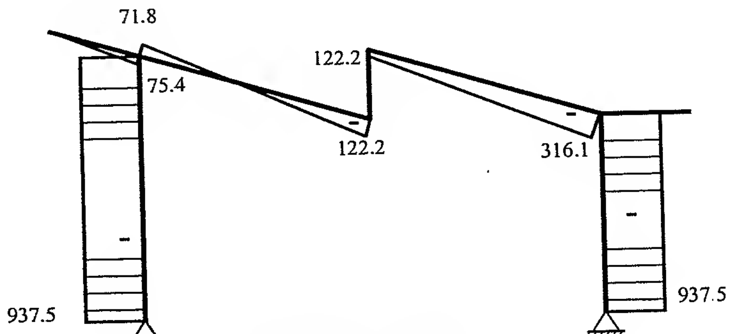
where $\theta = \tan^{-1}\left(\frac{2.7}{9}\right) = 16.7^\circ$, Positive direction is given in the figure below



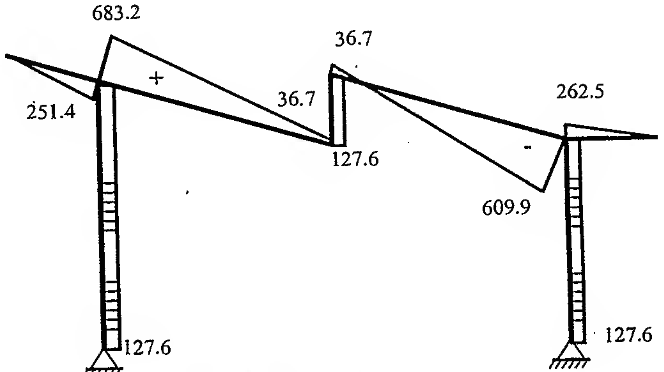


section	H	V	N	Q
1	0.0	-262.5	75.4	-251.4
2	127.56	675.0	-71.8	683.2
3	127.56	0.0	122.2	36.7
4	127.56	-675.0	316.1	-609.9

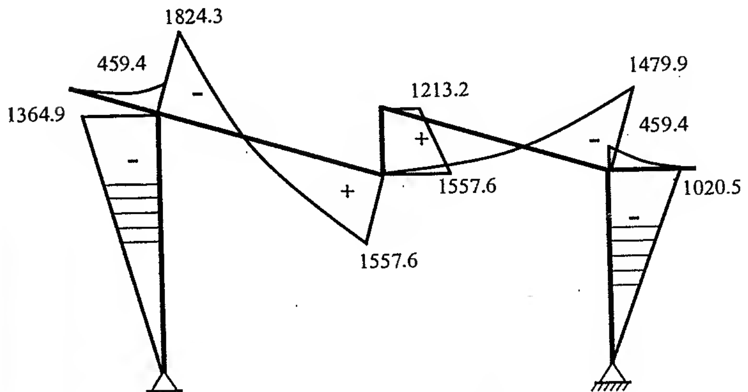
The normal, shear and bending moment are shown in the following figures.



Normal force diagram



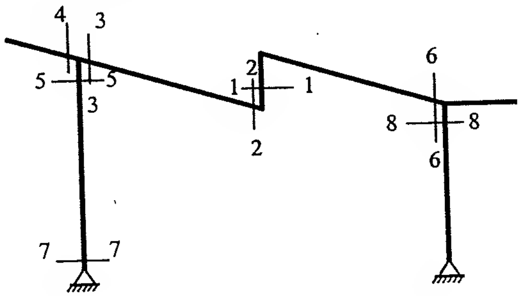
Shear force diagram



Bending moment diagram

Step 3: Flexural design of the critical section

Seven sections need to be designed as follows



Critical section locations

Step 3.1: Design of section 1 (350x1200)

The girder section is given as 350x1200 mm.

Section 1 is a rectangular section and is subjected to the following straining actions

M_u= 1557.6 kN.m

P_u= 0 kN

Assuming 70 mm concrete cover to the c.g. of the reinforcement

$$d = t - 70mm = 1200 - 70 = 1130\ mm$$

To use the R-ω, calculate R

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{1557.6 \times 10^6}{30 \times 350 \times 1130^2} = 0.116$$

From the chart with R=0.116, the reinforcement index ω=0.158

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.158 \times \frac{30}{360} \times 350 \times 1130 = 5207\ mm^2$$

$$A_{s\,\min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{30}}{360} \times 350 \times 1130 = 1354\ mm^2 \ \hookleftarrow < A_s \quad o.k \\ 1.3 A_s = 1.3 \times 5207 = 6769\ mm^2 \end{cases}$$

Use 12Φ25 (5890 mm^2) & 6Φ16 (about 20% of A_s) as secondary steel

---

Step 3.2: Design of section 2 (350x1200)

Section 2 is a T-section and is subjected to the following straining actions

M_u= 1557.6 kN.m

P_u=122.2 kN (compression)

The effective width B is taken as

$$B\ smaller\ of \rightarrow \begin{cases} 16 t_s + b \\ \dfrac{0.7 \times L}{5} + b = \\ CL \rightarrow CL \end{cases} \begin{cases} 16 \times 120 + 350 = 2270\ mm \\ \dfrac{0.7 \times 18000}{5} + 350 = 2870\ mm \\ 5000\ mm \end{cases}$$

B= 2270 mm

If P_u/f_{cu} b t is less than 0.04, the normal force can be neglected

$$\frac{P_u}{f_{cu} \times b \times t} = \frac{122.2 \times 1000}{30 \times 350 \times 1200} = 0.0097 < 0.04 \text{ ....neglect normal force}$$

The design will be carried out as if the section is subjected to moment only.

Assuming that the neutral axis is within the flange and using C-J curve gives

$$C1 = d / \sqrt{\frac{M_u}{f_{cu} \times B}} = 1130 / \sqrt{\frac{1557.6 \times 10^6}{30 \times 2270}} = 7.47$$

The point is outside the curve, thus c/d)$_{\min}$=0.125 and j=0.825

$$a = 0.8 \times c = 0.1 \times d = 0.1 \times 1130 = 113\ mm$$

Since a(113 mm)<t_s*(120 mm)*, our assumption is valid.

$$A_s = \frac{M_u}{f_y \times j \times d} = \frac{1557.6 \times 10^6}{360 \times 0.825 \times 1130} = 4641\ mm^2$$

$$A_{s\,\min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{30}}{360} \times 350 \times 1130 = 1354\ mm^2 \ \hookleftarrow < A_s \quad o.k \\ 1.3 A_s = 1.3 \times 4641 = 6033\ mm^2 \end{cases}$$

Use 10Φ25 (4909 mm^2) (actual detailing=12Φ25)

Step 3.3: Design of section 3 (350x1200)

Section 3 is a rectangular section with the following straining actions

M_u= 1824.3 kN.m

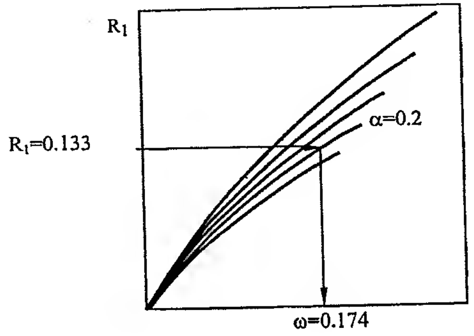
P_u= 71.8 kN *(tension)*

Since the normal force is tension, it cannot be neglected. M_{us} approach is used to calculate the area of steel

$$M_{us} = M_u + P_u\ (\frac{t}{2} - cover) = 1824.3 + (-71.8) \times \left(\frac{1200}{2} - 70\right) \times \frac{1}{1000} = 1786\ kN.m$$

$$R1 = \frac{M_{us}}{f_{cu}\, b\, d^2} = \frac{1786\ x\ 10^6}{30 \times 350\ \times 1130^2} = 0.133$$

The resulting R1 is greater than the singly reinforced section (α=0). Thus use simple bending curves with compression steel (α=0.2) and d′/d = 0.1



ω = 0.174

$$A_s = \omega \frac{f_{cu}}{f_y} b \times d + \frac{P_u}{f_y / \gamma_s} = 0.174 \times \frac{30}{360} 350 \times 1130 + \frac{71.8 \times 1000}{360/1.15} = 5964\ mm^2$$

$$A'_s = \alpha\ \omega \frac{f_{cu}}{f_y} b \times d = 0.2 \times 0.174 \times \frac{30}{360} \times 350 \times 1130 = 1147\ mm^2$$

Use 14Φ25 (6872 mm^2) Top

Use 4Φ25 (1963 mm^2) Bottom

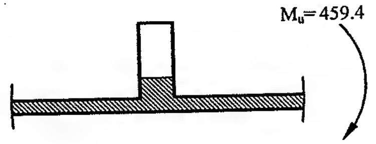
Step 3.4: Design of section 4 (350x900)

Because section 4 has an inverted slab and is subjected to negative bending, it is designed as T-section as shown in figure

M_u= 459.4 kN.m

P_u=75.4 kN (compression)

The effective width B is taken as



$$B\ smaller\ of \rightarrow \begin{cases} 16 t_s + b \\ CL \rightarrow CL = \end{cases} \begin{cases} 16 \times 120 + 350 = 2270\ mm \\ 5000\ mm \end{cases}$$

B= 2270 mm

$$d = t - cover = 900 - 50 = 850\ mm$$

If $P_u/f_{cu}bt$ is less than 0.04, the normal force can be neglected

$$\frac{P_u}{f_{cu} \times b \times t} = \frac{75.4 \times 1000}{30 \times 350 \times 900} = 0.0079 < 0.04\ \text{........neglect normal force}$$

The design will be carried out as if the section is subjected to moment only. Assuming that the neutral axis is within the flange and using C-J curve gives

$$C1 = d \Big/ \sqrt{\frac{M_u}{f_{cu} \times B}} = 850 \Big/ \sqrt{\frac{459.4 \times 10^6}{30 \times 2270}} = 10.3$$

The point is outside the curve, thus c/d)$_{min}$=0.125 and j=0.825

$$a = 0.8 \times c = 0.8 \times 0.125 \times 850 = 85\ mm$$

Since a(85 mm)<t_s*(120 mm)*, our assumption is valid

$$A_s = \frac{M_u}{f_y \times j \times d} = \frac{459.4 \times 10^6}{360 \times 0.825 \times 850} = 1819\ mm^2$$

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\, d = \dfrac{0.225\sqrt{30}}{360} \times 350 \times 850 = 1018\ mm^2 \ \lrcorner < A_s \quad o.k \\ 1.3 A_s = 1.3 \times 1819 = 2365\ mm^2 \end{cases}$$

Use 4Φ25 (1963 mm^2) (Actual detailing 6Φ25)

Step 3.5: Design of section 5 (350x1200)

$$t_1 \simeq 0.8 t_g \rightarrow t_g \simeq 1.2 m$$

$$t_2 = 0.67 t_1 \rightarrow 0.75 t_1 \simeq 0.90 m$$

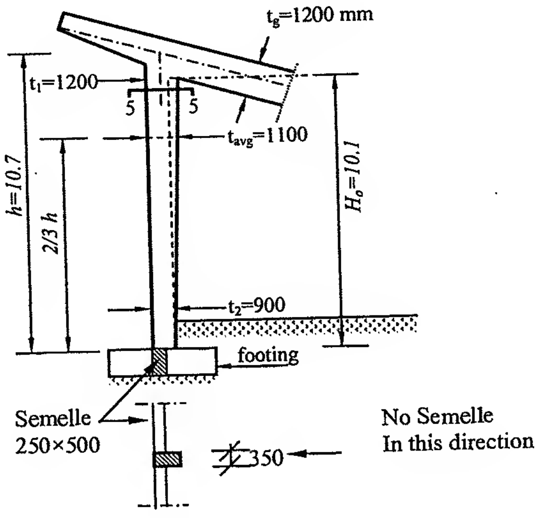
The column has a variable cross section starting from 900 mm at the footing and 1200 mm at the top. The column is subjected to the following forces

M_u= 1364.9 kN.m

P_u=937.5 kN

For buckling calculations; an average column width measured at 2/3*h* is used.

$$t_{avg} = t_2 + \frac{2}{3}(t_1 - t_2) = 900 + \frac{2}{3}(1200 - 900) = 1100\ mm$$



Buckling analysis in the out-of-plane direction

From Fig EX9.2 H_o = 4.8 m (from the foundation level to the wall beam)

Since this direction is braced, the effective length factor k can be obtained from Table 8.3 with case 1 top and case 1 at bottom (the semelle is larger than the column dimension). Thus, k=0.75.

$$H_e = k \times H_o = 0.75 \times 5.3 = 3.975\ m$$

$$\lambda = \frac{H_e}{b} = \frac{3.975}{0.35} = 11.36 < 15 \text{ (case of braced columns)}$$

Thus no additional moments are induced in the out-of-plane direction

Buckling analysis in the in-plane direction

- The height of the column is measured from the bottom of the beam to the base (H_o), as shown in Figure above.

$$H_o \cong h - \frac{t_g}{2} = 10.7 - \frac{1.2}{2} = 10.1 m$$

- Since the column is unbraced in this direction, the effective length factor k can be obtained from Table 8.4. The top part of the column is considered case 1, and the bottom part is considered case 3 (isolated footing with no semelle). Thus k=1.6, and the effective length H_e equals

$$H_e = k \times H_o = 1.6 \times 10.1 = 16.16\ m$$

- The slenderness ratio λ is calculated using the average column thickness not the actual section width, thus λ equals

$$\lambda = \frac{H_e}{t_{avg}} = \frac{16.16}{1.1} = 14.7$$

Since λ greater than 10, the column is considered long and additional moment calculation is required.

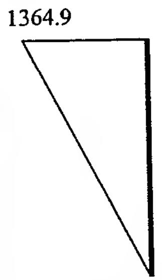
$$\delta = \frac{\lambda^2 \times t_{avg}}{2000} = \frac{14.7^2 \times 1.1}{2000} = 0.119\ m$$

$$e_{\min} = 0.05 \times t_{avg} = 0.05 \times 1.1 = 0.055\ m < \delta$$

Thus the lateral deflection due to buckling is larger than the code minimum eccentricity. The column is subjected to axial force of P_u=937.5 kN and the additional moment equals

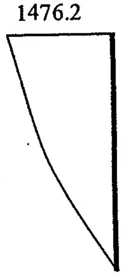
$$M_{add} = P_u \times \delta = 937.5 \times 0.119 = 111.3\ kN.m$$

$$M_{tot} = M_u + M_{add} = 1364.9 + 111.3 = 1476.2\ kN.m$$



Original moment (M_u)

Additional moment (M_{add})



Total moment (M_{tot})

$$\frac{P_u}{f_{cu} \times b \times t} = \frac{937.5 \times 1000}{30 \times 350 \times 1200} = 0.074$$

$$\frac{M_u}{f_{cu} \times b \times t^2} = \frac{1476.2 \times 10^6}{30 \times 350 \times 1200^2} = 0.098$$

The factor ζ equals

$$\zeta = \frac{t - 2 \times cover}{t} = \frac{1200 - 2 \times 70}{1200} = 0.87$$

Using interaction diagram with f_y=360N/mm^2, α=0.4, and

At ζ=0.8 ρ=3.0

At ζ=0.9 ρ=2.5

using interpolation for ζ=0.87 gives ρ=2.65

$$\mu = \rho \times f_{cu} \times 10^{-4} = 2.65 \times 30 \times 10^{-4} = 0.00795$$

$$A_s = \mu \times b \times t = 0.00795 \times 350 \times 1200 = 3339\ mm^2 \qquad (8\Phi 25,\ 3927\ \text{mm}^2)$$

$$A'_s = \alpha \cdot A_s = 0.4 \times 3339 = 1336\ mm^2 \qquad (4\Phi 22,\ 1520\ \text{mm}^2)$$

$$A_{s,total} = A'_s + A_s = 3927 + 1520 = 5447\ mm^2$$

Since the column is long the minimum reinforcement ratio is

$$\mu_{\min} = 0.25 + 0.052\ \lambda = 0.25 + 0.052 \times 14.7 = 1.014\%$$

$$A_{s,\min} = \mu_{\min} \times b \times t = 0.01014 \times 350 \times 1200 = 4258\ mm^2 < A_{s,tot} \ldots\ldots o.k$$

Step 3.6: Design of section 6 (350x1200)

Section 6 is a rectangular section and is subjected to the following straining actions

M_u= 1479.9 kN.m P_u= 316.1 kN

If P_u/f_{cu} b t is less than 0.04, the normal force can be neglected

$$\frac{P_u}{f_{cu} \times b \times t} = \frac{316.1 \times 1000}{30 \times 350 \times 1200} = 0.025 < 0.04 \ \ldots\ldots\text{neglect normal force}$$

The design will be carried out as if the section is subjected to moment only.

To use the R-ω, calculate R

$$R = \frac{M_u}{f_{cu} \times b \times d^2} = \frac{1479.9 \times 10^6}{30 \times 350 \times 1130^2} = 0.1103$$

From the chart with R=0.1103, the reinforcement index ω=0.148

$$A_s = \omega \times \frac{f_{cu}}{f_y} \times b \times d = 0.148 \times \frac{30}{360} \times 350 \times 1130 = 4877\ mm^2$$

$$A_{s\min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} b\ d = \dfrac{0.225\sqrt{30}}{360} \times 350 \times 1130 = 1354\ mm^2 \ \lrcorner < A_s \quad o.k \\ 1.3A_s = 1.3 \times 4877 = 6341\ mm^2 \end{cases}$$

Step 3.7: Design of section 7 (350x900)

This section is subjected to pure compression (P_u= 937.5 kN). From the detailing of the frame the section will be reinforced with 4Φ25 and 4Φ22

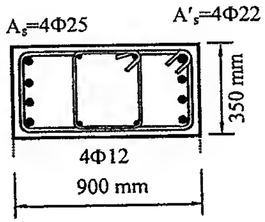
$$A_{sc} = 4 \times 491 + 4 \times 380 = 3484\ mm^2$$

The minimum percentage of reinforcement μ for this long column is 1.01% (refer to step 3.5)

$$A_{s,\min} = \frac{1.01}{100} \times b \times t = \frac{1.01}{100} \times 350 \times 900 = 3194\ mm^2 < A_{sc} \ldots\ldots \text{o.k}$$

$$P_u = 0.35 \times f_{cu} \times A_c + 0.67 \times f_y \times A_{sc}$$

$$P_u = \frac{1}{1000}\left(0.35 \times 30 \times (350 \times 900) + 0.67 \times 360 \times 3484\right) = 4147\ \ kN > (937.5) \ldots\ldots o.k$$



Step 3.8: Design of section 8 (350x1200)

This is a column section and the procedure of calculations is similar to those presented in section 5. (M_u=1020.5 , P_u=937.5 kN)→→A_s=6Φ25

Step 4: Design for shear

The critical section for shear is at d/2 from the face of the support

$$Q_u = Q - w_u \times \left(\frac{t_{col}}{2} + \frac{d}{2}\right) = 683.2 - 75 \times \left(\frac{1.2}{2} + \frac{1.13}{2}\right) = 595.82\ kN$$

$$q_u = \frac{Q_u}{b \times d} = \frac{595.82 \times 1000}{350 \times 1130} = 1.506\ \text{N/mm}^2$$

The applied shear q_u should be less than the maximum shear stress $q_{u,max}$

$$q_{u,\max} = 0.7\sqrt{\frac{f_{cu}}{1.5}} = 0.70\sqrt{\frac{30}{1.5}} = 3.13\ N/mm^2 \ldots > q_u(1.506) \ldots o.k$$

The presence of the tension force (P_u=71.8 kN) decrease the concrete shear capacity of the frame by amount of δ_t where

$$\delta_t = 1 - 0.3 \times \frac{P_u}{A_c} = 1 - 0.3 \times \frac{71.8 \times 1000}{350 \times 1200} = 0.949$$

$$q_{cu} = \delta_t \times 0.24 \sqrt{\frac{f_{cu}}{1.5}} = 0.949 \times 0.24 \sqrt{\frac{30}{1.5}} = 1.019\, N/mm^2$$

Since $q_u > q_{cu}$, the frame has to be provided with special reinforcement for shear.

$$q_{su} = q_u - \frac{q_{cu}}{2} = 1.506 - \frac{1.019}{2} = 0.997\ N/mm^2$$

Assume that steel yield strength of the stirrups is 360 N/mm^2. Try *two* branches of stirrups with 10 mm in diameter (78.5 mm^2)

$$A_{st} = 2 \times 78.5 = 157\ mm^2$$

The required stirrup spacing can be obtained using the following

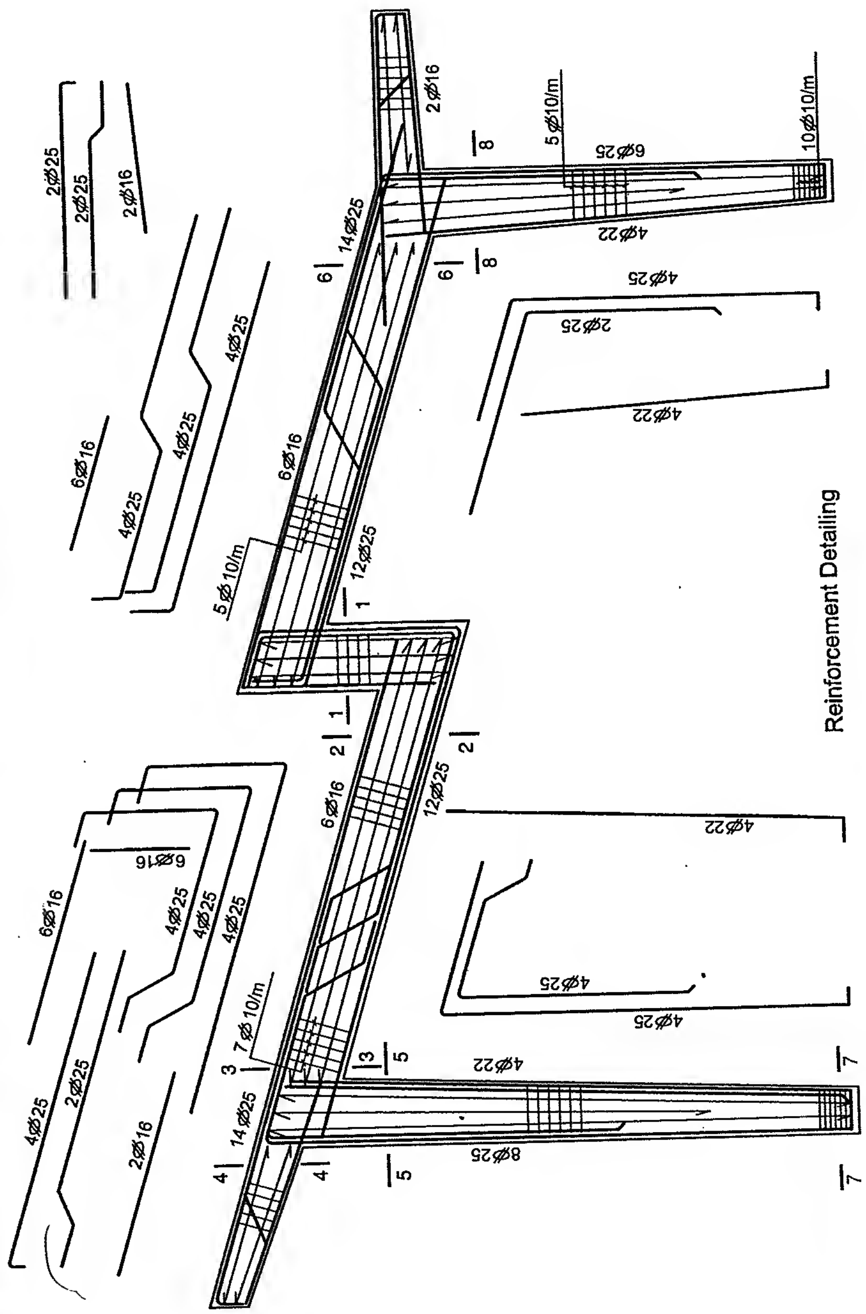
$$s \times q_{su} \times b = A_{st} \times \frac{f_{yst}}{1.15}$$

$$s \times 0.997 \times 350 = 157 \times \frac{360}{1.15} \quad \rightarrow\rightarrow \quad s=142.8 \text{ mm.}$$

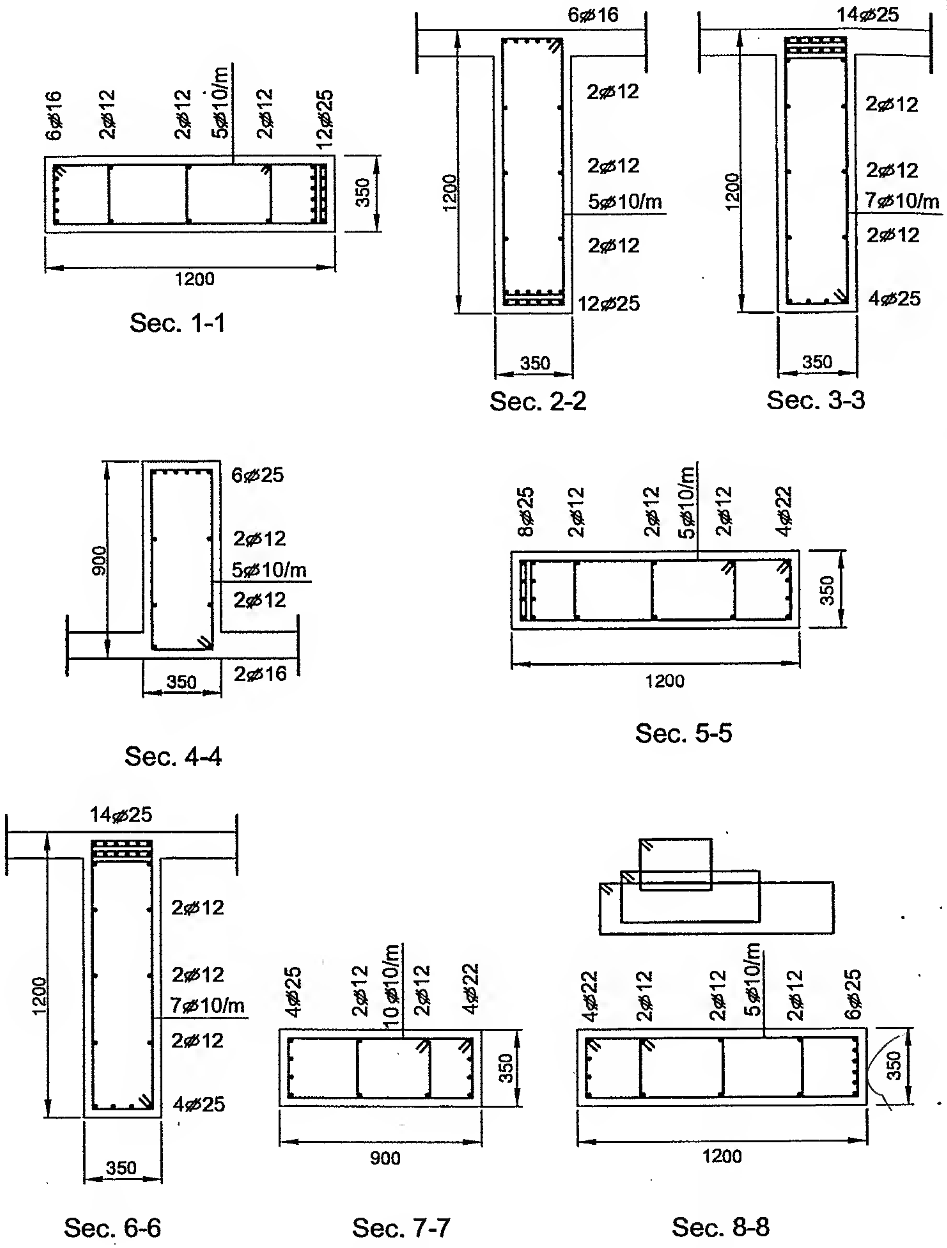
Use 7Φ10/m′ (s=142 mm). This amount is greater than the minimum given by

$$A_{st,\min} = \frac{0.4}{f_y} \times b \times s = \frac{0.4}{360} \times 350 \times 142 = 55.2\ mm^2 < A_{st}(157\ mm^2)..o.k$$

Note: For the middle third part of the girder and columns where the shear force is small, an amount of 5 Φ 10 /m′ is used. At the intersection of the column and the footing the stirrups is increased to 10 Φ 10 /m′.



Reinforcement Detailing



Sec. 1-1

Sec. 2-2

Sec. 3-3

Sec. 4-4

Sec. 5-5

Sec. 6-6

Sec. 7-7

Sec. 8-8

Reinforcement Detailing of the Frame (Continued)

APPENDIX A

Design Charts for Sections Subjected to Flexure

Area of Steel Bars in cm ^{2} (used in Egypt)

Φ	Weight	Cross sectional area (cm^2)											
mm	kg/m'	1	2	3	4	5	6	7	8	9	10	11	12
6	0.222	0.28	0.57	0.85	1.13	1.41	1.70	1.98	2.26	2.54	2.83	3.11	3.39
8	0.395	0.50	1.01	1.51	2.01	2.51	3.02	3.52	4.02	4.52	5.03	5.53	6.03
10	0.617	0.79	1.57	2.36	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.42
12	0.888	1.13	2.26	3.39	4.52	5.65	6.79	7.92	9.05	10.18	11.31	12.44	13.57
14	1.208	1.54	3.08	4.62	6.16	7.70	9.24	10.78	12.32	13.85	15.39	16.93	18.47
16	1.578	2.01	4.02	6.03	8.04	10.05	12.06	14.07	16.08	18.10	20.11	22.12	24.13
18	1.998	2.54	5.09	7.63	10.18	12.72	15.27	17.81	20.36	22.90	25.45	27.99	30.54
20	2.466	3.14	6.28	9.42	12.57	15.71	18.85	21.99	25.13	28.27	31.42	34.56	37.70
22	2.984	3.80	7.60	11.40	15.21	19.01	22.81	26.61	30.41	34.21	38.01	41.81	45.62
25	3.853	4.91	9.82	14.73	19.63	24.54	29.45	34.36	39.27	44.18	49.09	54.00	58.90
28	4.834	6.16	12.32	18.47	24.63	30.79	36.95	43.10	49.26	55.42	61.58	67.73	73.89
32	6.313	8.04	16.08	24.13	32.17	40.21	48.25	56.30	64.34	72.38	80.42	88.47	96.51
38	8.903	11.34	22.68	34.02	45.36	56.71	68.05	79.39	90.73	102.1	113.4	124.8	136.1

Area of Other Steel Bars in cm ^{2}

Φ	Weight	Cross sectional area (cm^2)											
mm	kg/m'	1	2	3	4	5	6	7	8	9	10	11	12
6	0.222	0.28	0.57	0.85	1.13	1.41	1.70	1.98	2.26	2.54	2.83	3.11	3.39
8	0.395	0.50	1.01	1.51	2.01	2.51	3.02	3.52	4.02	4.52	5.03	5.53	6.03
10	0.617	0.79	1.57	2.36	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.42
13	1.042	1.33	2.65	3.98	5.31	6.64	7.96	9.29	10.62	11.95	13.27	14.60	15.93
16	1.578	2.01	4.02	6.03	8.04	10.05	12.06	14.07	16.08	18.10	20.11	22.12	24.13
19	2.226	2.84	5.67	8.51	11.34	14.18	17.01	19.85	22.68	25.52	28.35	31.19	34.02
22	2.984	3.80	7.60	11.40	15.21	19.01	22.81	26.61	30.41	34.21	38.01	41.81	45.62
25	3.853	4.91	9.82	14.73	19.63	24.54	29.45	34.36	39.27	44.18	49.09	54.00	58.90
28	4.834	6.16	12.32	18.47	24.63	30.79	36.95	43.10	49.26	55.42	61.58	67.73	73.89
32	6.313	8.04	16.08	24.13	32.17	40.21	48.25	56.30	64.34	72.38	80.42	88.47	96.5
38	8.903	11.34	22.68	34.02	45.36	56.71	68.05	79.39	90.73	102.1	113.4	124.8	136.1

Area of Steel Bars in mm ^{2} (used in Egypt)

Φ	Weight	Cross sectional area (mm^2)											
mm	kg/m'	1	2	3	4	5	6	7	8	9	10	11	12
6	0.222	28.3	56.5	84.8	113	141	170	198	226	254	283	311	339
8	0.395	50.3	101	151	201	251	302	352	402	452	503	553	603
10	0.617	78.5	157	236	314	393	471	550	628	707	785	864	942
12	0.888	113	226	339	452	565	679	792	905	1018	1131	1244	1357
14	1.208	154	308	462	616	770	924	1078	1232	1385	1539	1693	1847
16	1.578	201	402	603	804	1005	1206	1407	1608	1810	2011	2212	2413
18	1.998	254	509	763	1018	1272	1527	1781	2036	2290	2545	2799	3054
20	2.466	314	628	942	1257	1571	1885	2199	2513	2827	3142	3456	3770
22	2.984	380	760	1140	1521	1901	2281	2661	3041	3421	3801	4181	4562
25	3.853	491	982	1473	1963	2454	2945	3436	3927	4418	4909	5400	5890
28	4.834	616	1232	1847	2463	3079	3695	4310	4926	5542	6158	6773	7389
32	6.313	804	1608	2413	3217	4021	4825	5630	6434	7238	8042	8847	9651
38	8.903	1134	2268	3402	4536	5671	6805	7939	9073	10207	11341	12475	13609

Area of Other Steel Bars in mm ^{2}

Φ	Weight	Cross sectional area (mm^2)											
mm	kg/m'	1	2	3	4	5	6	7	8	9	10	11	12
6	0.222	28.3	56.5	84.8	113.1	141.4	170	198	226	254	283	311	339
8	0.395	50.3	100.5	151	201	251	302	352	402	452	503	553	603
10	0.617	79	157	236	314	393	471	550	628	707	785	864	942
13	1.042	133	265	398	531	664	796	929	1062	1195	1327	1460	1593
16	1.578	201	402	603	804	1005	1206	1407	1608	1810	2011	2212	2413
19	2.226	284	567	851	1134	1418	1701	1985	2268	2552	2835	3119	3402
22	2.984	380	760	1140	1521	1901	2281	2661	3041	3421	3801	4181	4562
25	3.853	491	982	1473	1963	2454	2945	3436	3927	4418	4909	5400	5890
28	4.834	616	1232	1847	2463	3079	3695	4310	4926	5542	6158	6773	7389
32	6.313	804	1608	2413	3217	4021	4825	5630	6434	7238	8042	8847	9651
38	8.903	1134	2268	3402	4536	5671	6805	7939	9073	10207	11341	12475	13609

Values of α and β for solid slab with live loads less than 5 kN/m²

r	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
α	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
β	0.35	0.29	0.25	0.21	0.18	0.16	0.14	0.12	0.11	0.09	0.08

α and β values for two-way hollow block slabs (Marcus values)

r	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	0.396	0.473	0.543	0.606	0.660	0.706	0.746	0.778	0.806	0.830	0.849
β	0.396	0.333	0.262	0.212	0.172	0.140	0.113	0.093	0.077	0.063	0.053

α and β values for paneled beam slabs (Grashoff's values)

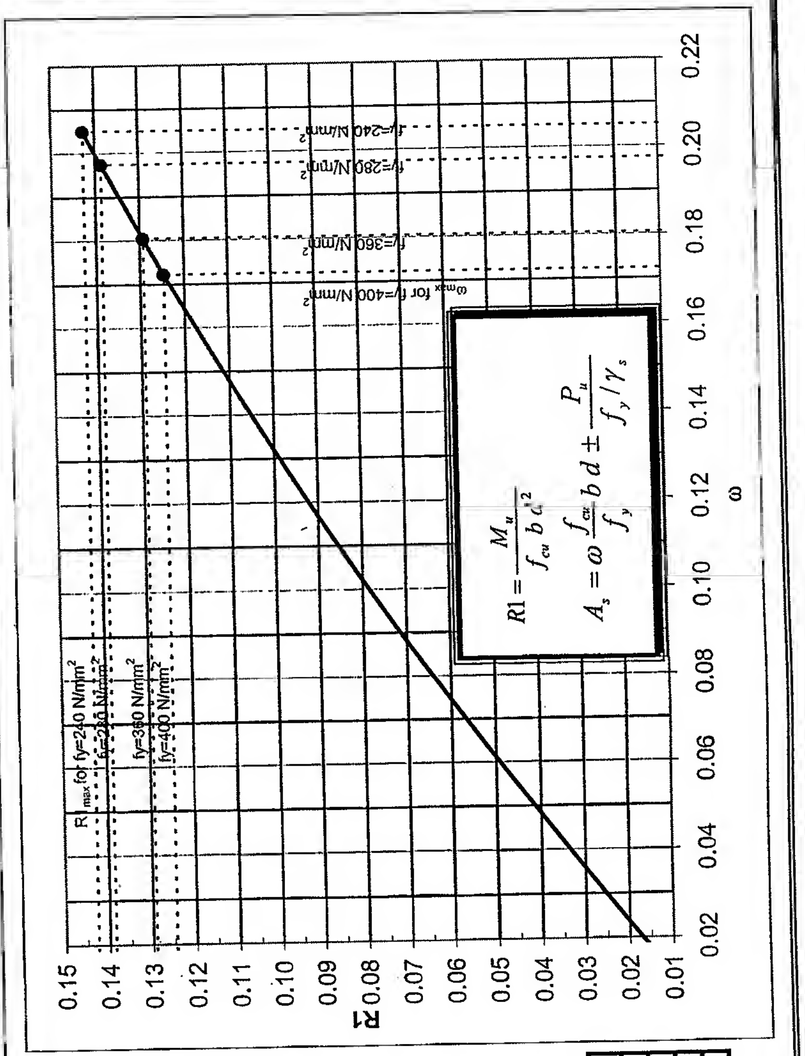
r	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	0.500	0.595	0.672	0.742	0.797	0.834	0.867	0.893	0.914	0.928	0.941
β	0.500	0.405	0.328	0.258	0.203	0.166	0.133	0.107	0.086	0.072	0.059

Coefficients of equivalent uniform loads on beams

$L/2x$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	0.667	0.725	0.769	0.803	0.830	0.853	0.870	0.885	0.897	0.908	0.917
β	0.500	0.554	0.582	0.615	0.642	0.667	0.688	0.706	0.722	0.737	0.750

DESIGN CHART FOR SECTIONS SUBJECTED TO SIMPLE BENDING

(Table 4-1)



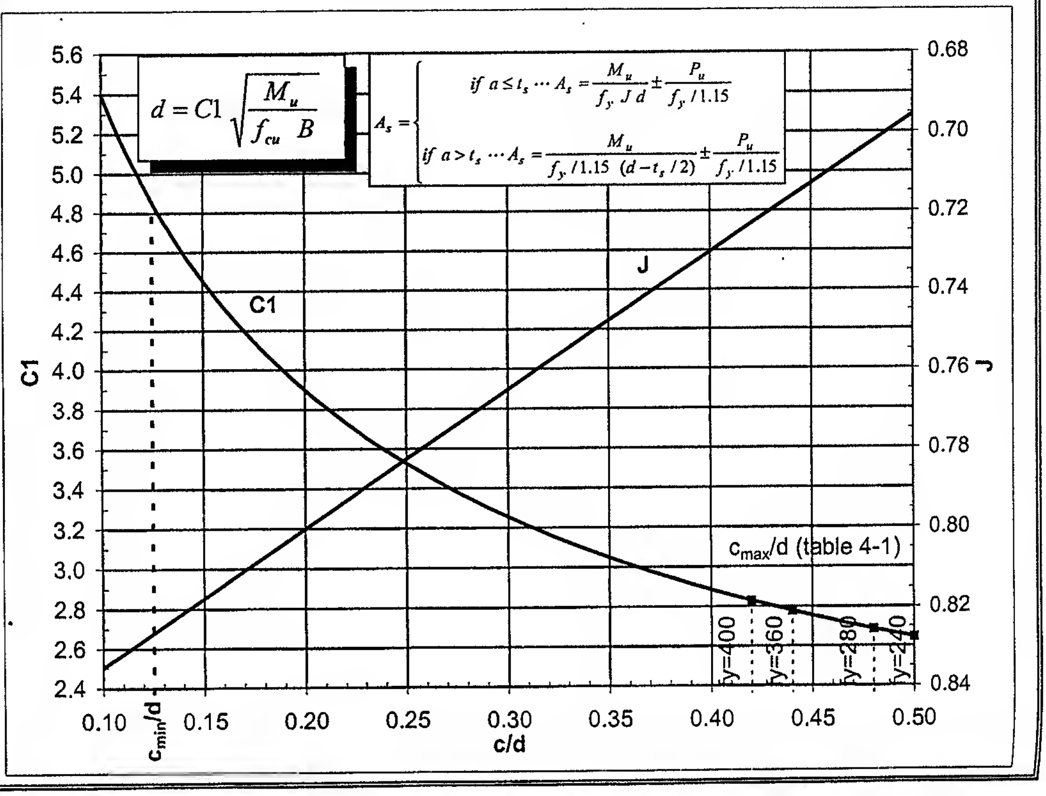
$$R1 = \frac{M_u}{f_{cu}\, b\, d^2}$$

$$A_s = \omega \frac{f_{cu}}{f_y} b\, d \pm \frac{P_u}{f_y/\gamma_s}$$

R1	ω	
0.015	0.018	
0.020	0.024	
0.025	0.030	
0.030	0.036	
0.035	0.042	
0.040	0.048	
0.045	0.055	
0.050	0.061	
0.055	0.068	
0.060	0.074	
0.065	0.081	
0.070	0.088	
0.075	0.095	
0.080	0.102	
0.085	0.109	
0.090	0.117	
0.095	0.124	
0.100	0.132	
0.105	0.140	
0.110	0.148	
0.115	0.156	
0.120	0.164	$R1_{max}$
0.125	0.173	fy=400
0.129	0.180	fy=360
0.139	0.198	fy=280
0.143	0.206	fy=240

DESIGN CHART FOR SECTIONS SUBJECTED TO SIMPLE BENDING (R and T-sections)

FOR ALL GRADES OF STEEL AND CONCRETE



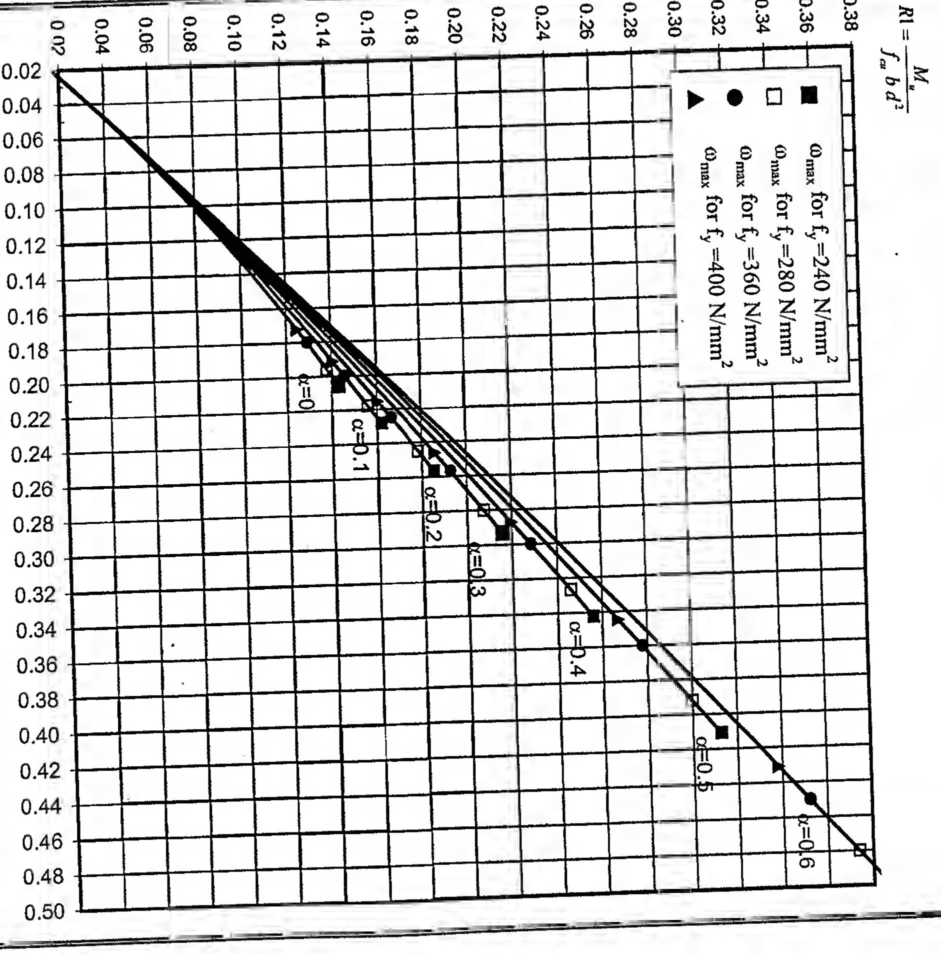
$$d = C1\sqrt{\frac{M_u}{f_{cu}\,B}}$$

$$A_s = \begin{cases} \text{if } a \le t_s \cdots A_s = \dfrac{M_u}{f_y\,J\,d} \pm \dfrac{P_u}{f_y/1.15} \\ \text{if } a > t_s \cdots A_s = \dfrac{M_u}{f_y/1.15\,(d - t_s/2)} \pm \dfrac{P_u}{f_y/1.15} \end{cases}$$

C1	J	c/d	c_{max}/d
2.65	0.696	0.500	fy=240
2.69	0.703	0.480	fy=280
2.78	0.717	0.440	fy=360
2.83	0.723	0.420	fy=400
2.90	0.732	0.395	
2.95	0.738	0.379	
3.00	0.743	0.364	
3.05	0.748	0.350	
3.10	0.753	0.337	
3.15	0.757	0.324	
3.20	0.761	0.312	
3.25	0.765	0.301	
3.30	0.768	0.291	
3.35	0.772	0.281	
3.40	0.775	0.272	
3.45	0.778	0.263	
3.50	0.781	0.254	
3.55	0.784	0.246	
3.60	0.787	0.239	
3.65	0.789	0.231	
3.70	0.791	0.225	
3.75	0.794	0.218	
3.80	0.796	0.212	
3.85	0.798	0.206	
3.90	0.800	0.200	
3.95	0.802	0.194	
4.00	0.804	0.189	
4.05	0.806	0.184	
4.10	0.807	0.179	
4.15	0.809	0.175	
4.20	0.810	0.170	
4.25	0.812	0.166	
4.30	0.813	0.162	
4.35	0.815	0.158	
4.40	0.816	0.154	
4.45	0.817	0.150	
4.50	0.818	0.147	
4.55	0.820	0.143	
4.60	0.821	0.140	
4.65	0.822	0.137	
4.70	0.823	0.134	
4.75	0.824	0.131	
4.80	0.825	0.128	
4.85	0.826	0.125	

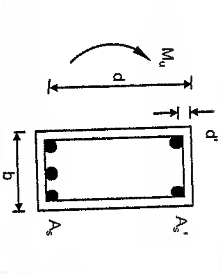
DESIGN CHART FOR DOUBLY REINFORCED SECTIONS SUBEJCED TO SIMPLE BENDING.

All types of steel (table 4-1). d'/d=0.05



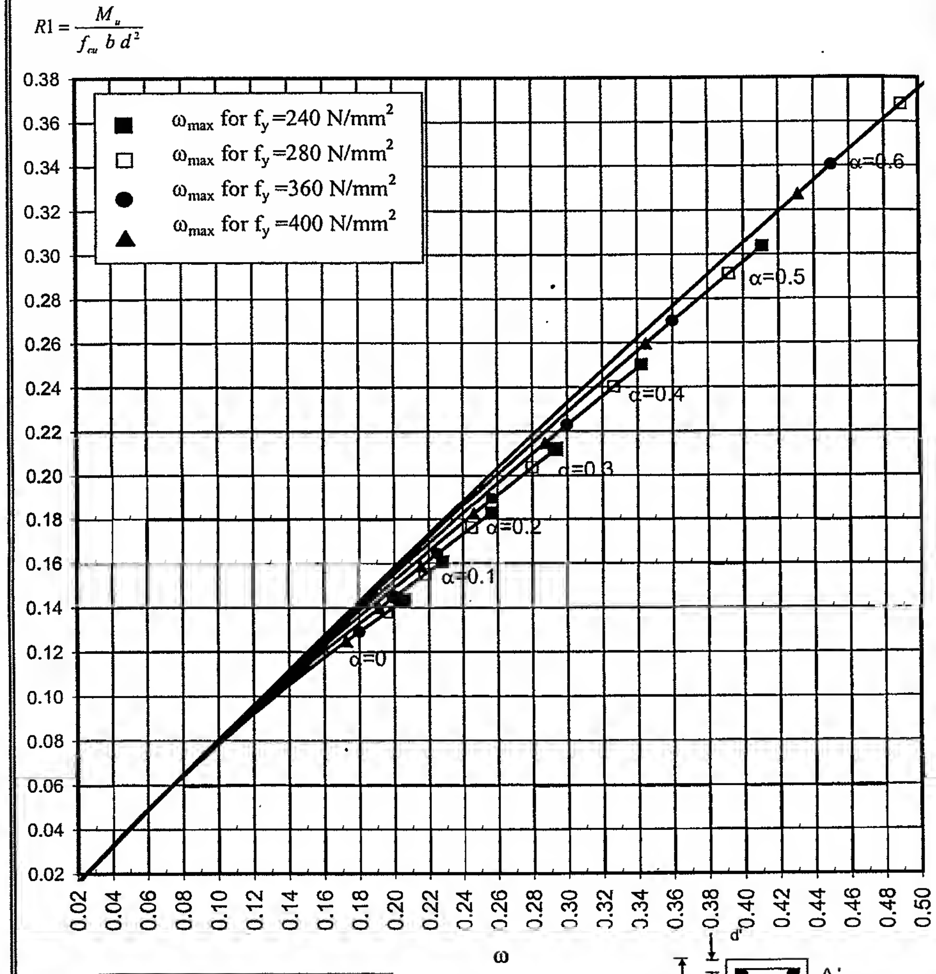
$$A_s = \omega\,b\,d\,\frac{f_{cu}}{f_y} \pm \frac{P_u}{f_y/\gamma_s}$$

$$A_s' = \alpha\,\omega\,b\,d\,\frac{f_{cu}}{f_y}$$



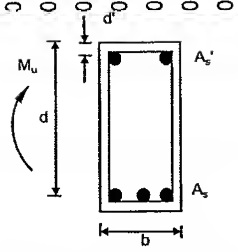
DESIGN CHART FOR DOUBLY REINFORCED SECTIONS SUBEJCED TO SIMPLE BENDING.

All types of steel (table 4-1). d'/d=0.10



$$A_s = \omega\, b\, d\, \frac{f_{cu}}{f_y} \pm \frac{P_u}{f_y/\gamma_s}$$

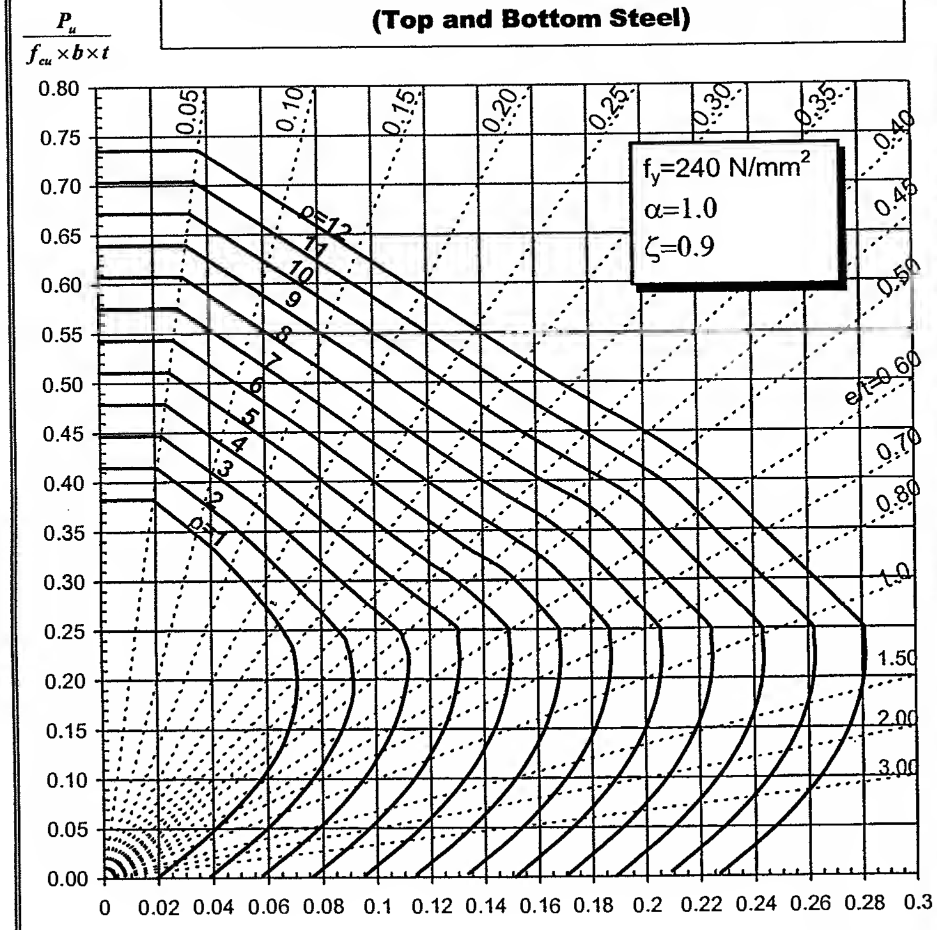
$$A_s' = \alpha\, \omega\, b\, d\, \frac{f_{cu}}{f_y}$$

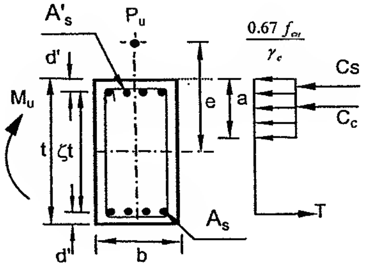


APPENDIX B

Interaction Diagrams (Top and bottom steel)

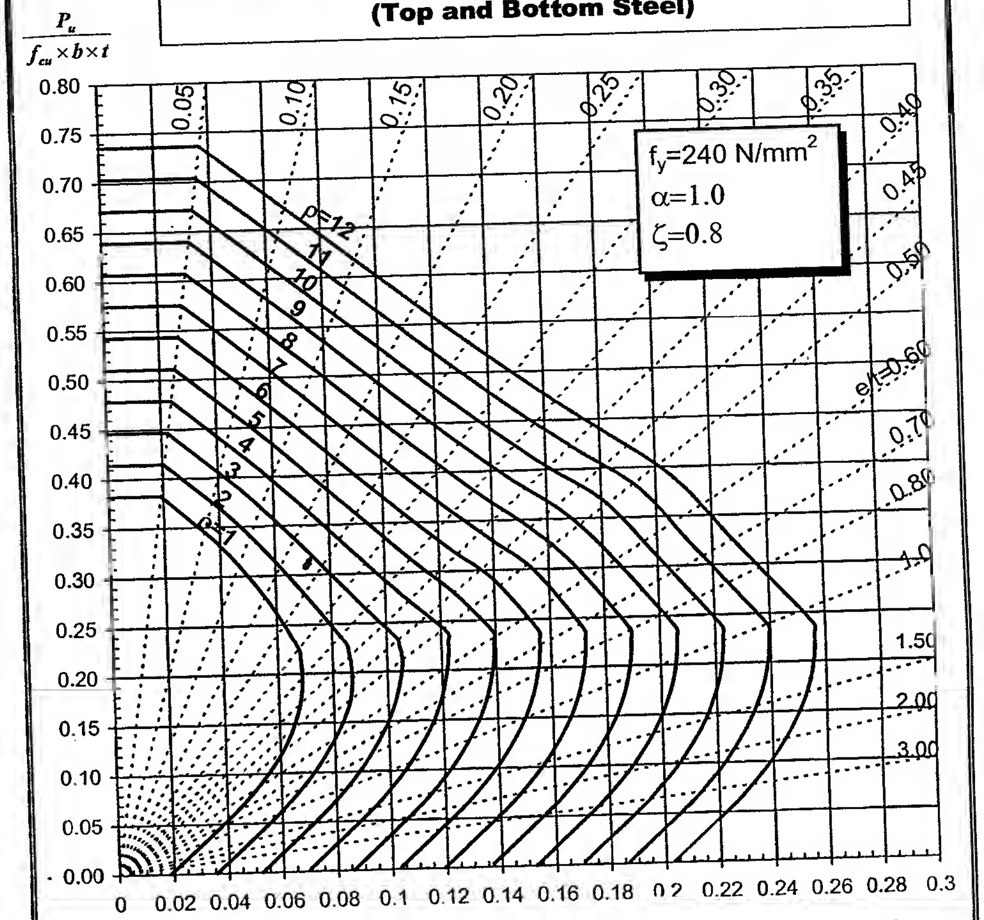
Interaction Diagram for Rectangular Sections
(Top and Bottom Steel)

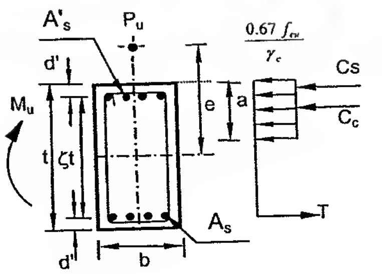




$$\mu = \rho\ f_{cu} \times 10^{-4}$$
$$A_s = \mu \times b \times t$$
$$A'_s = \alpha \times A_s$$
$$\zeta = \frac{d - d'}{t}$$

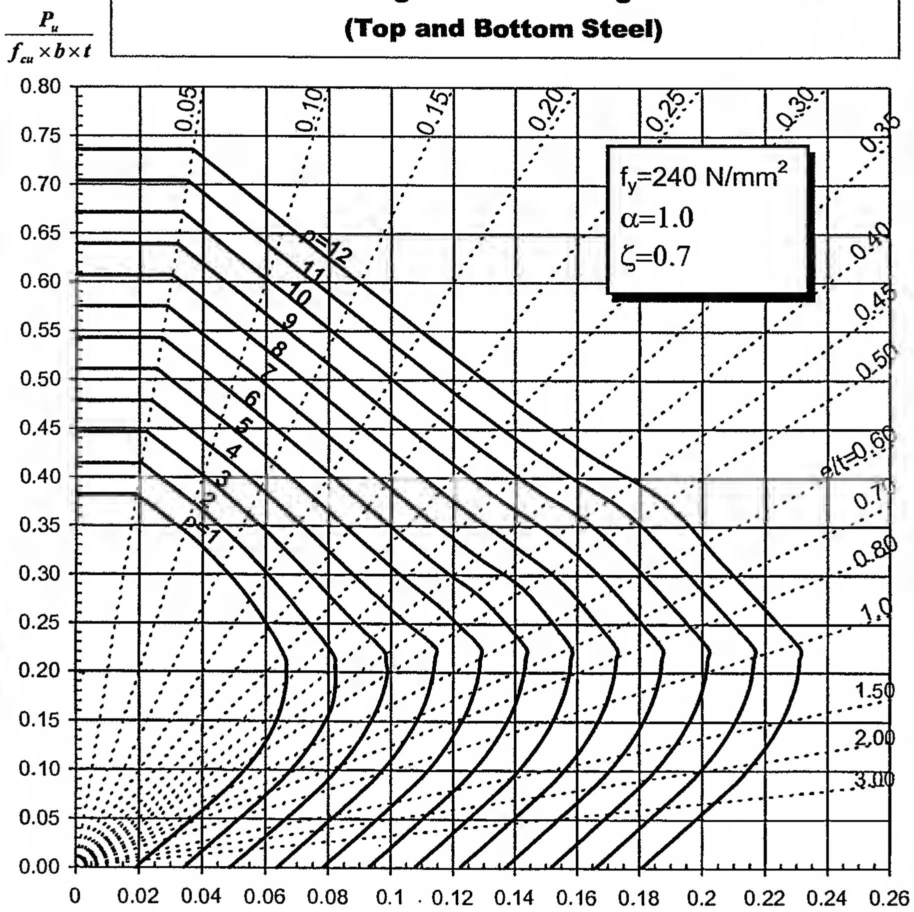
Interaction Diagram for Rectangular Sections
(Top and Bottom Steel)

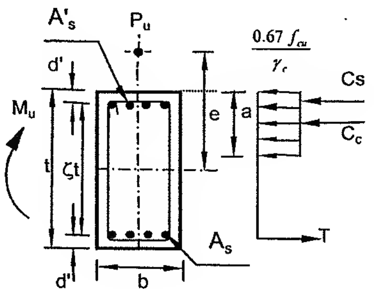




$$\mu = \rho\ f_{cu} \times 10^{-4}$$
$$A_s = \mu \times b \times t$$
$$A'_s = \alpha \times A_s$$
$$\zeta = \frac{d - d'}{t}$$

Interaction Diagram for Rectangular Sections
(Top and Bottom Steel)





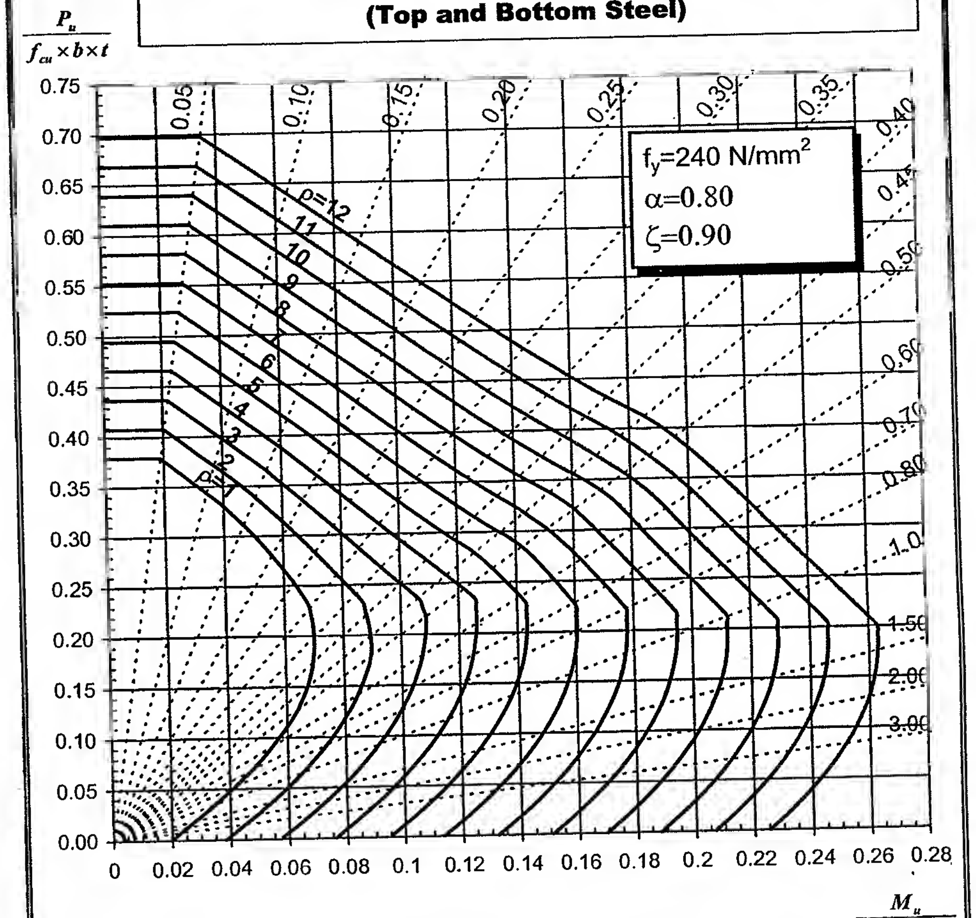
$\mu = \rho \; f_{cu} \times 10^{-4}$

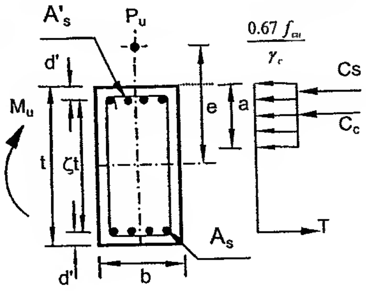
$A_s = \mu \times b \times t$

$A'_s = \alpha \times A_s$

$\zeta = \frac{d - d'}{t}$

Interaction Diagram for Rectangular Sections
(Top and Bottom Steel)





$\mu = \rho \; f_{cu} \times 10^{-4}$

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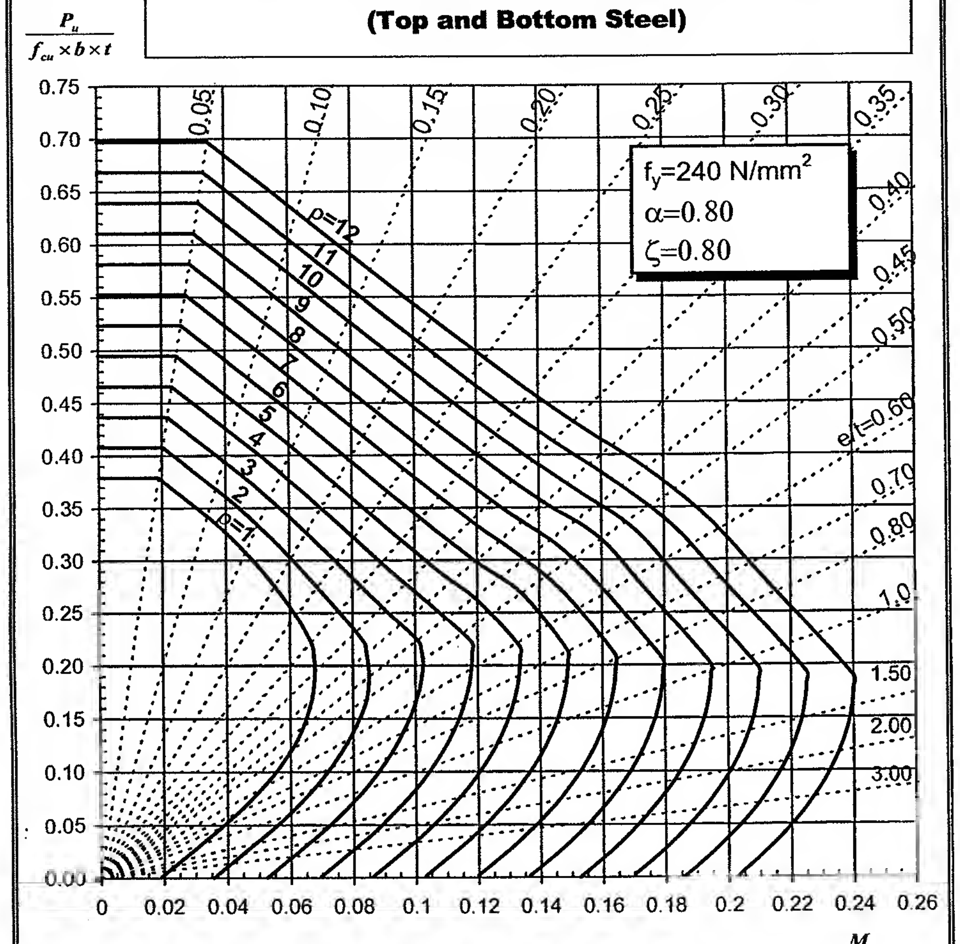
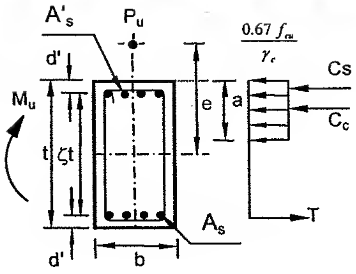
Interaction Diagram for Rectangular Sections
(Top and Bottom Steel)





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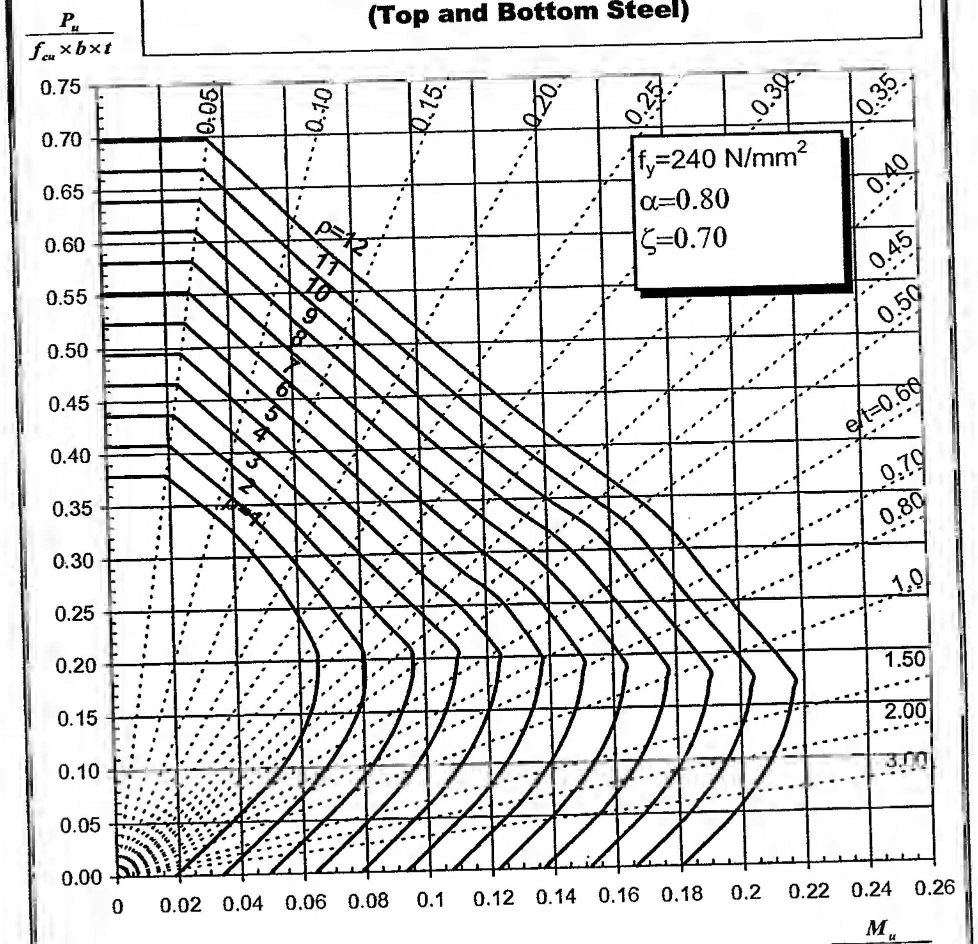
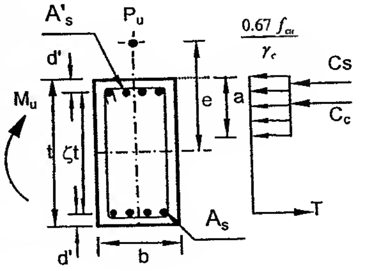
Interaction Diagram for Rectangular Sections
(Top and Bottom Steel)





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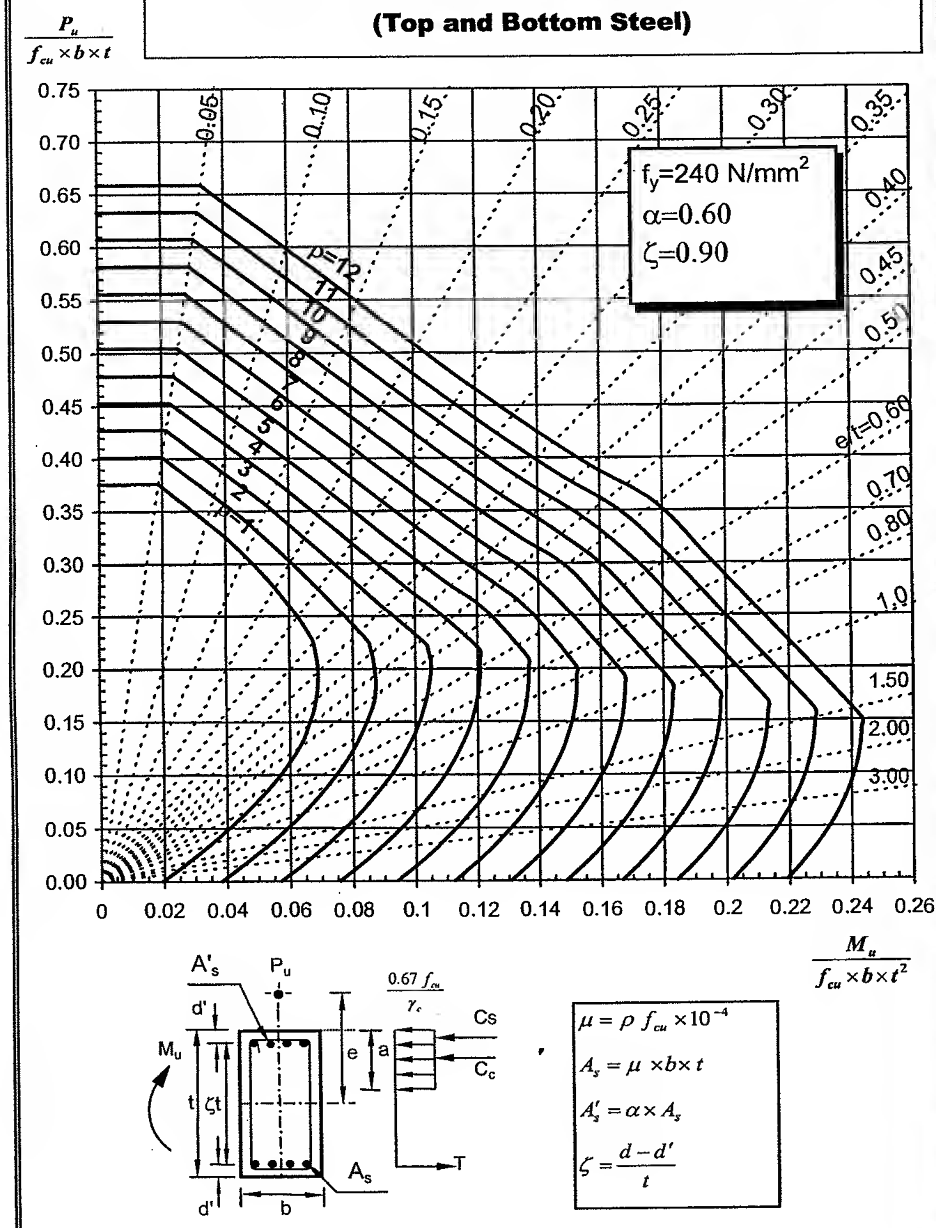

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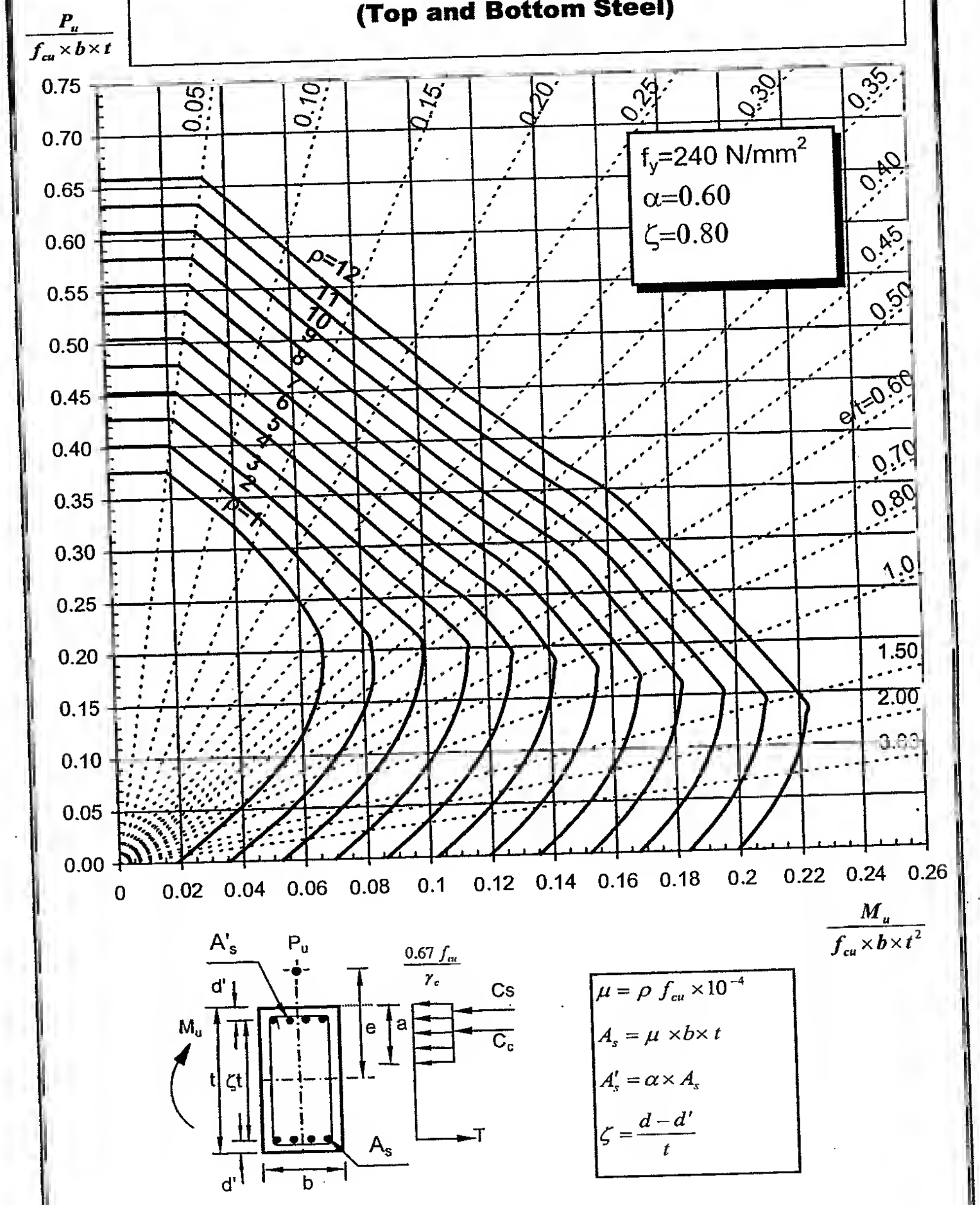
Interaction Diagram for Rectangular Sections (Top and Bottom Steel)



$\mu = \rho\ f_{cu} \times 10^{-4}$

$A_s = \mu \times b \times t$

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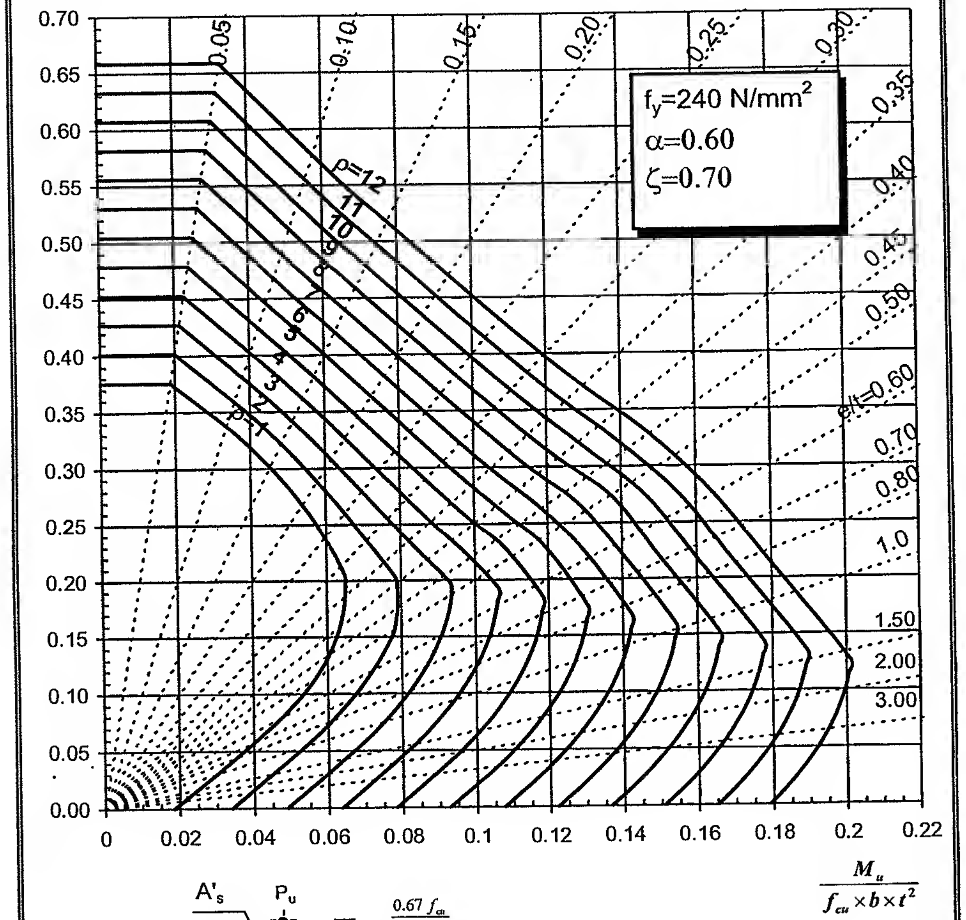
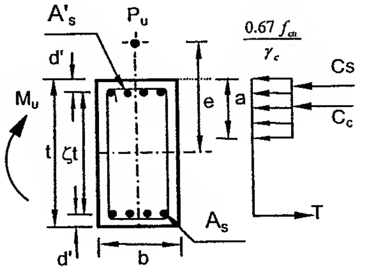
Interaction Diagram for Rectangular Sections
(Top and Bottom Steel)





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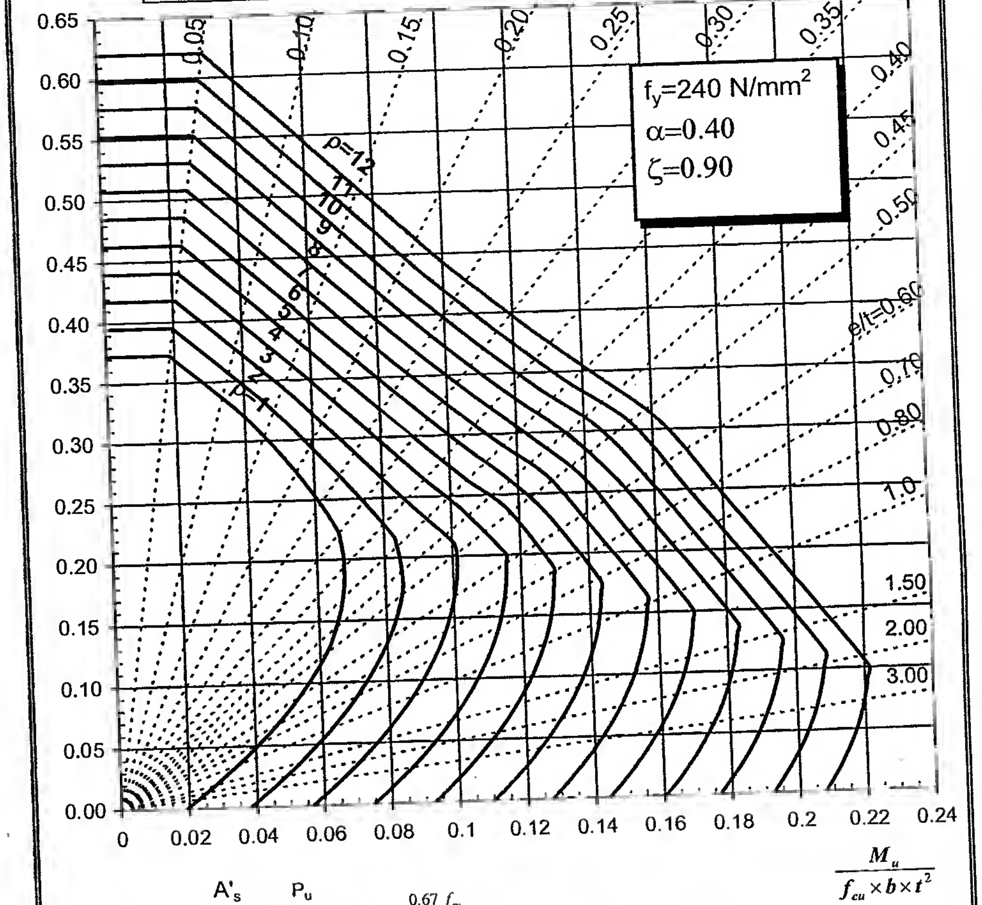
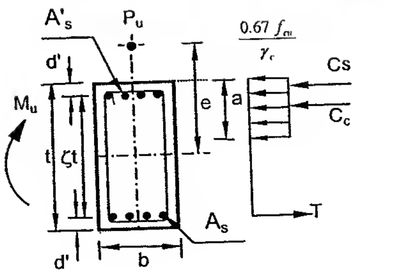
$\zeta = \frac{d - d'}{t}$

Interaction Diagram for Rectangular Sections
(Top and Bottom Steel)





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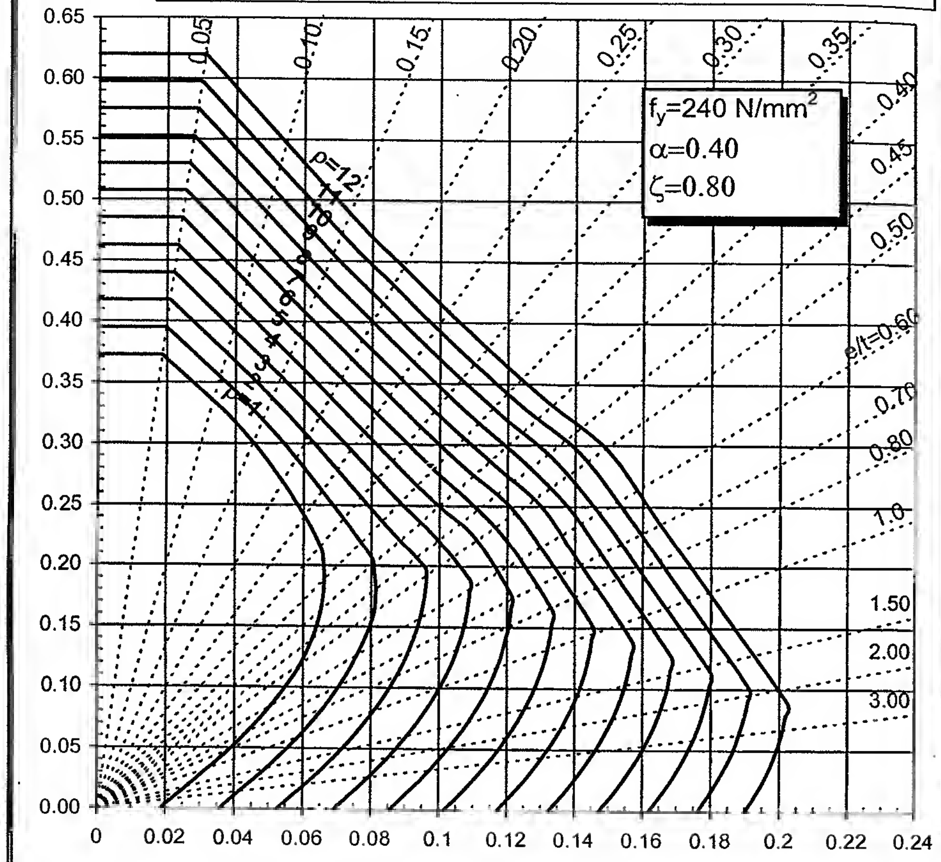
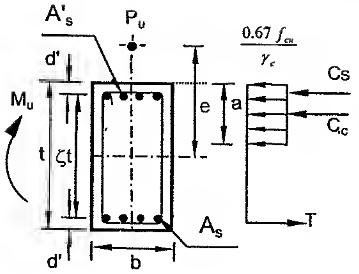
Interaction Diagram for Rectangular Sections (Top and Bottom Steel)





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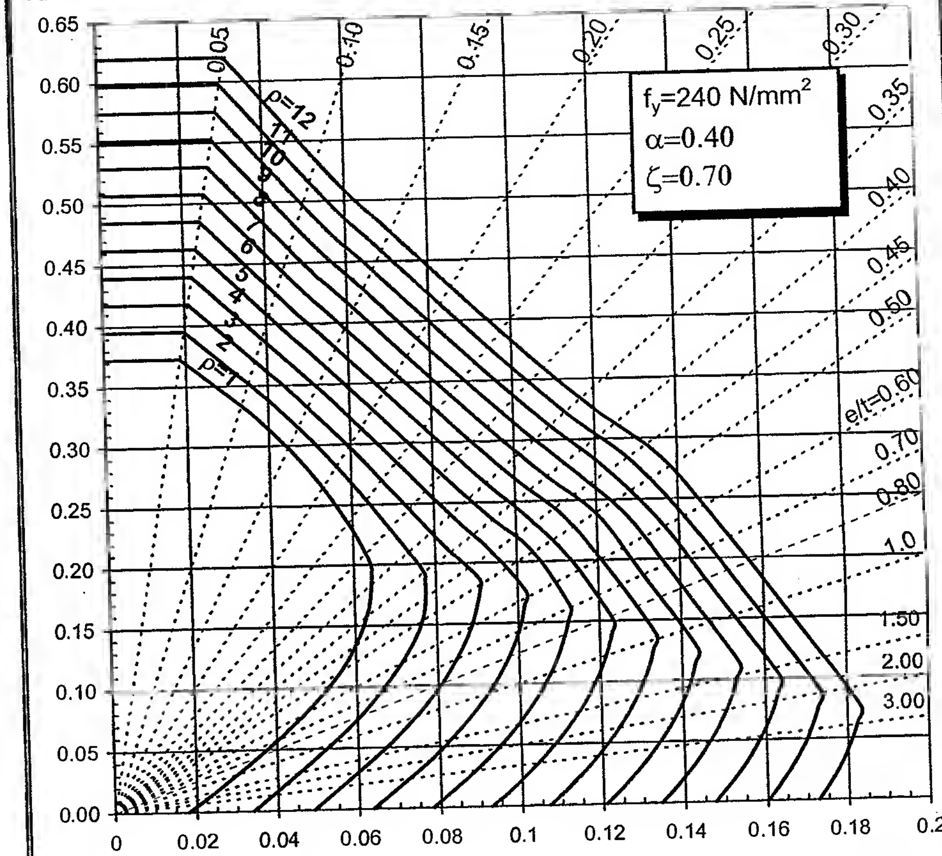
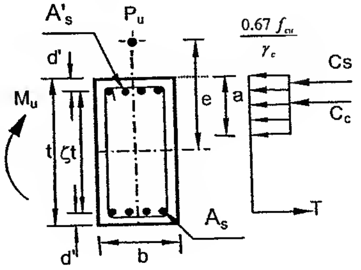
Interaction Diagram for Rectangular Sections (Top and Bottom Steel)





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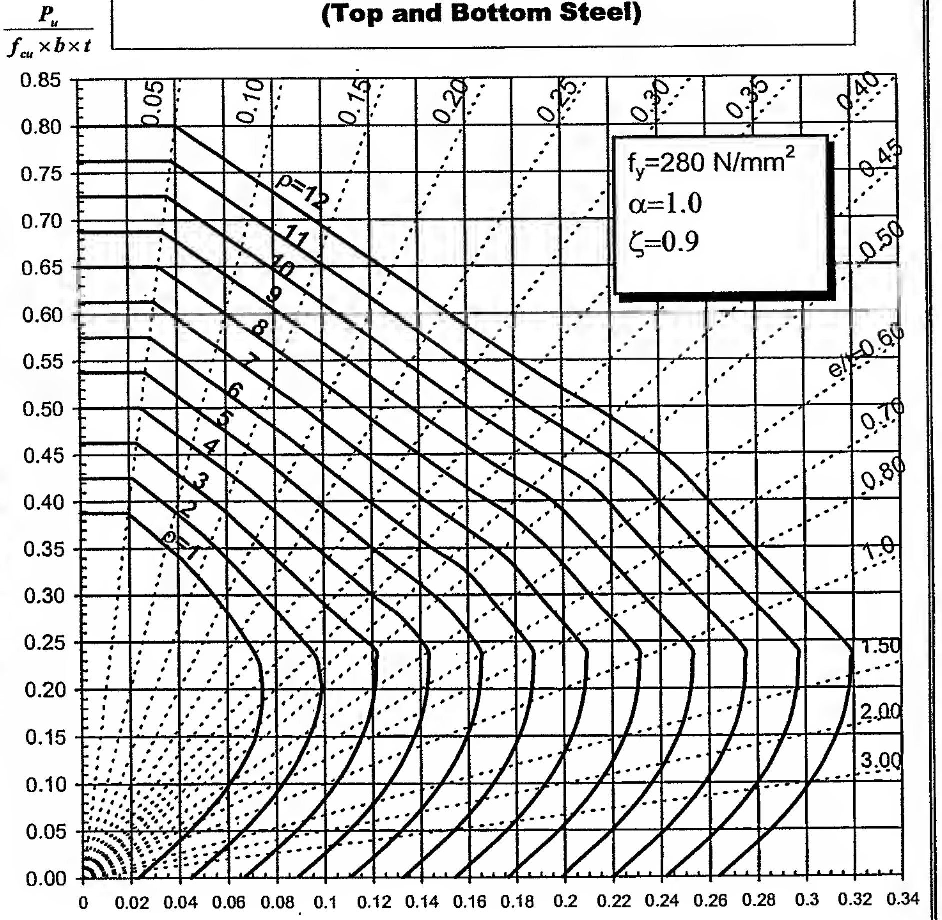
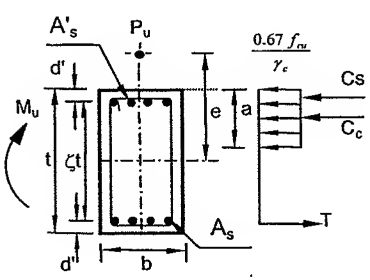
Interaction Diagram for Rectangular Sections (Top and Bottom Steel)





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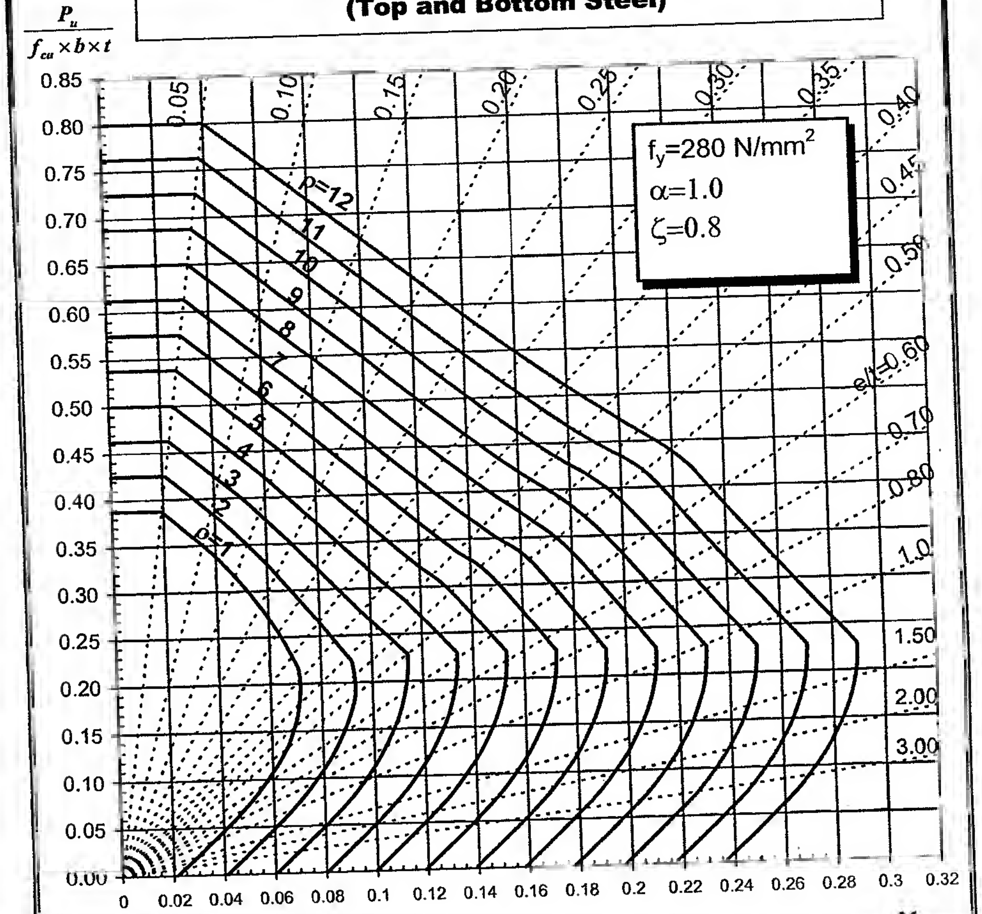
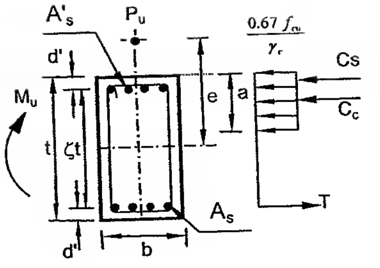
Interaction Diagram for Rectangular Sections (Top and Bottom Steel)





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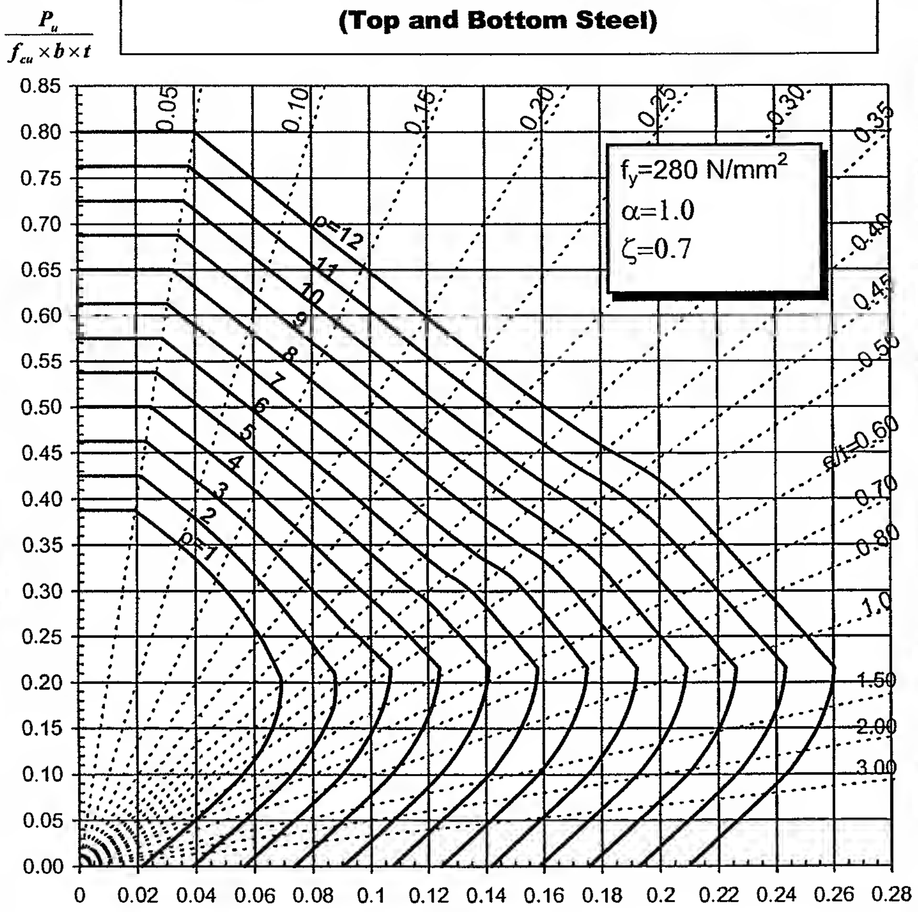
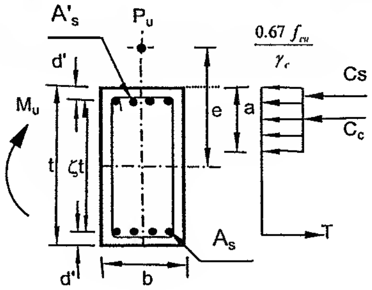
Interaction Diagram for Rectangular Sections
(Top and Bottom Steel)





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$A_s' = \alpha \times A_s$

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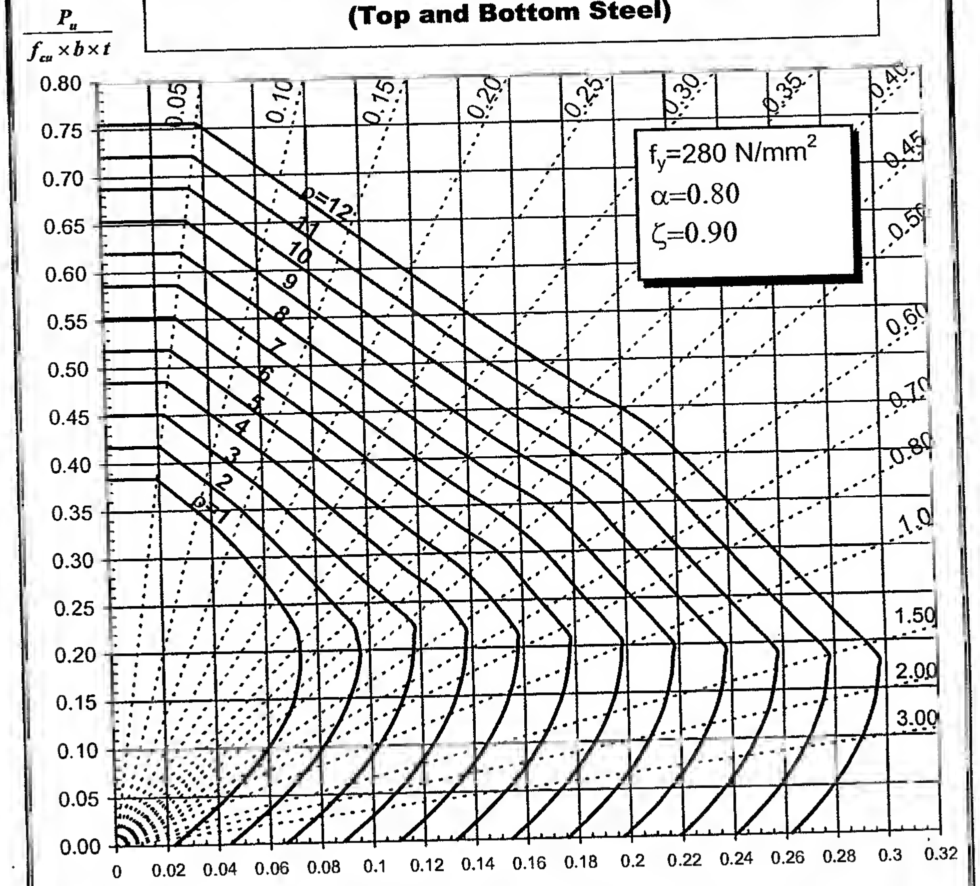
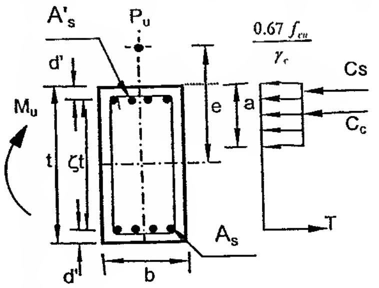
Interaction Diagram for Rectangular Sections
(Top and Bottom Steel)





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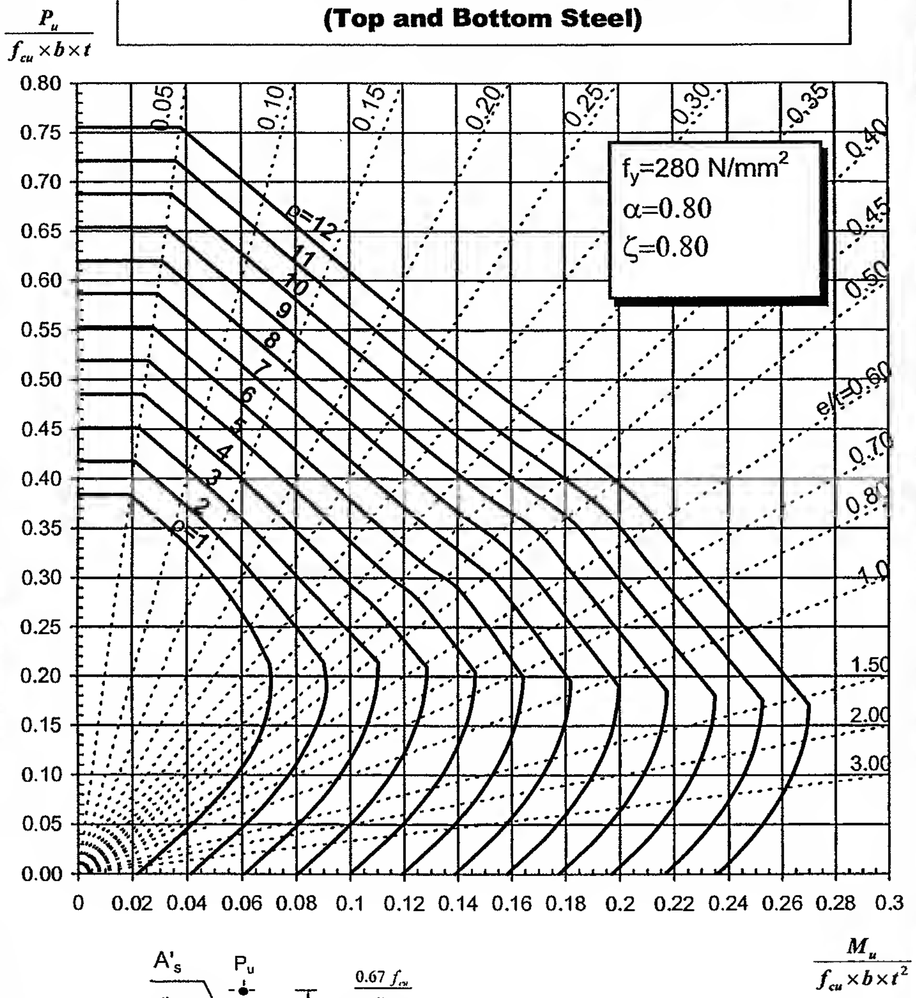
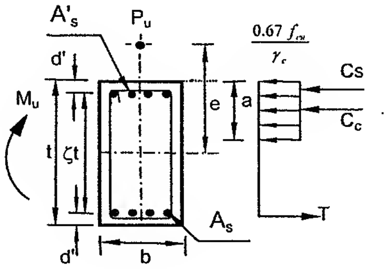
Interaction Diagram for Rectangular Sections (Top and Bottom Steel)





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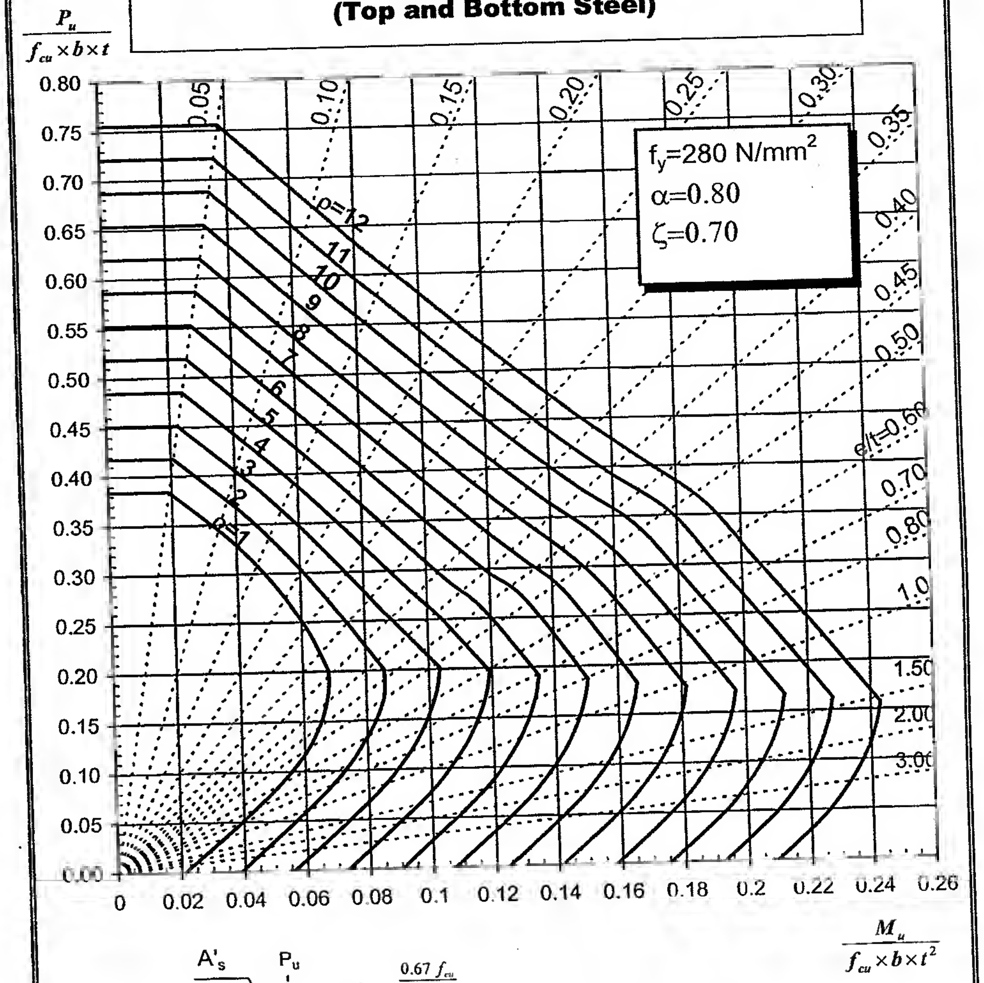
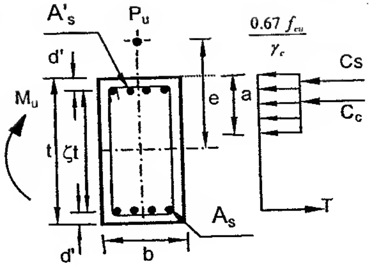
Interaction Diagram for Rectangular Sections (Top and Bottom Steel)





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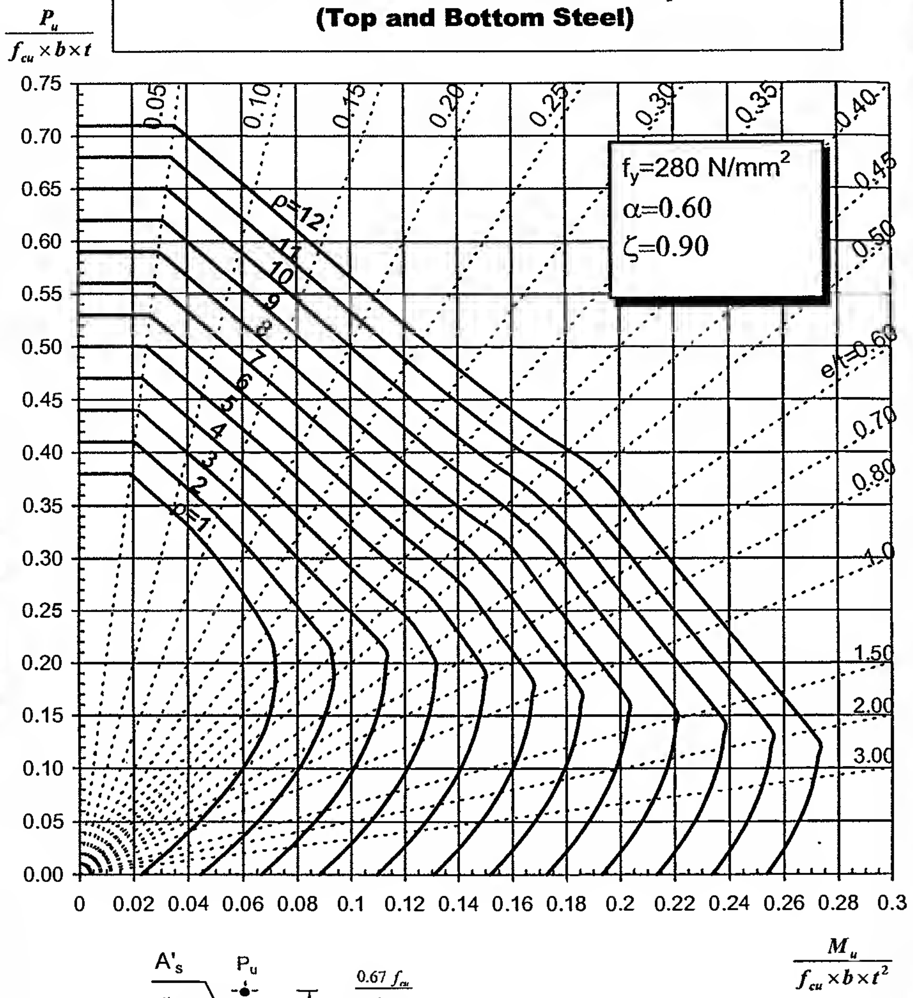
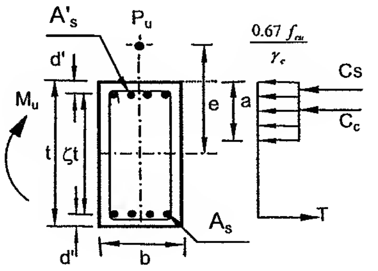
Interaction Diagram for Rectangular Sections (Top and Bottom Steel)





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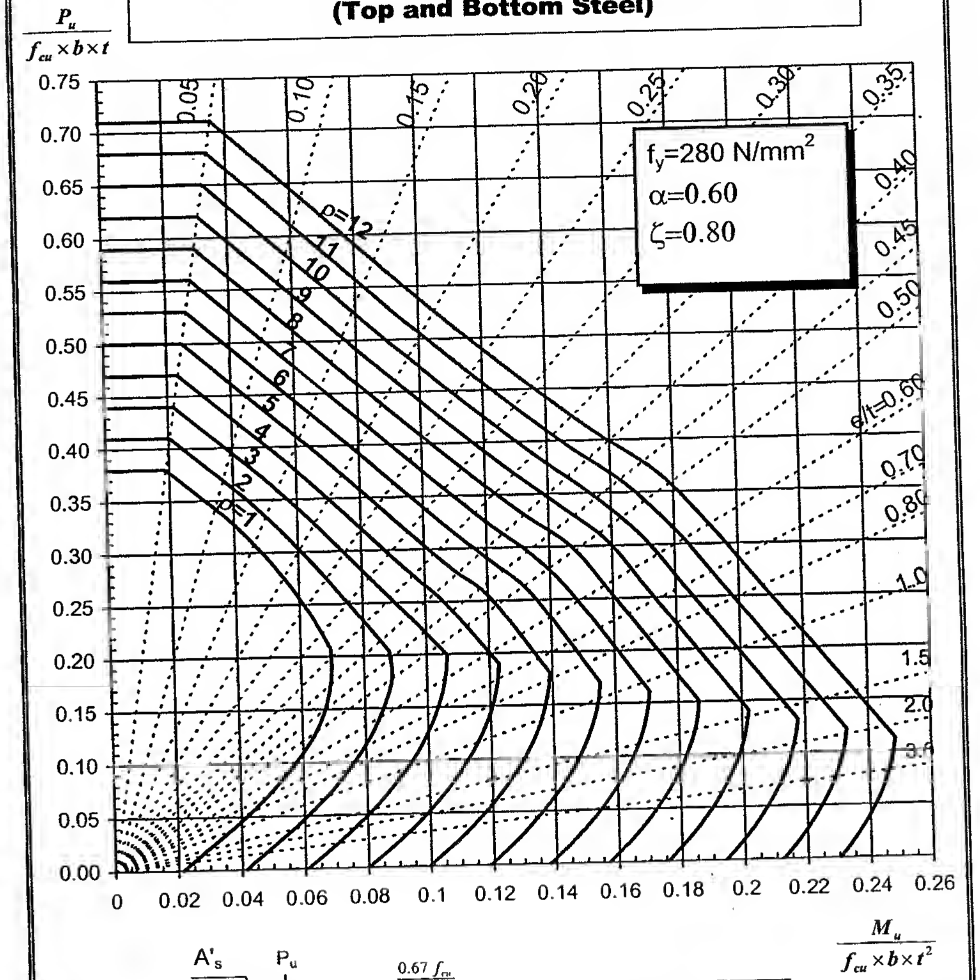
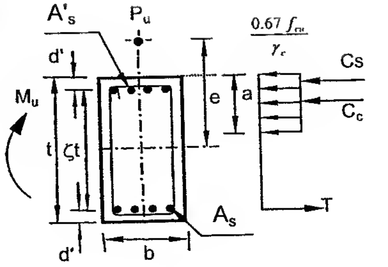
Interaction Diagram for Rectangular Sections (Top and Bottom Steel)





$\mu = \rho \, f_{cu} \times 10^{-4}$

$A_s = \mu \times b \times t$

$A'_s = \alpha \times A_s$

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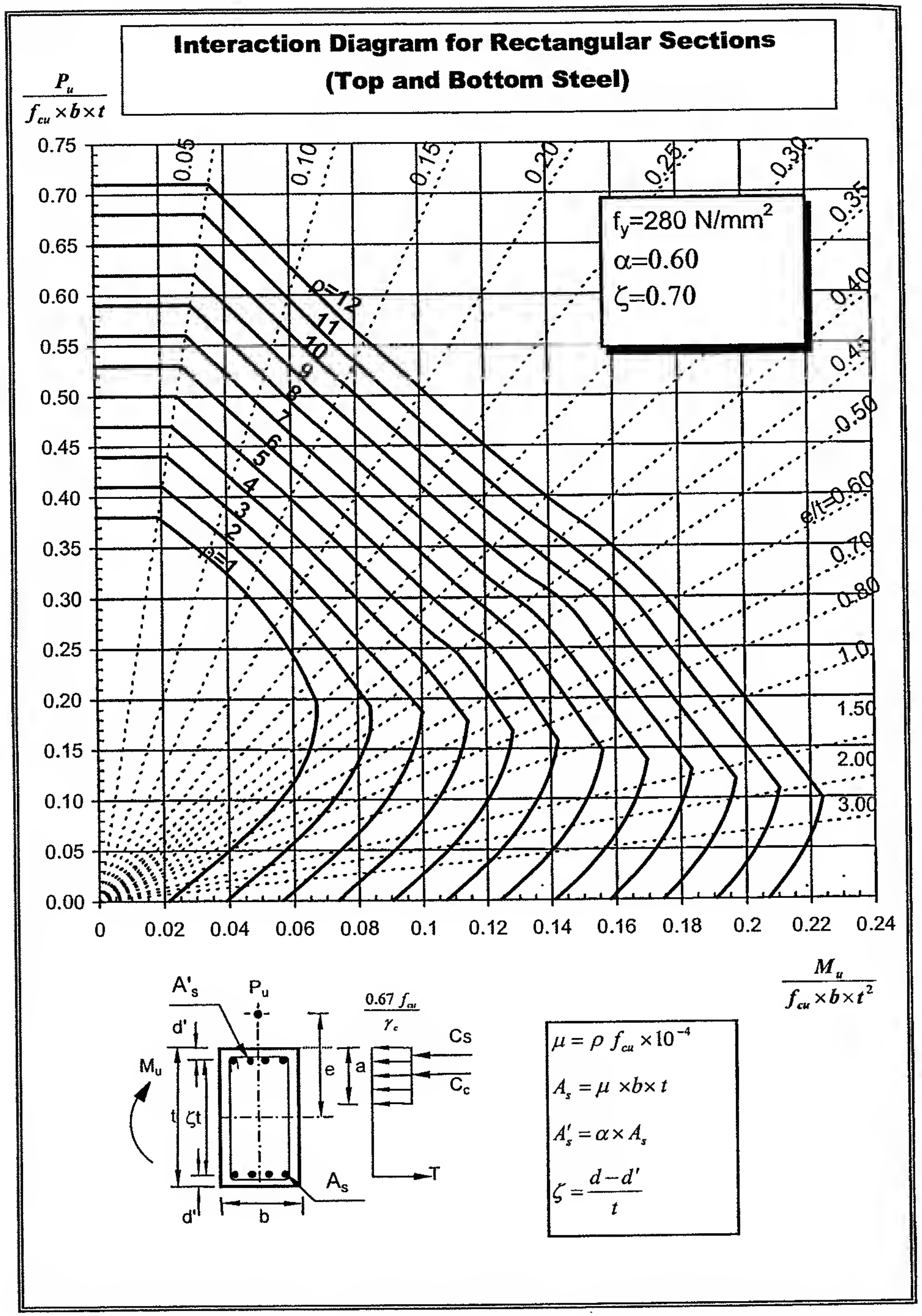
Interaction Diagram for Rectangular Sections
(Top and Bottom Steel)



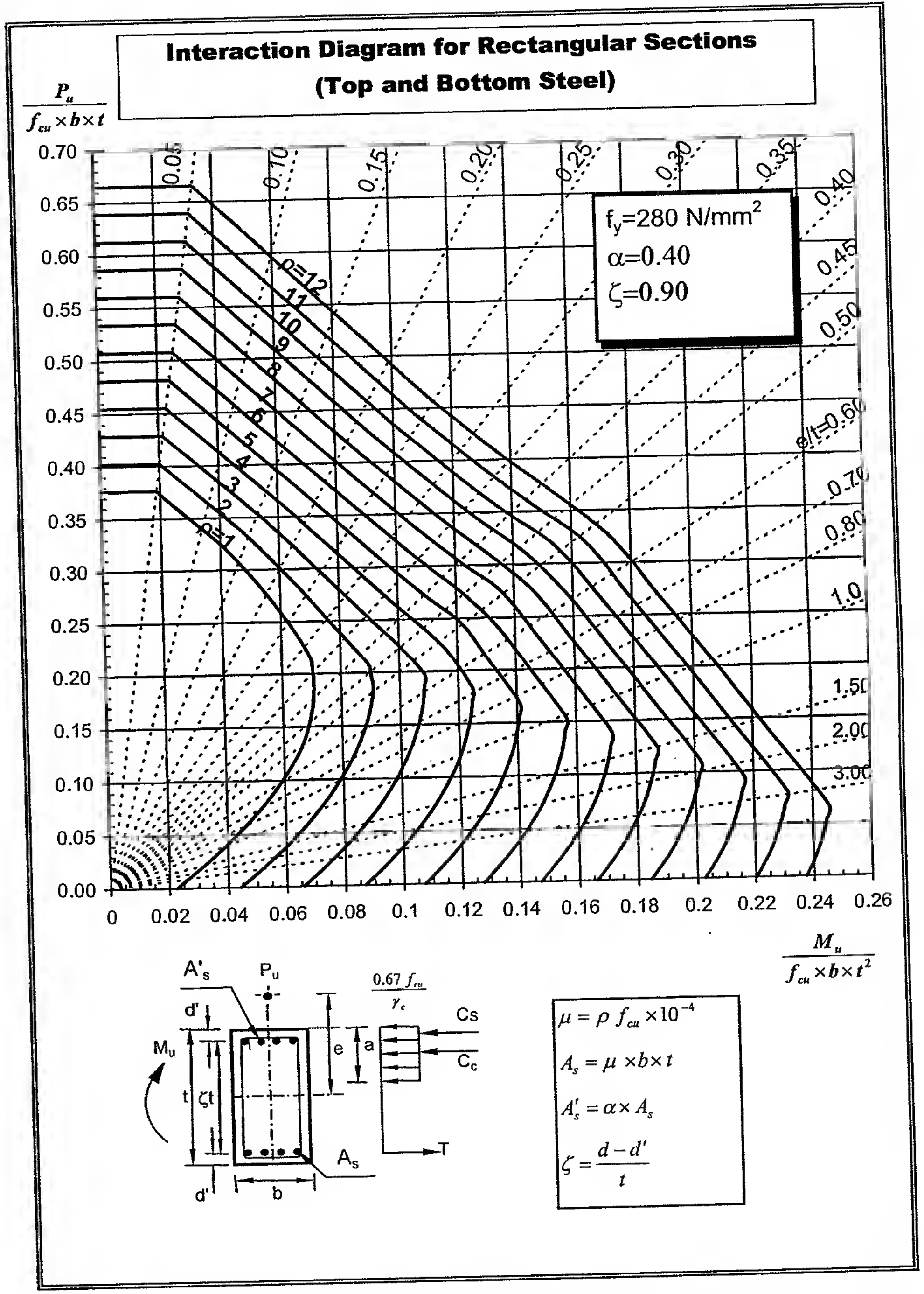
$\mu = \rho\ f_{cu} \times 10^{-4}$

$A_s = \mu \times b \times t$

$A'_s = \alpha \times A_s$

$\zeta = \frac{d - d'}{t}$

Interaction Diagram for Rectangular Sections
(Top and Bottom Steel)



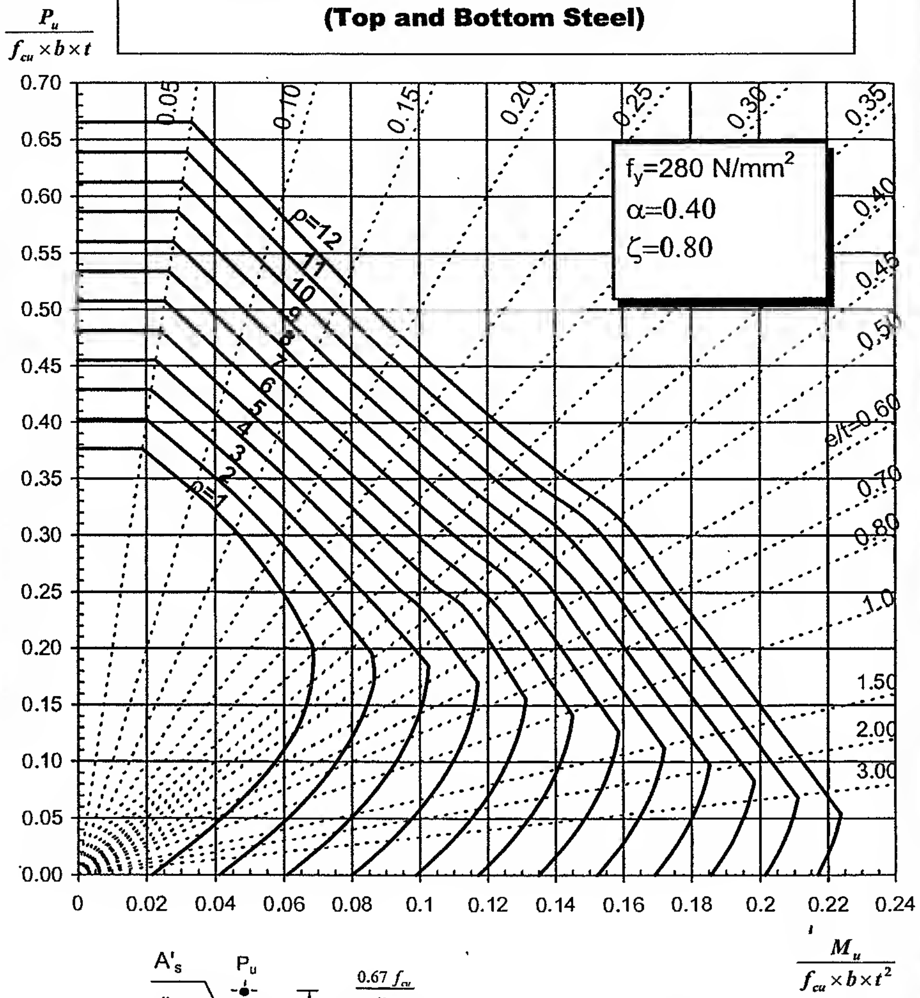
$\mu = \rho\ f_{cu} \times 10^{-4}$

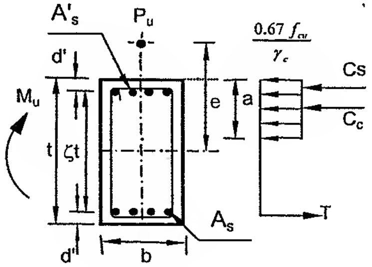
$A_s = \mu \times b \times t$

$A'_s = \alpha \times A_s$

$\zeta = \frac{d - d'}{t}$

Interaction Diagram for Rectangular Sections (Top and Bottom Steel)





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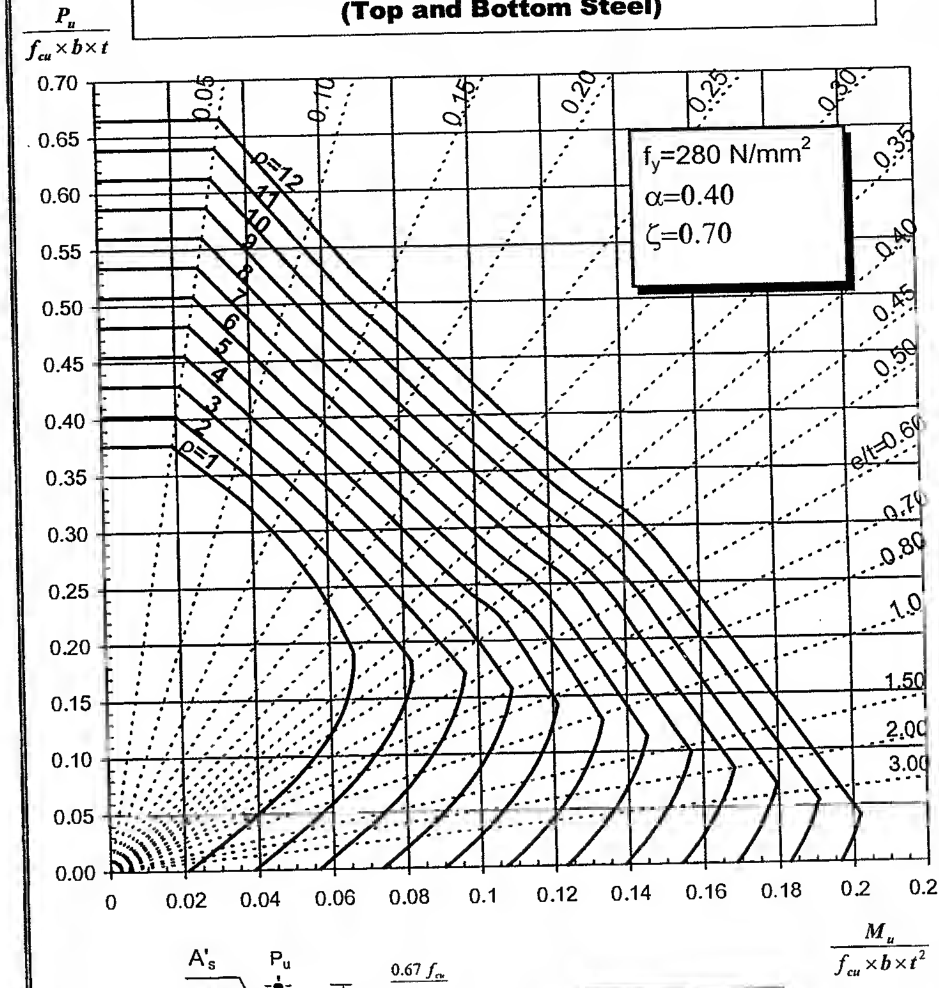

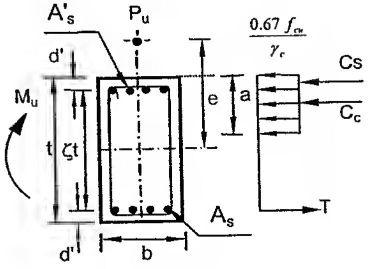

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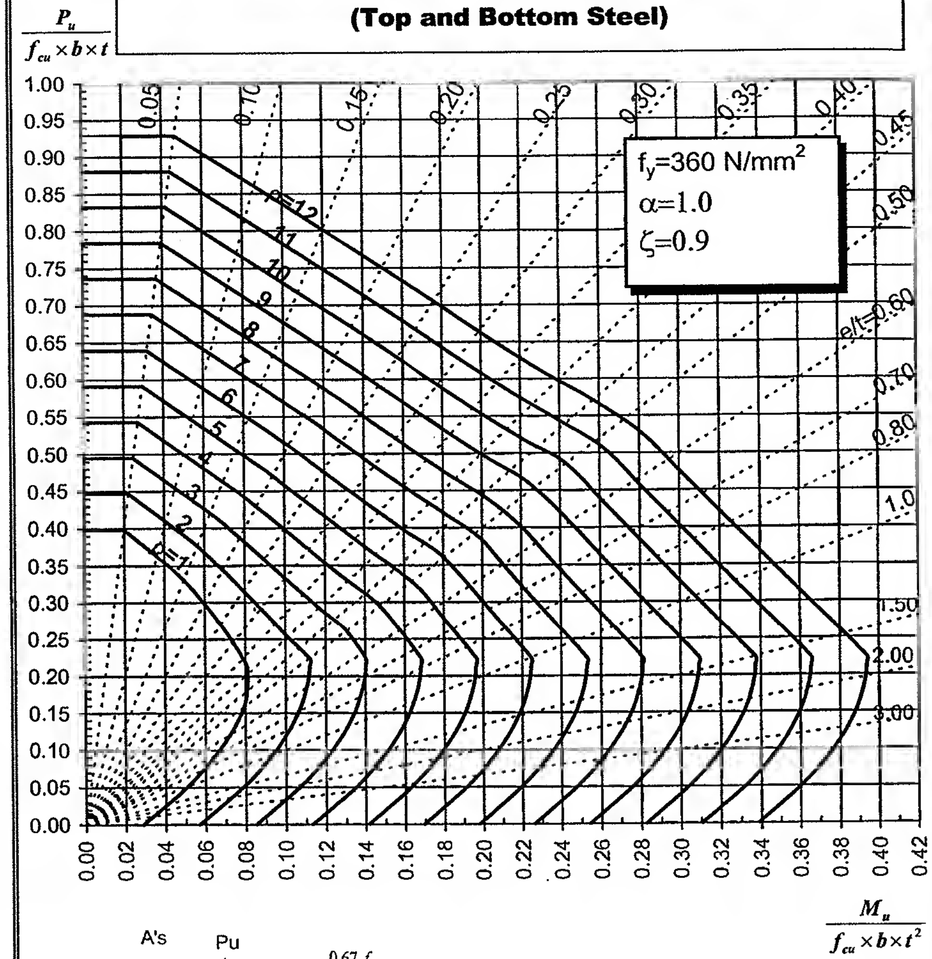

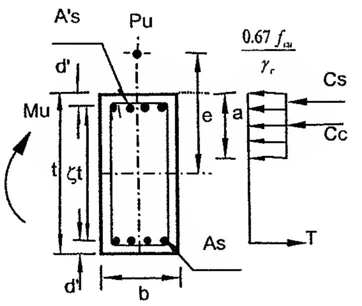

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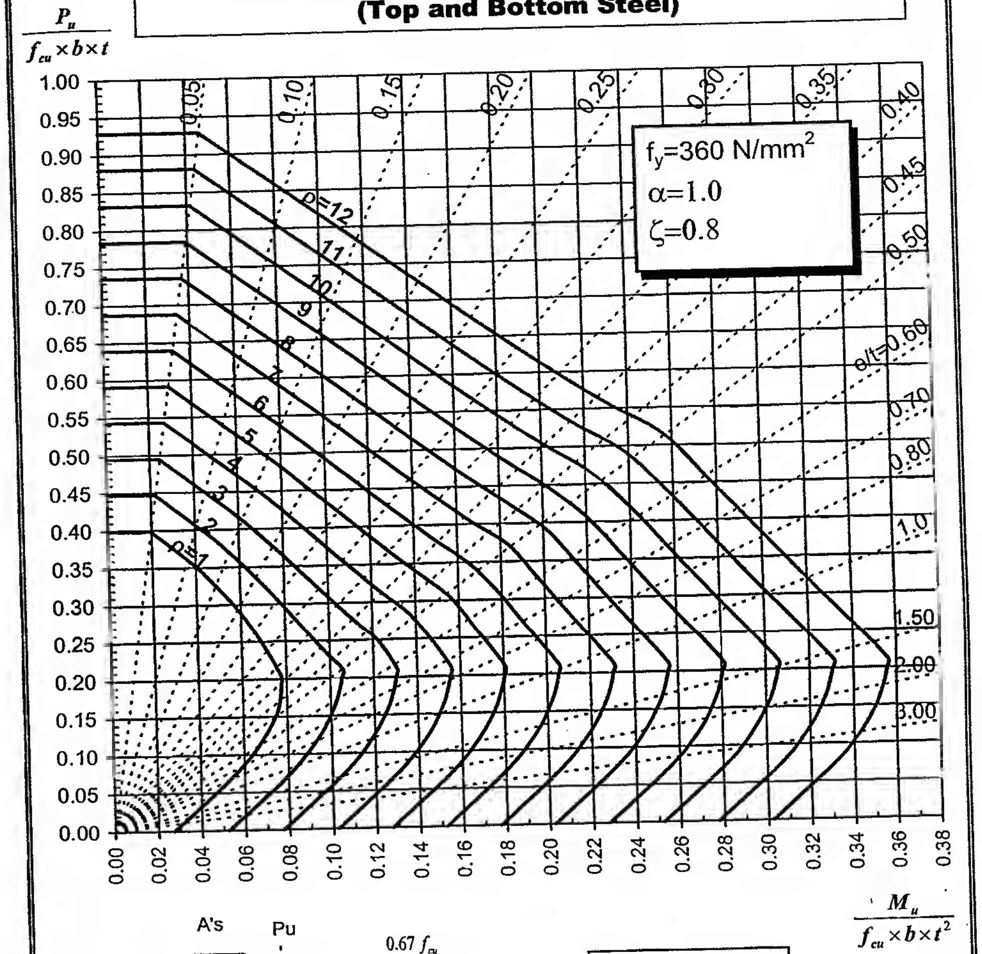

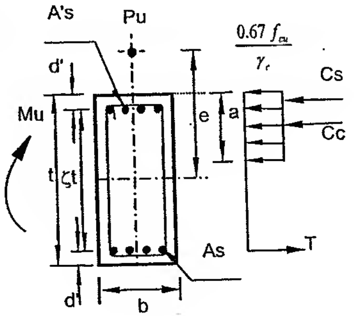

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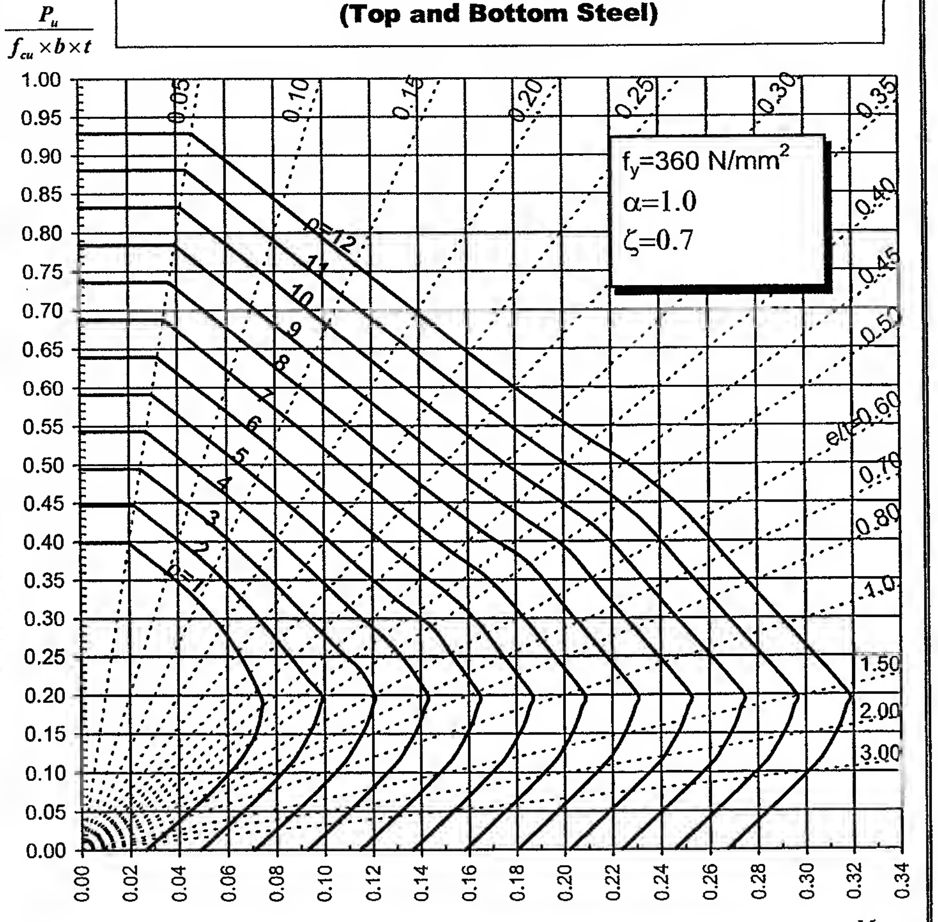

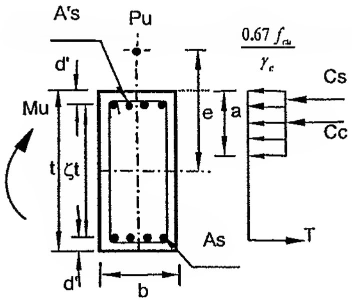

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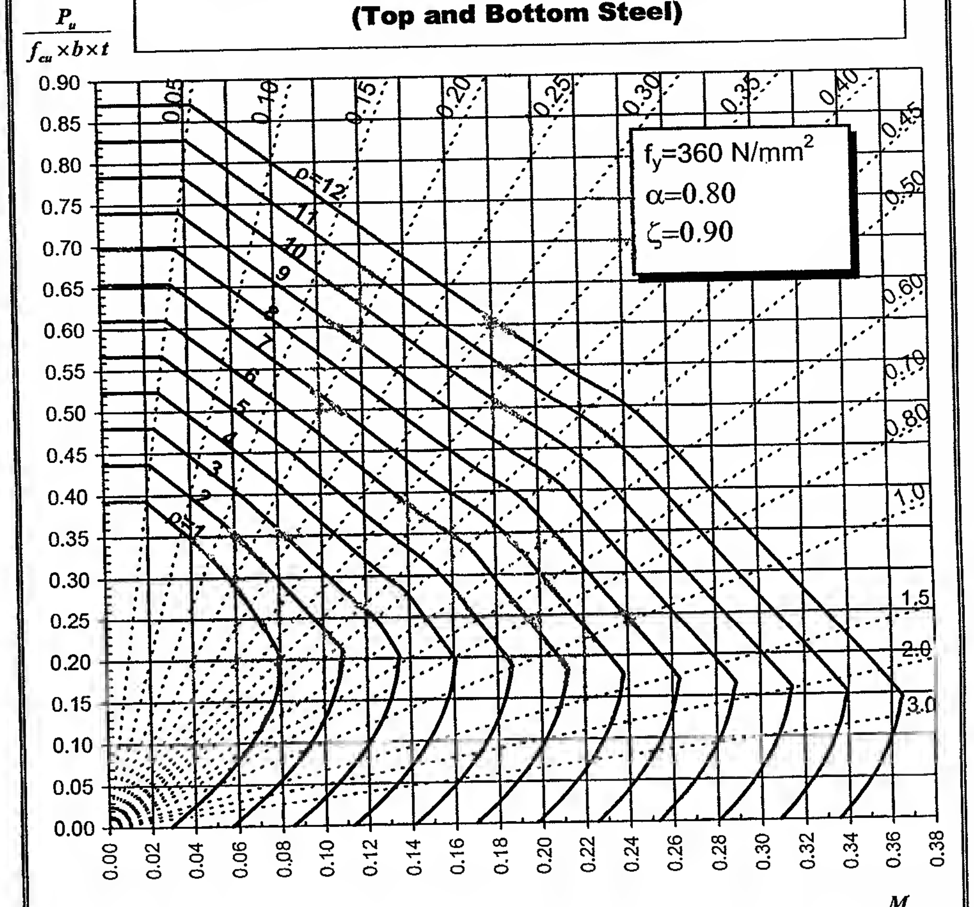

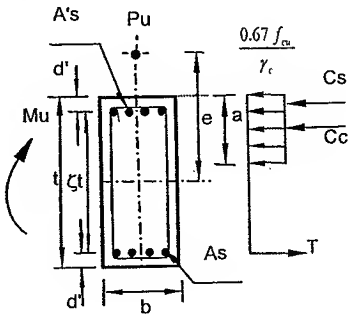

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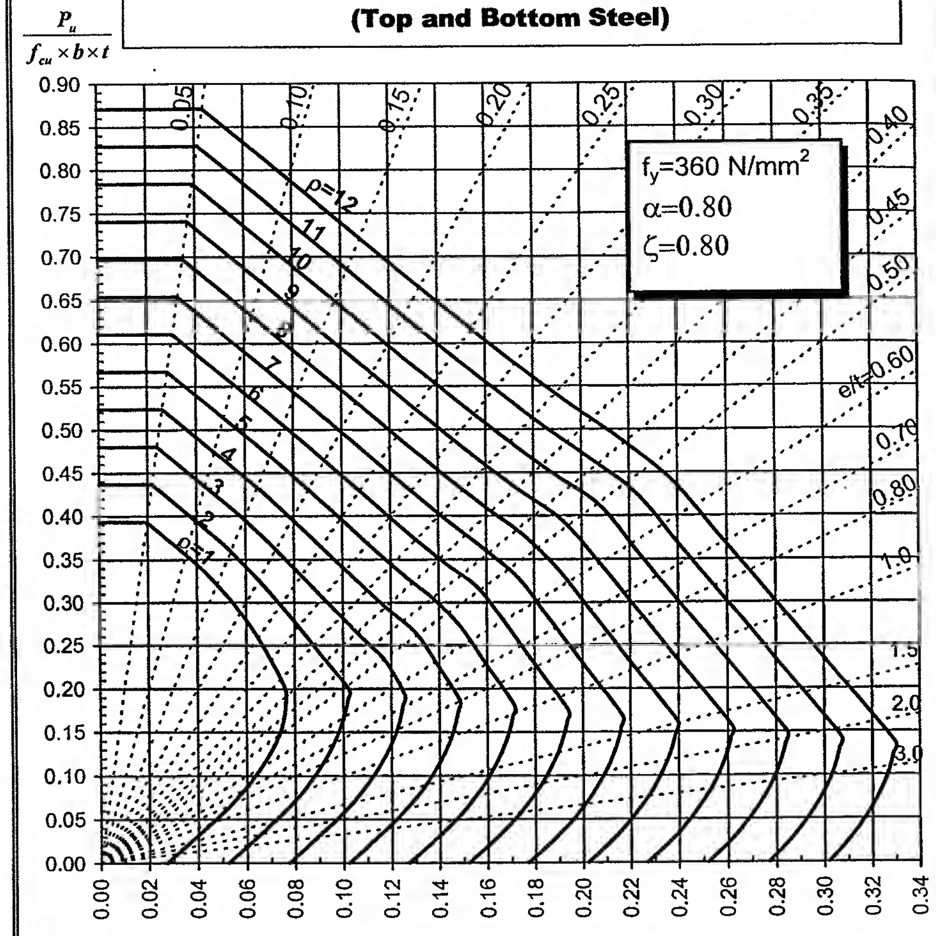

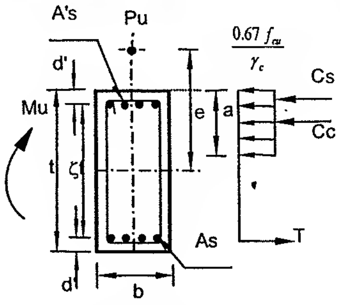

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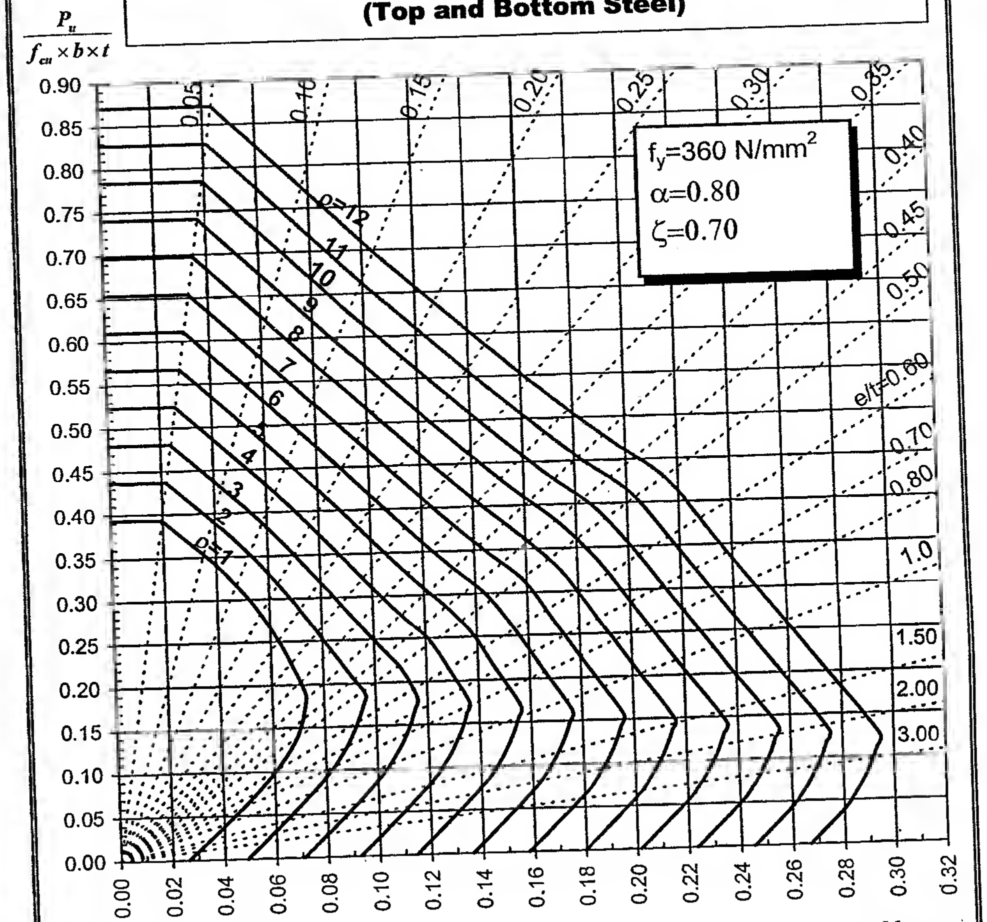

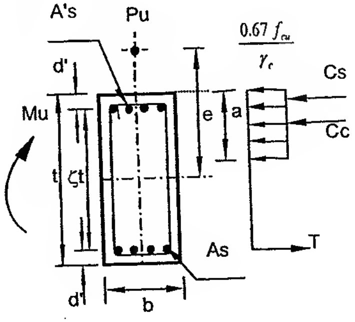

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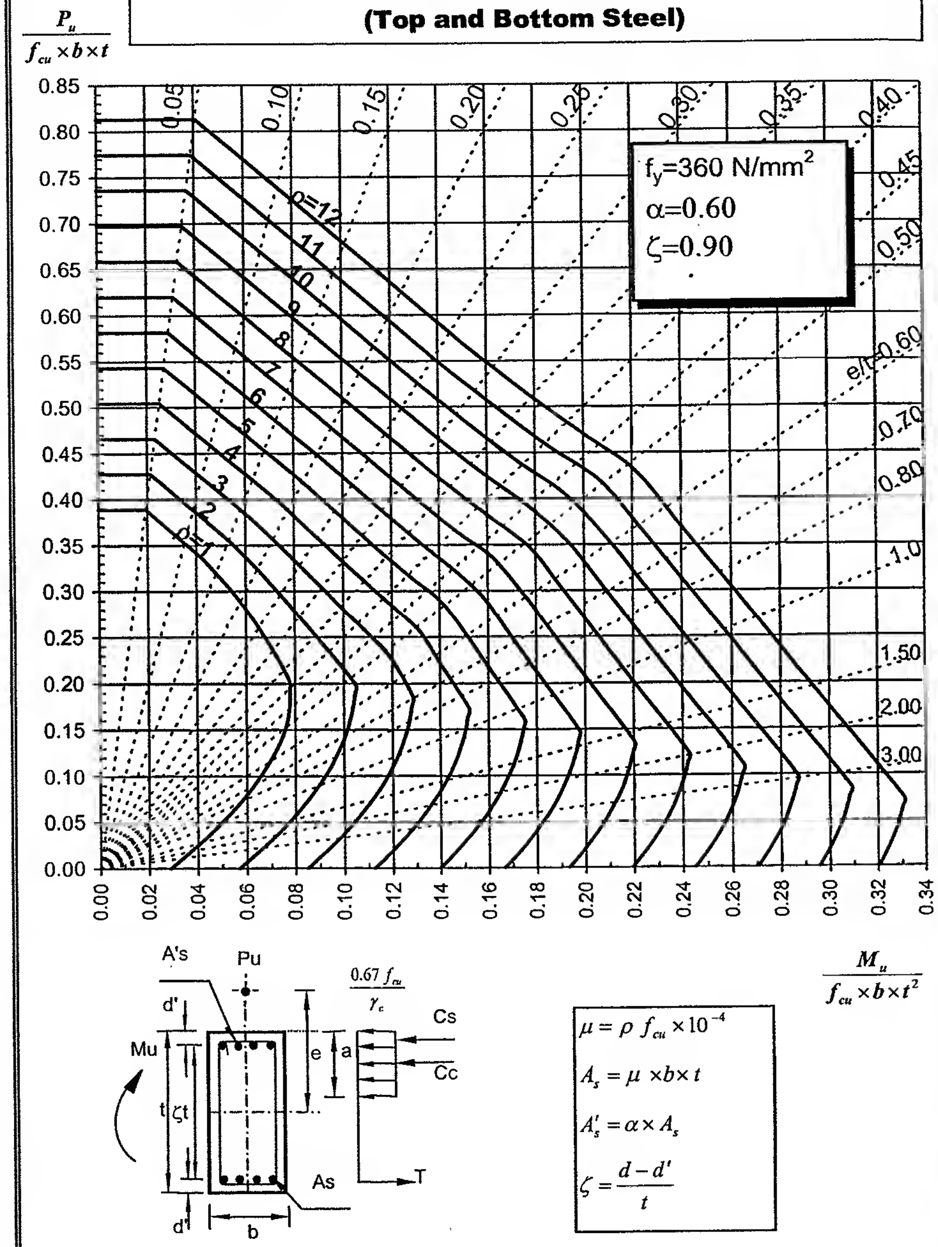
(Top and Bottom Steel)



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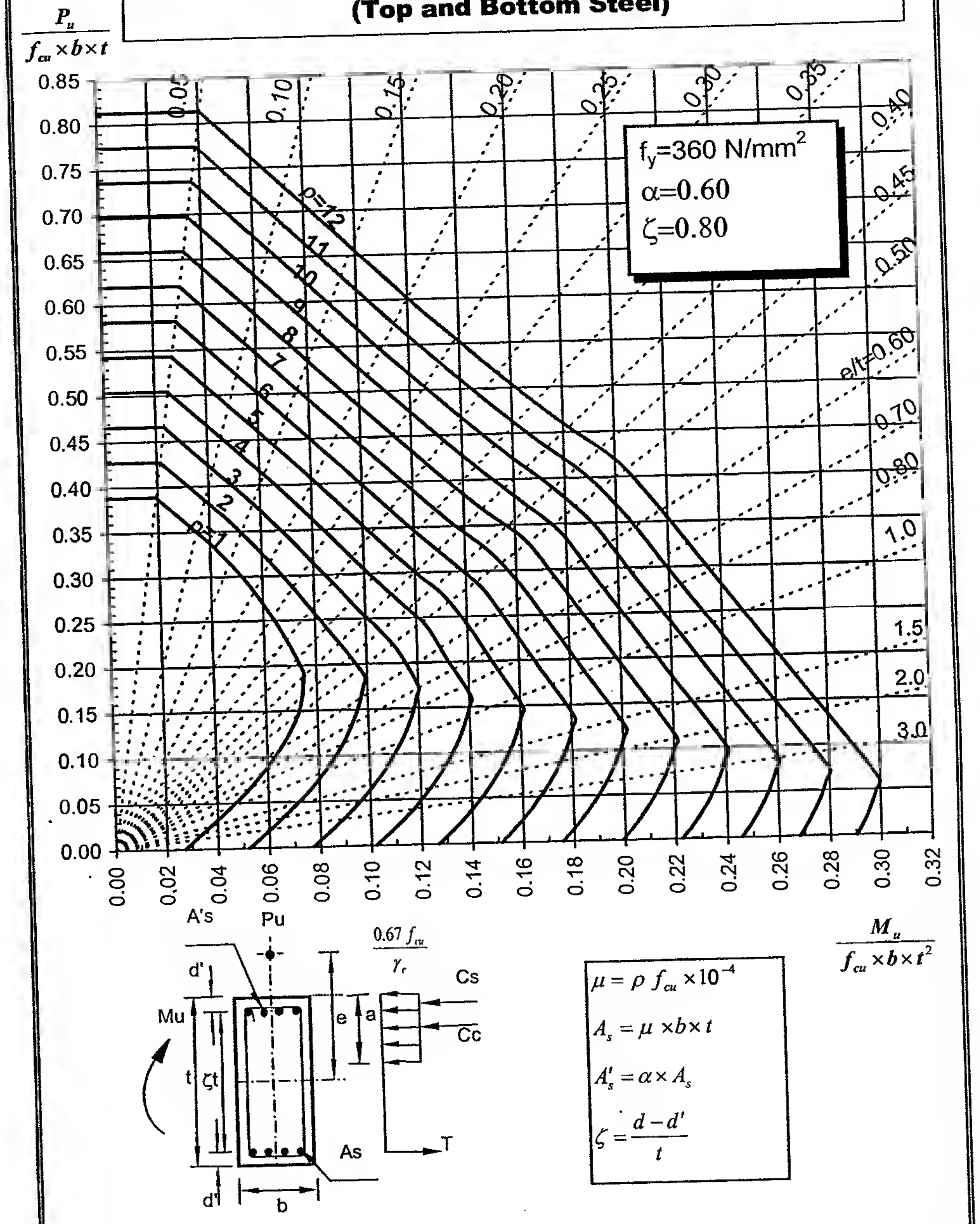
(Top and Bottom Steel)



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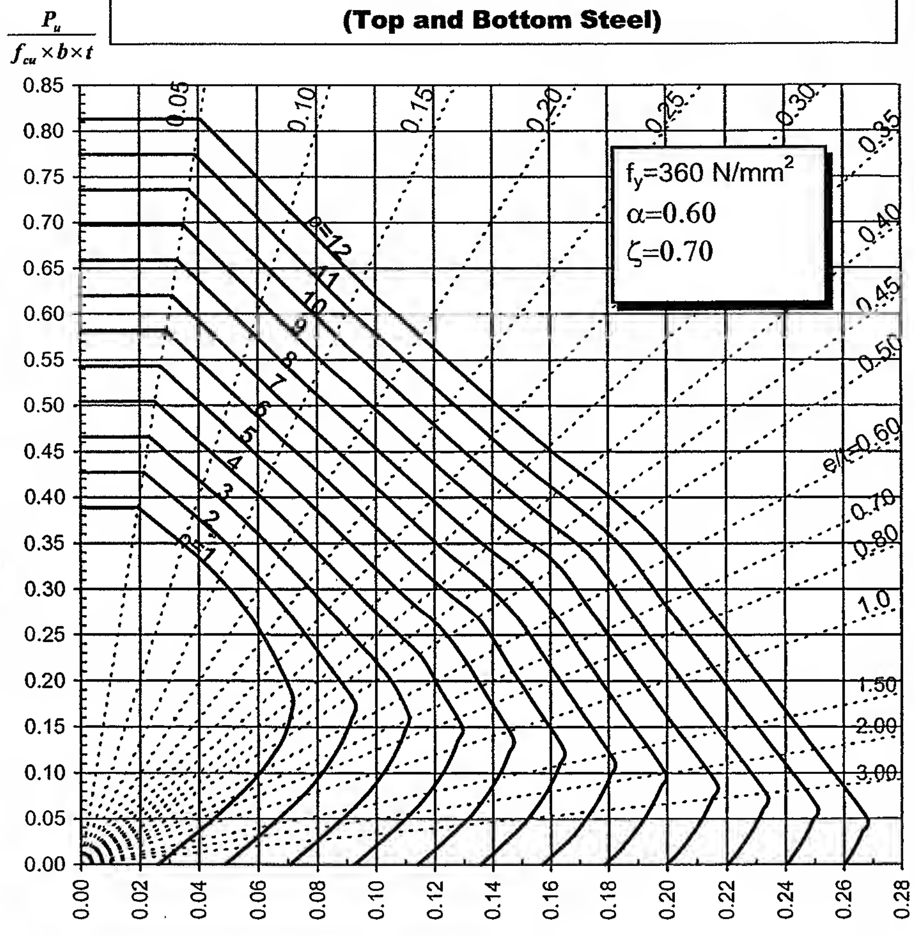

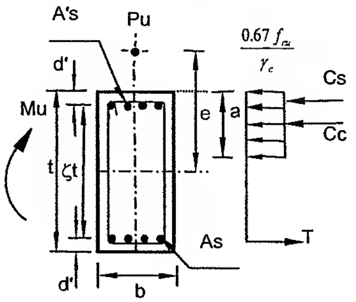

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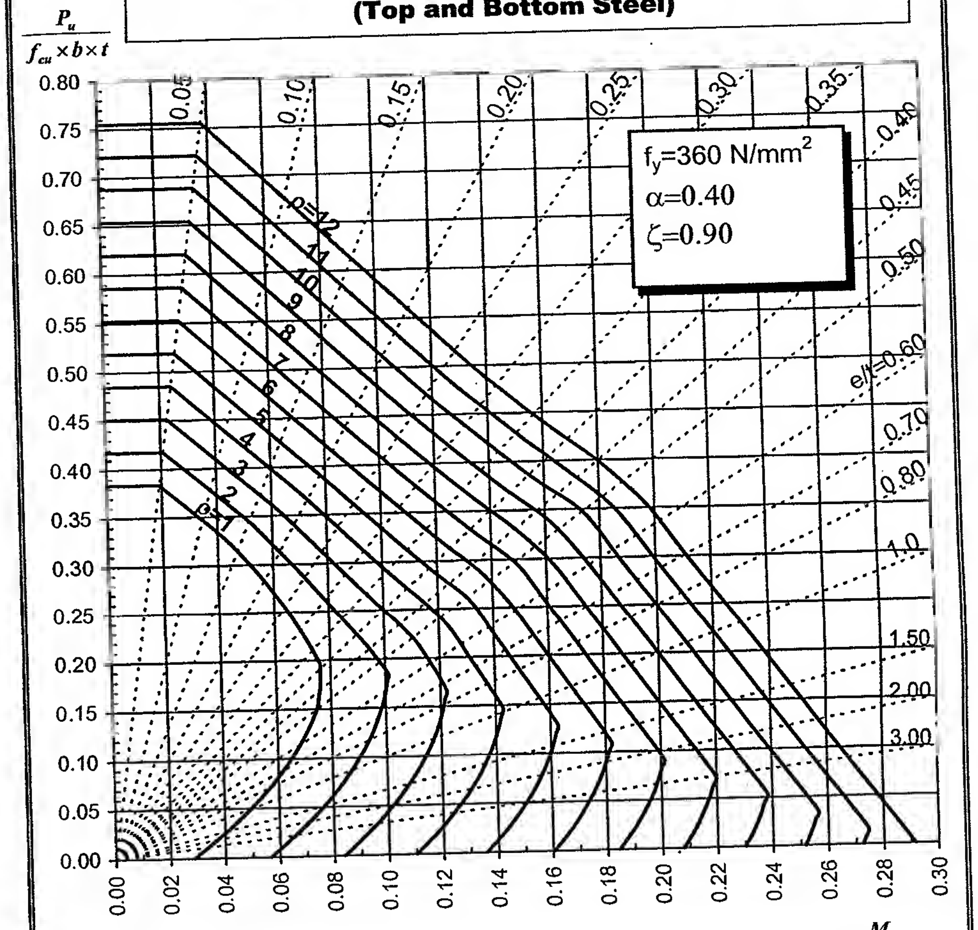

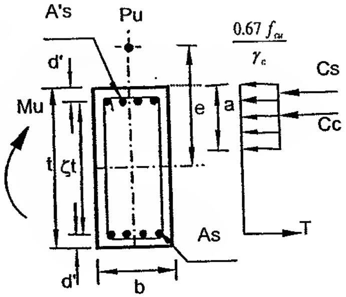

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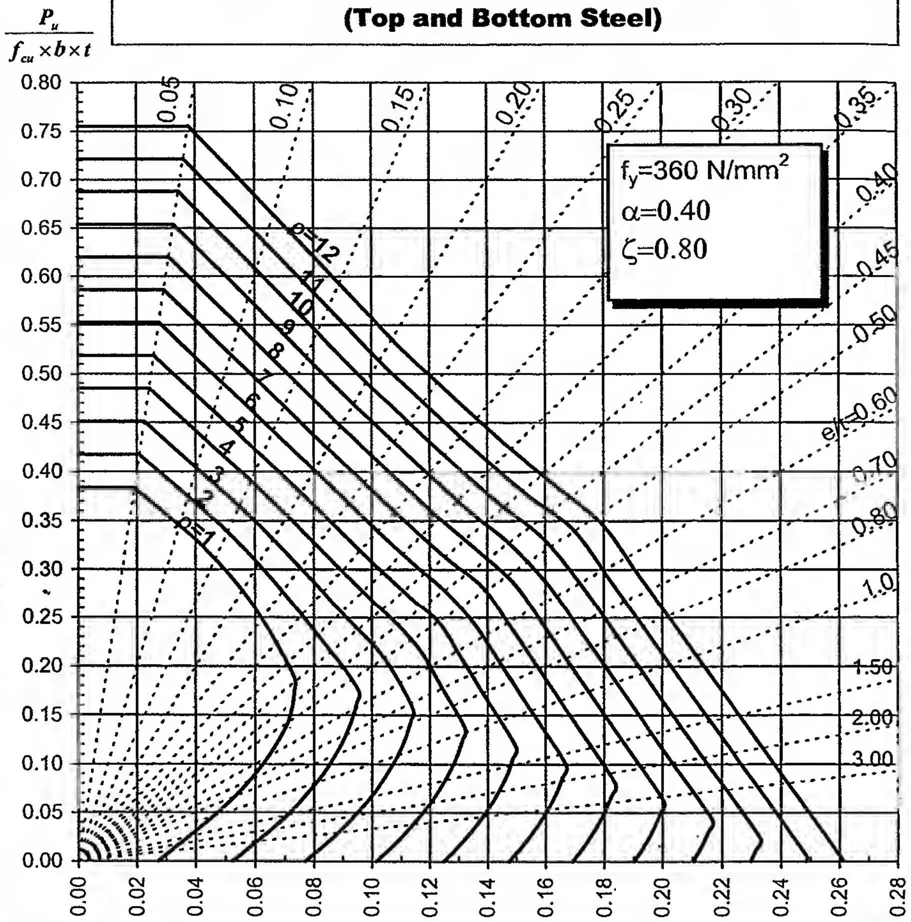

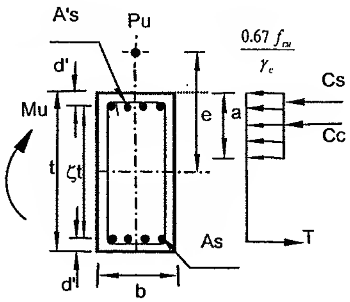

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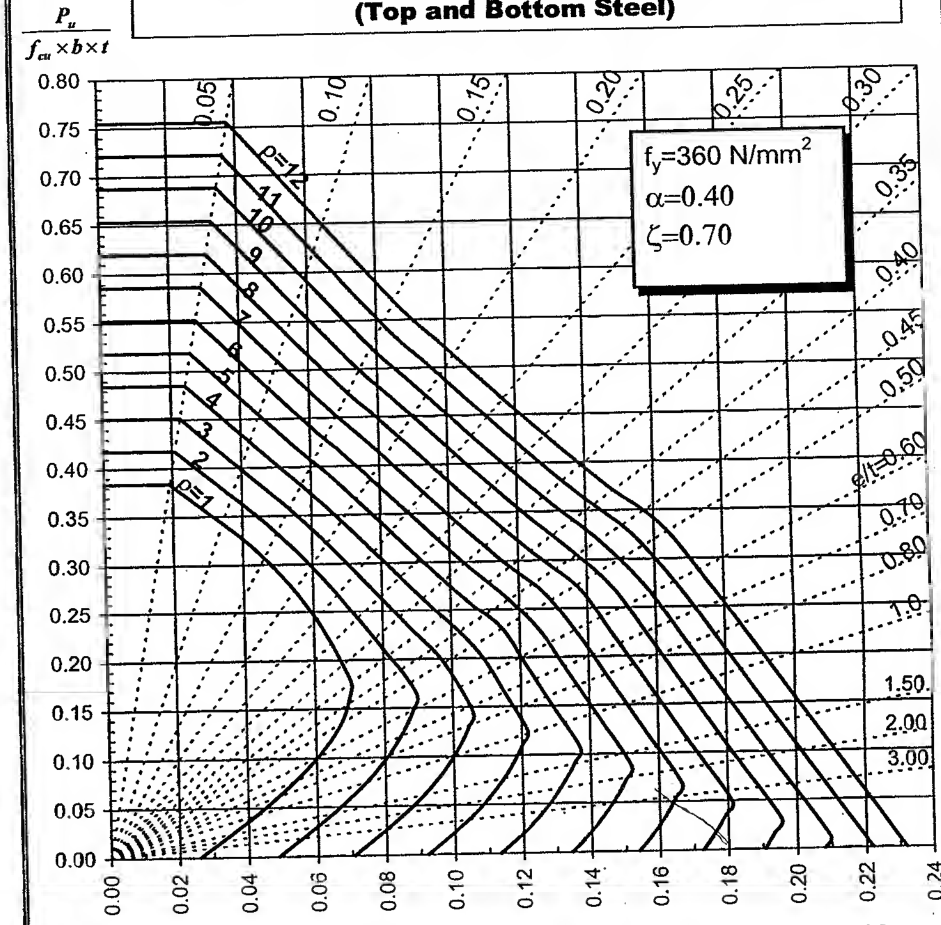
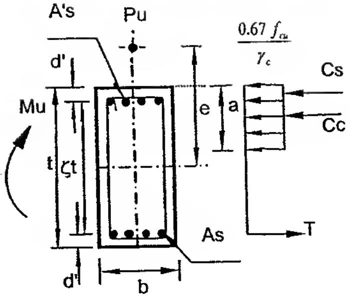
(Top and Bottom Steel)





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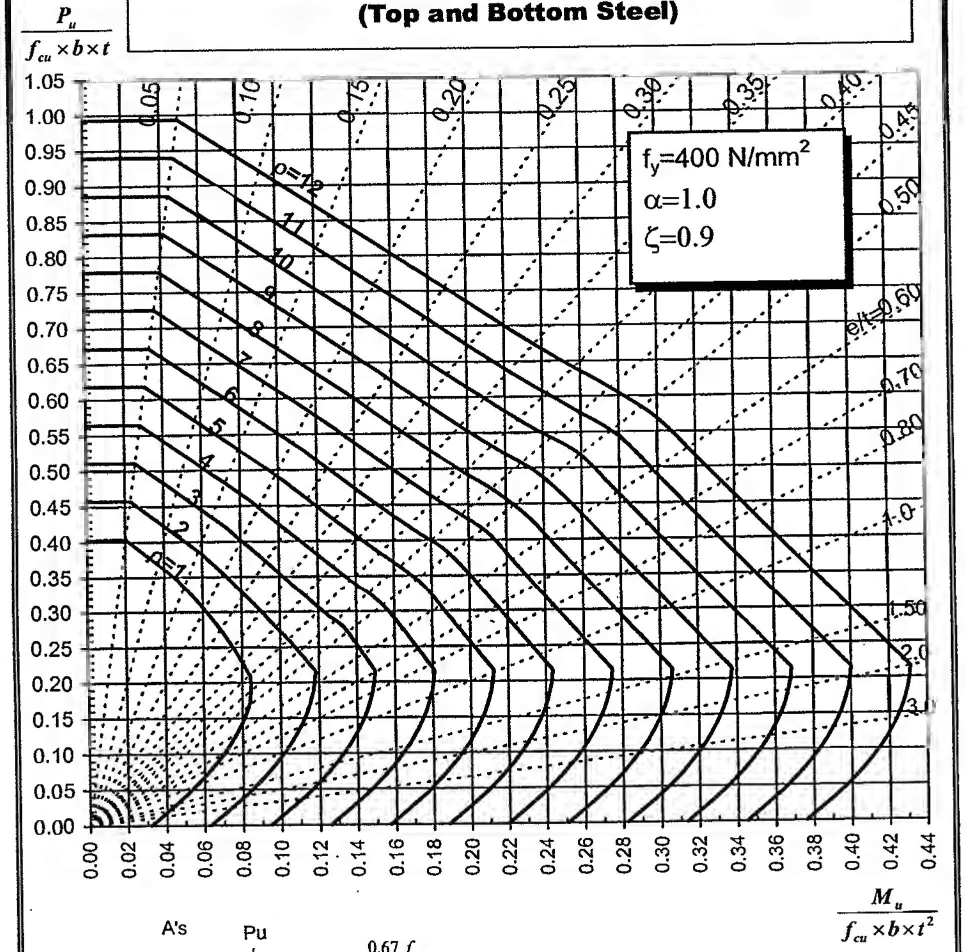
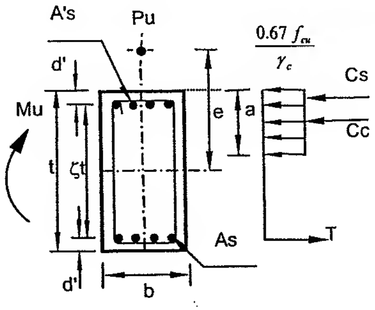
Interaction Diagram for Rectangular sections (Top and Bottom Steel)





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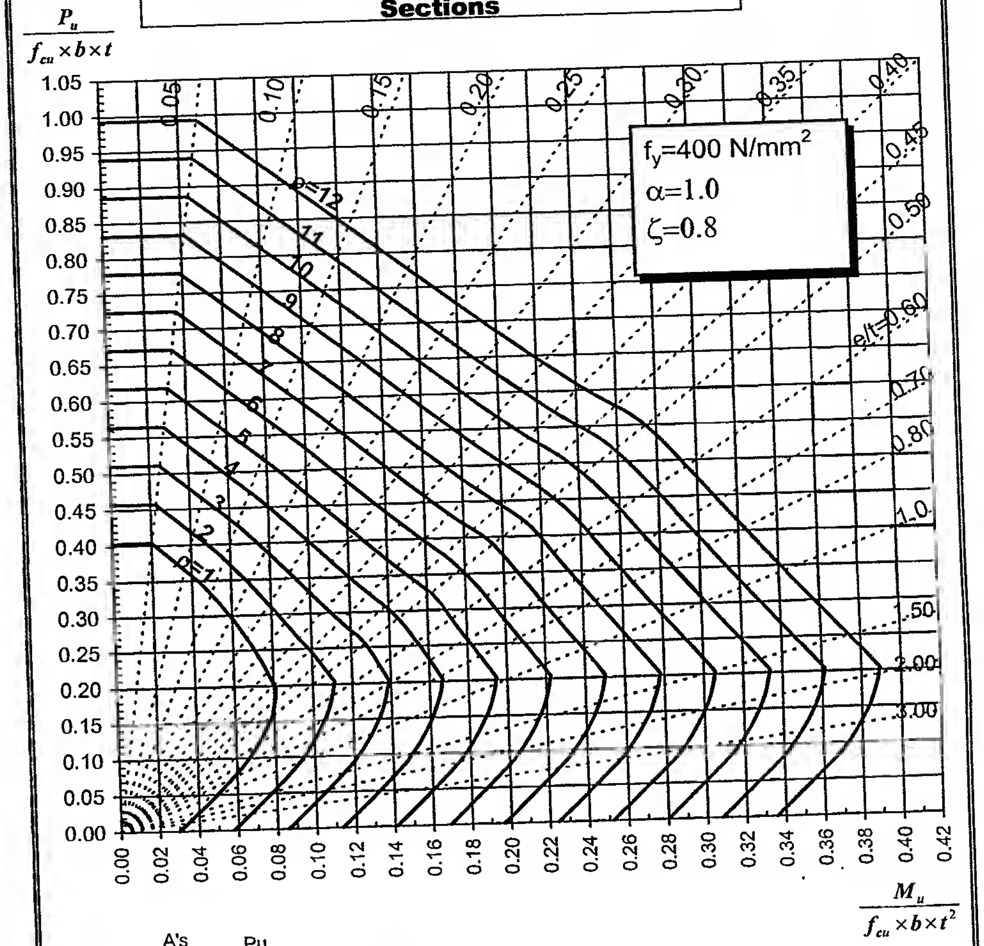
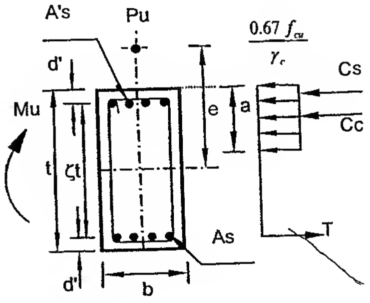
Interaction Diagram for Rectangular Sections





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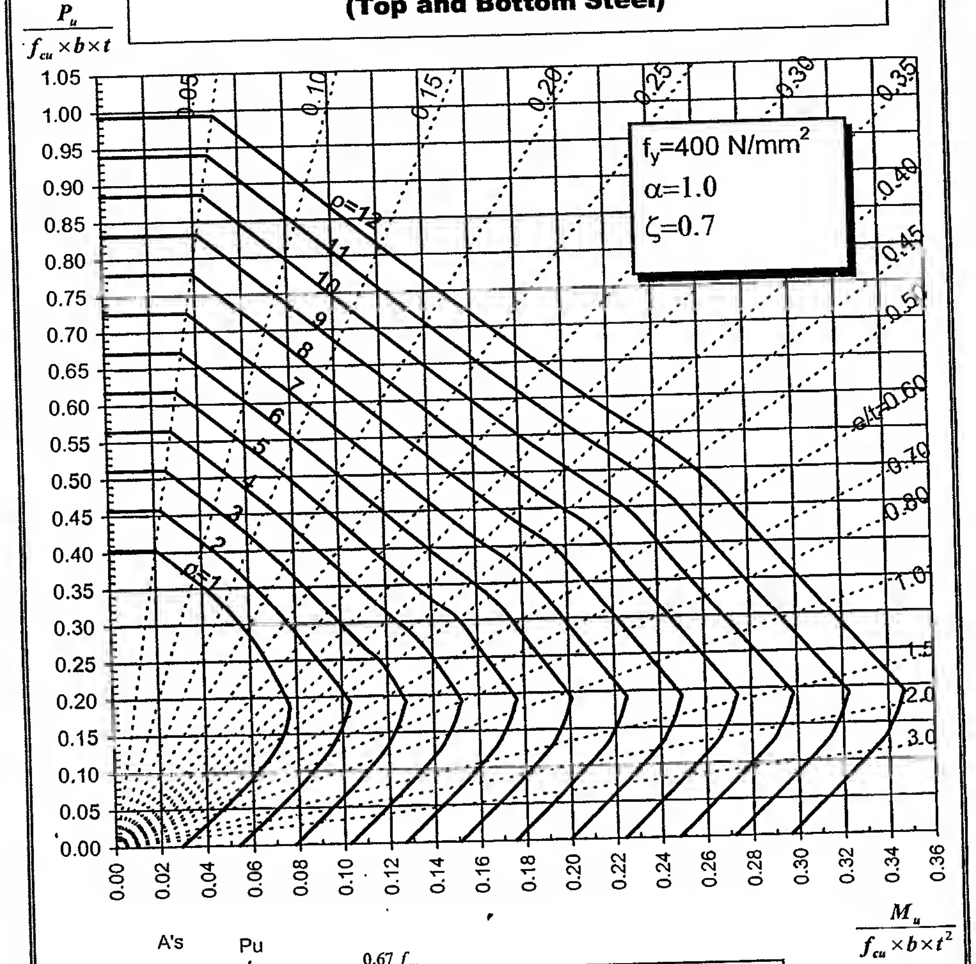

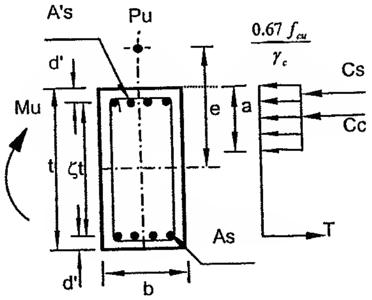

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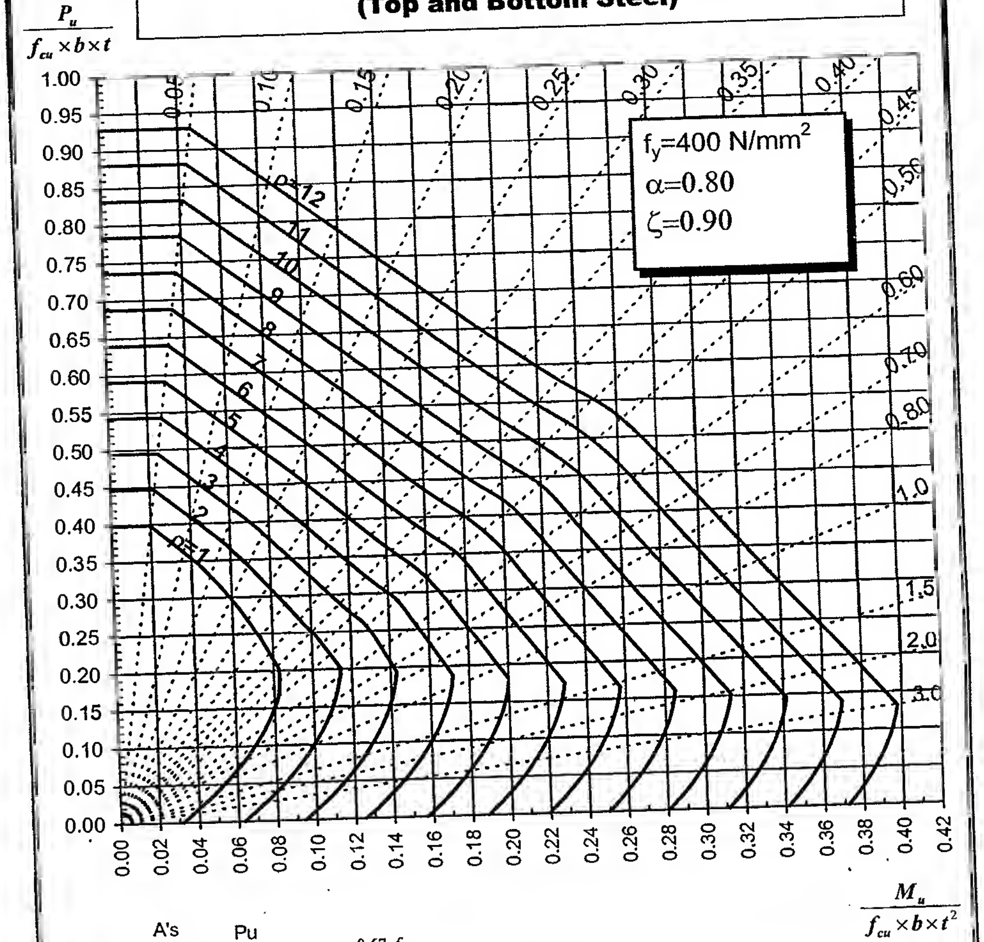

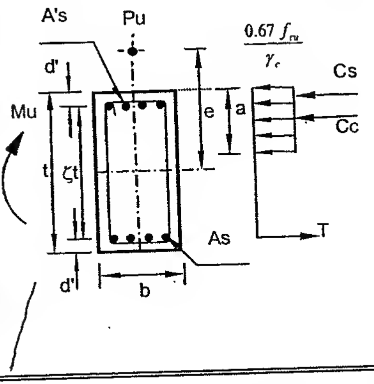

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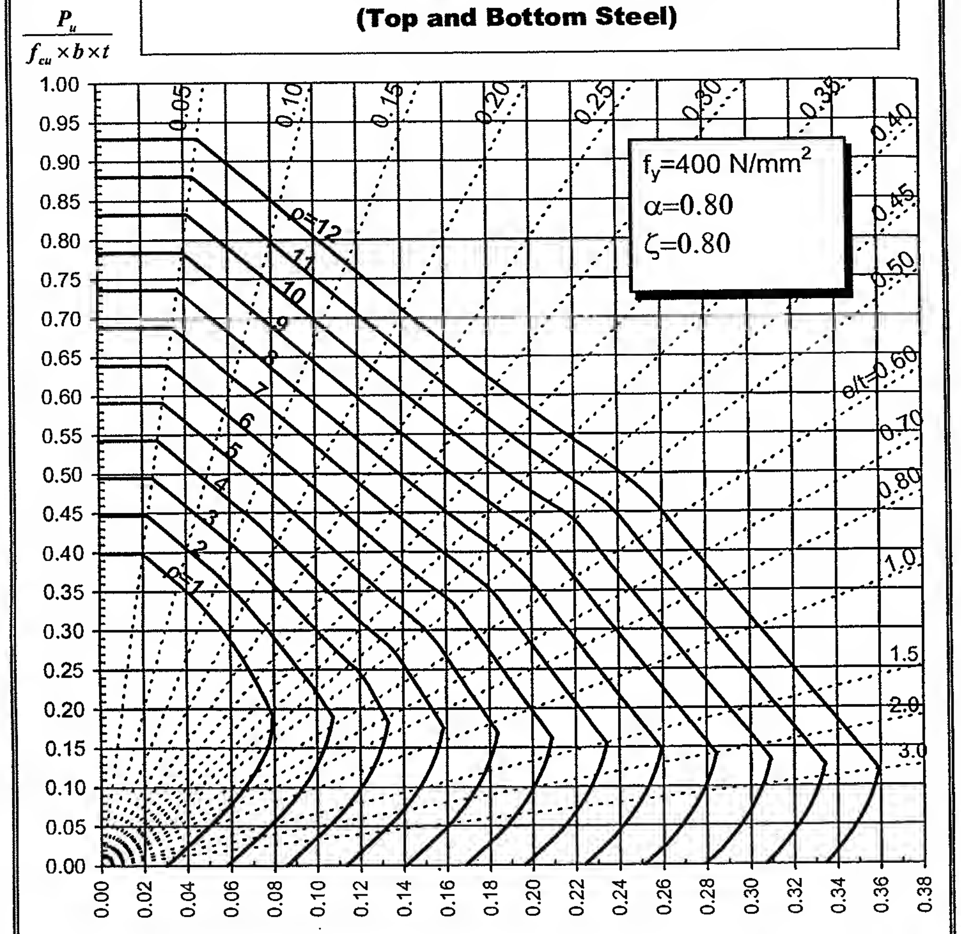

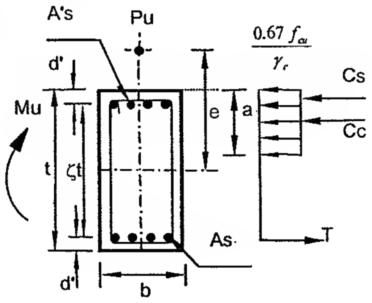

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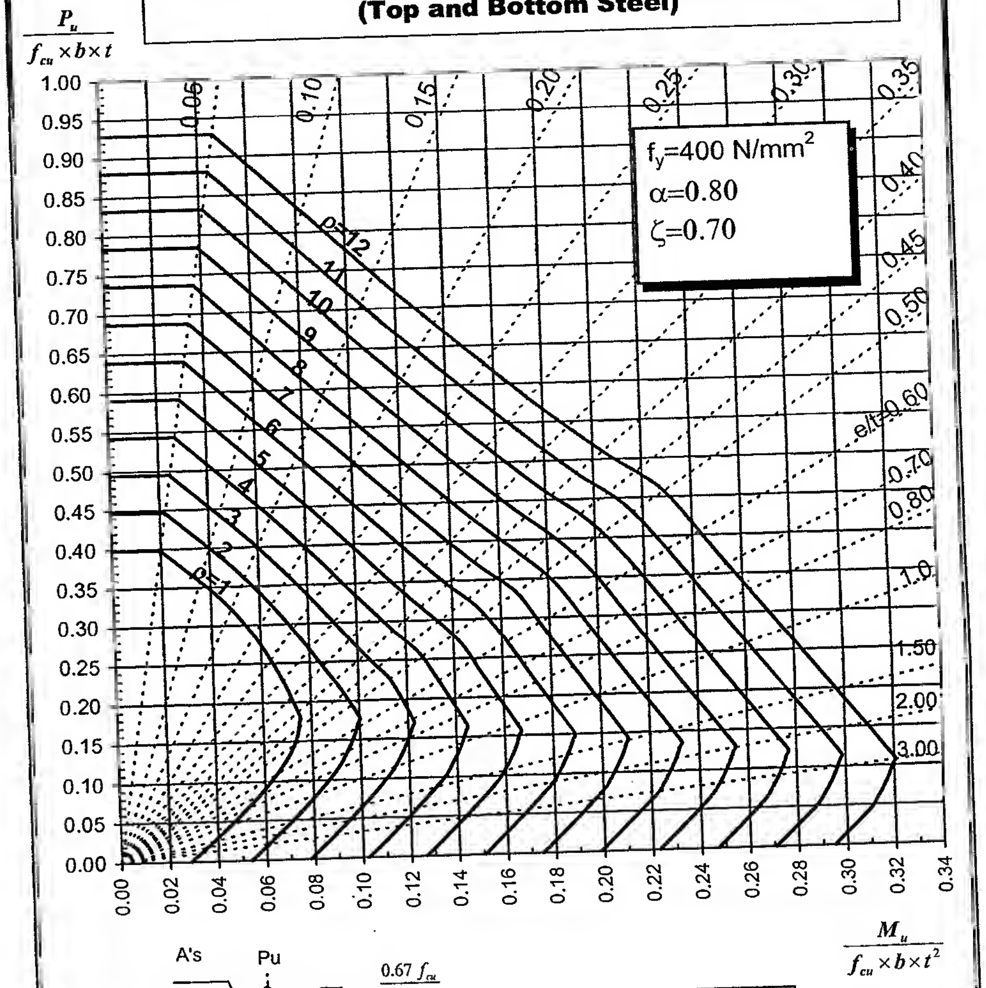

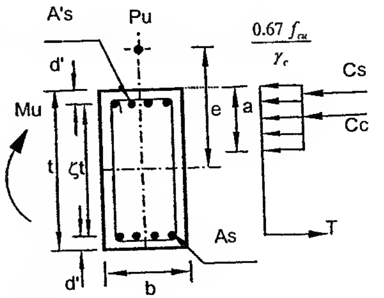

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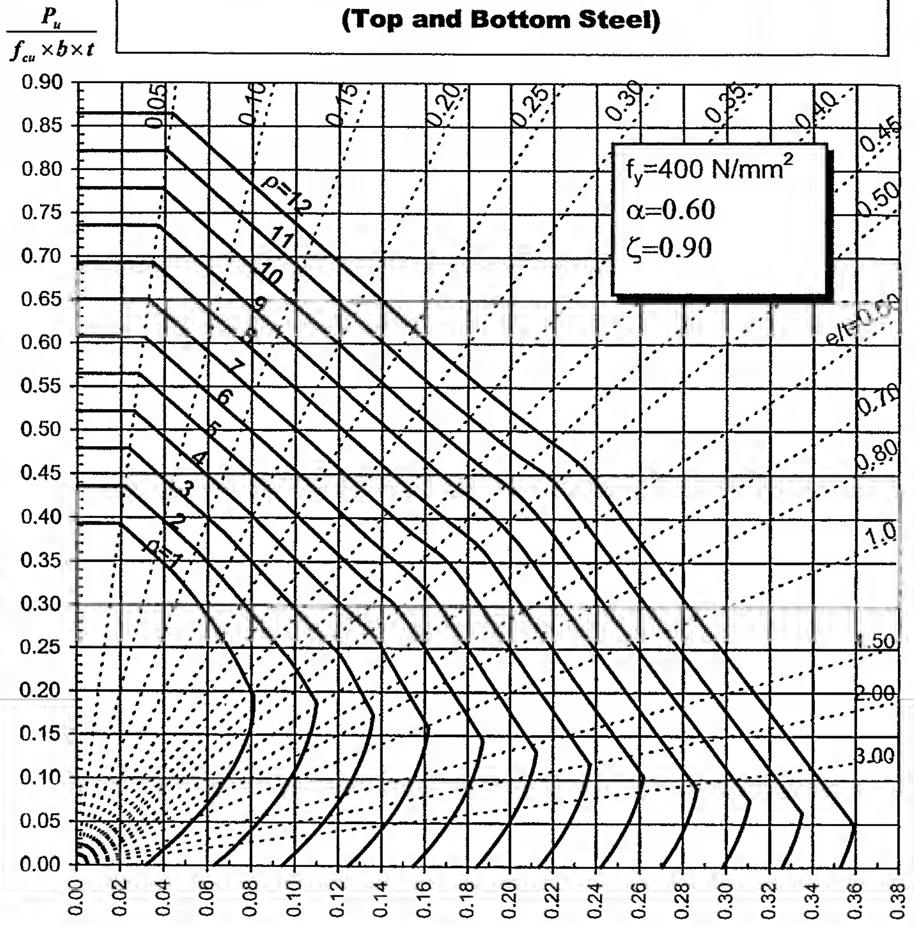

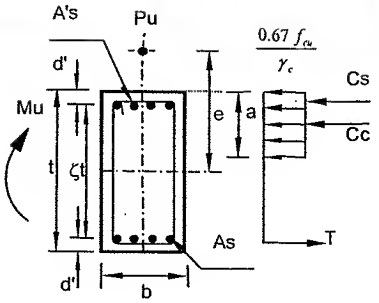

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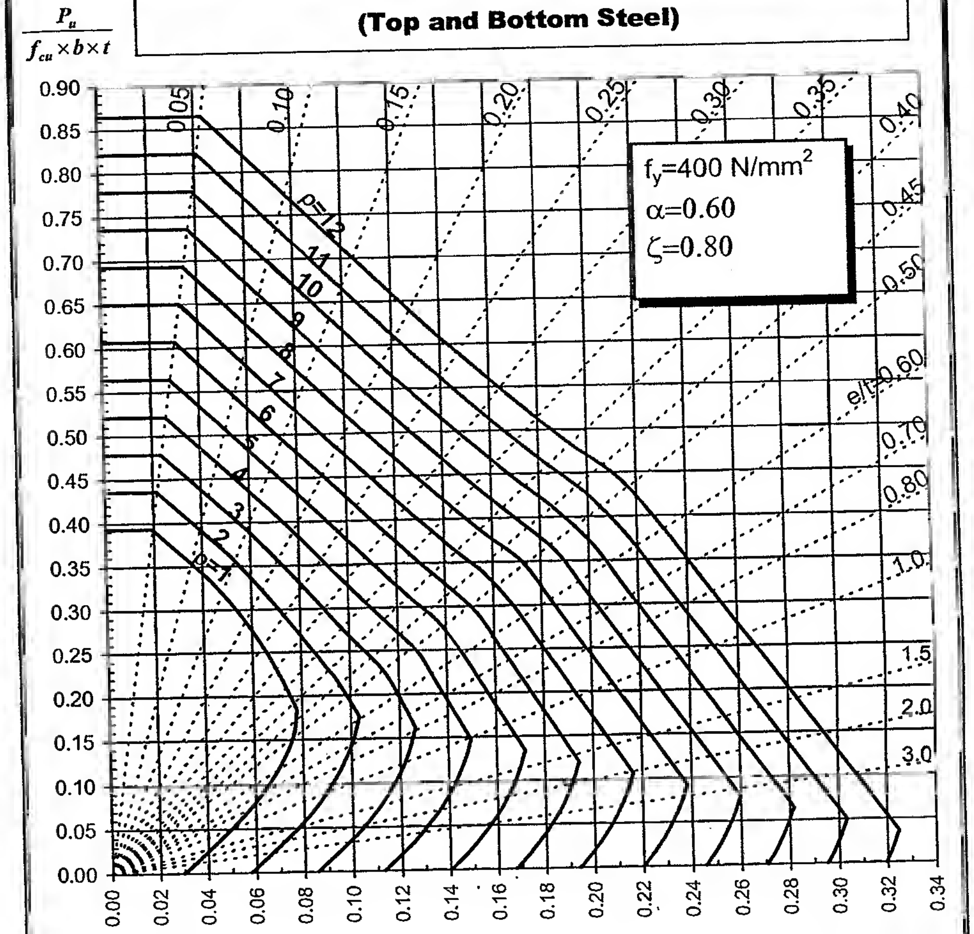

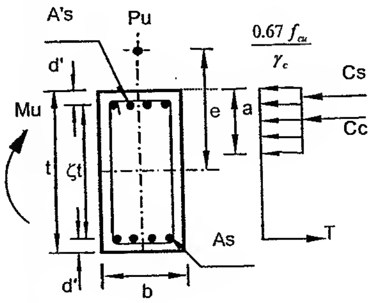

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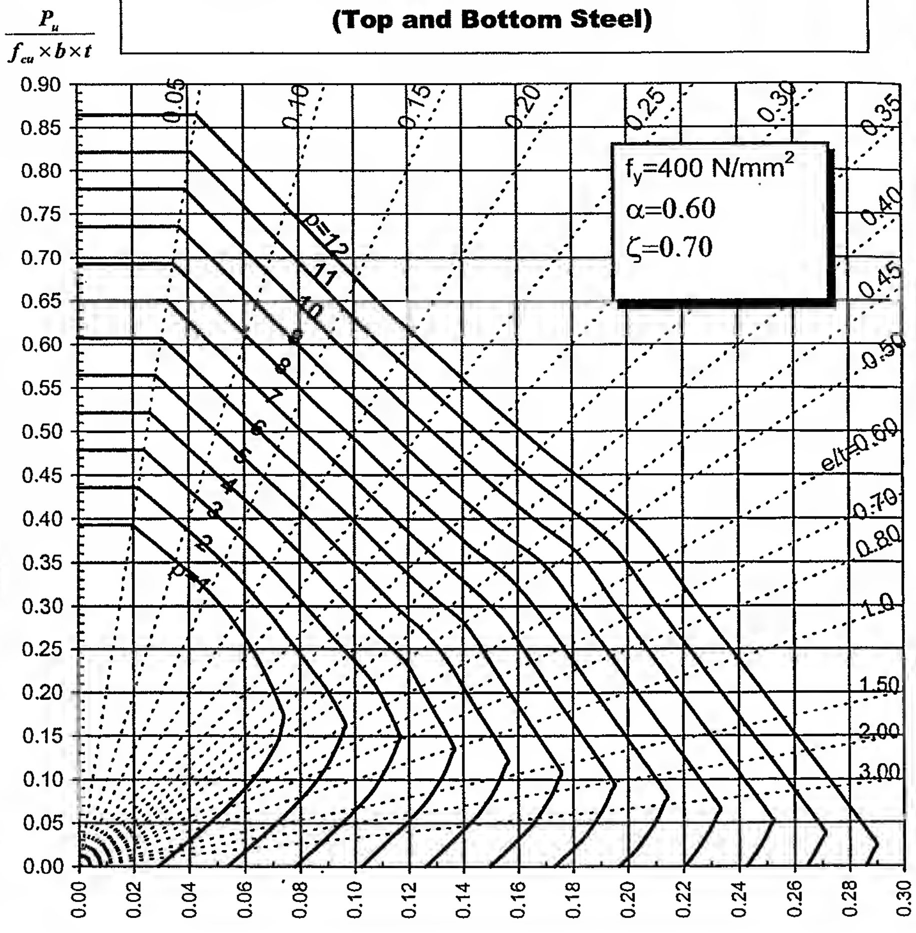

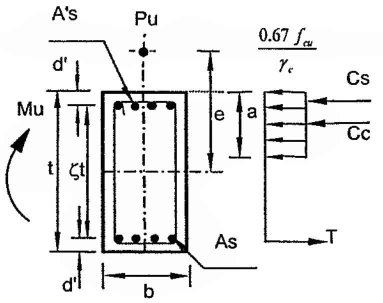

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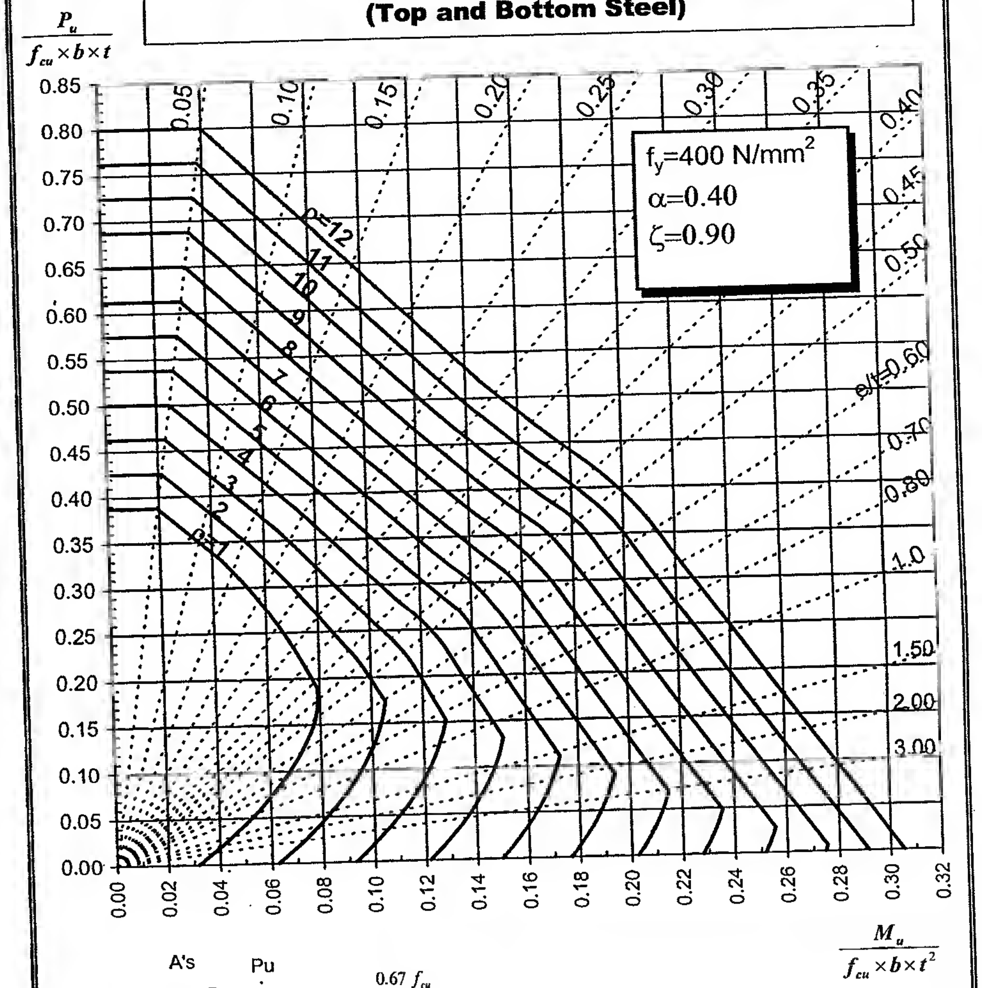

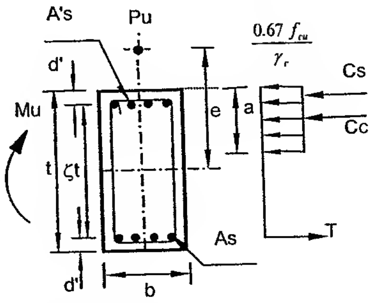

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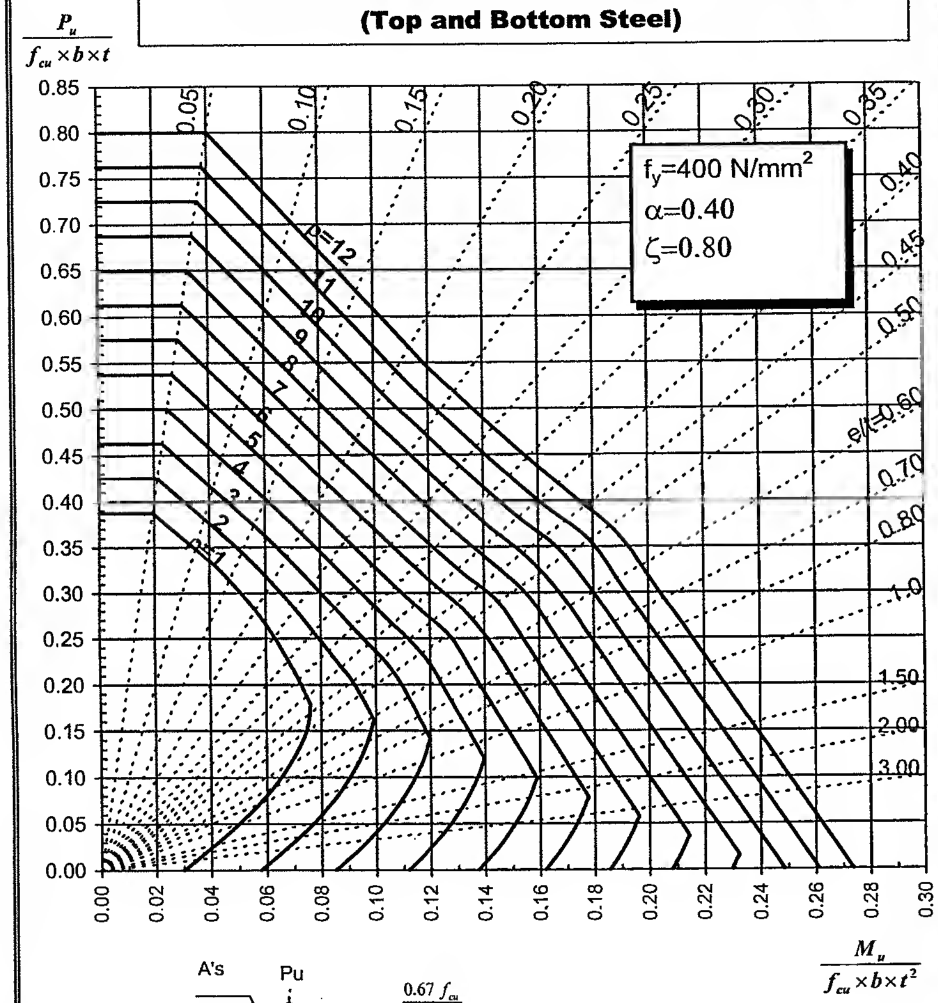

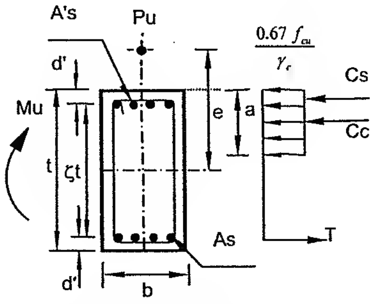

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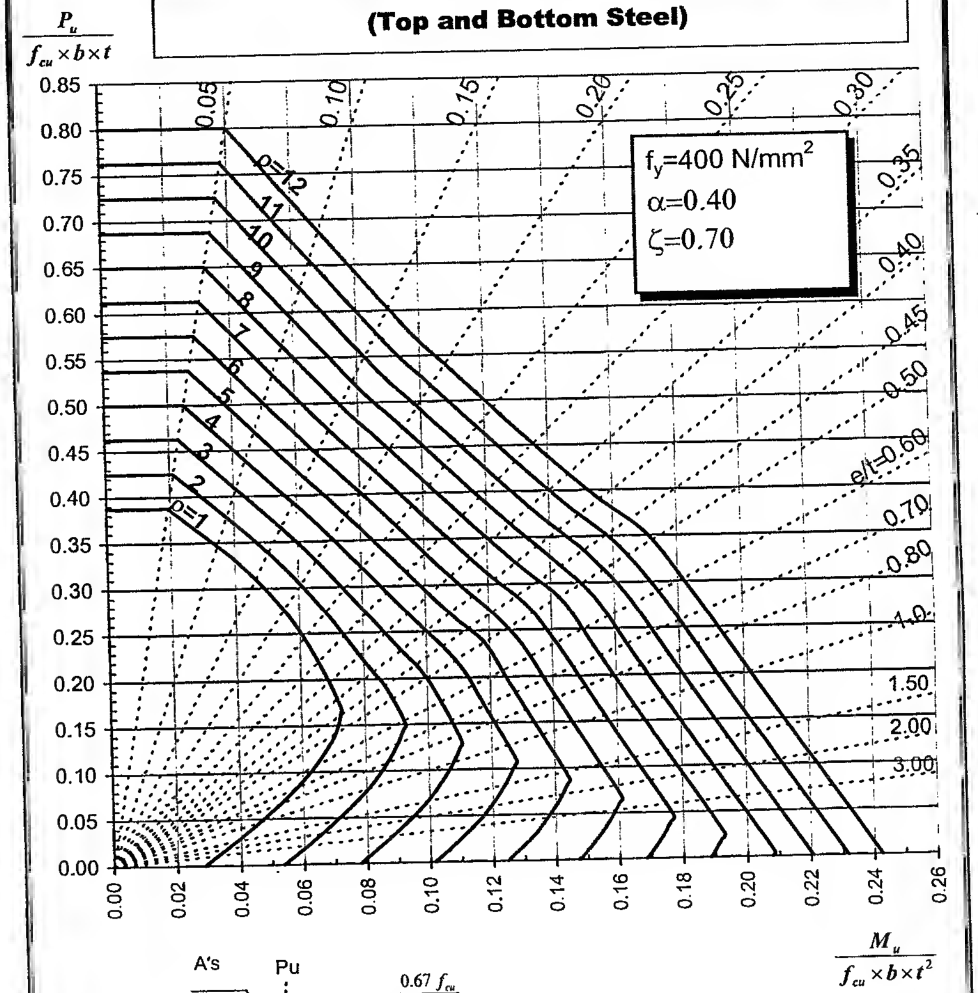

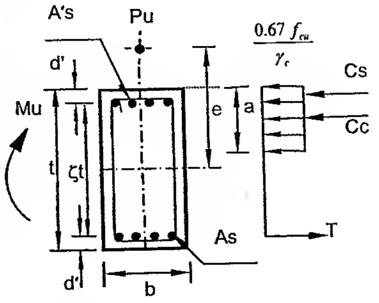

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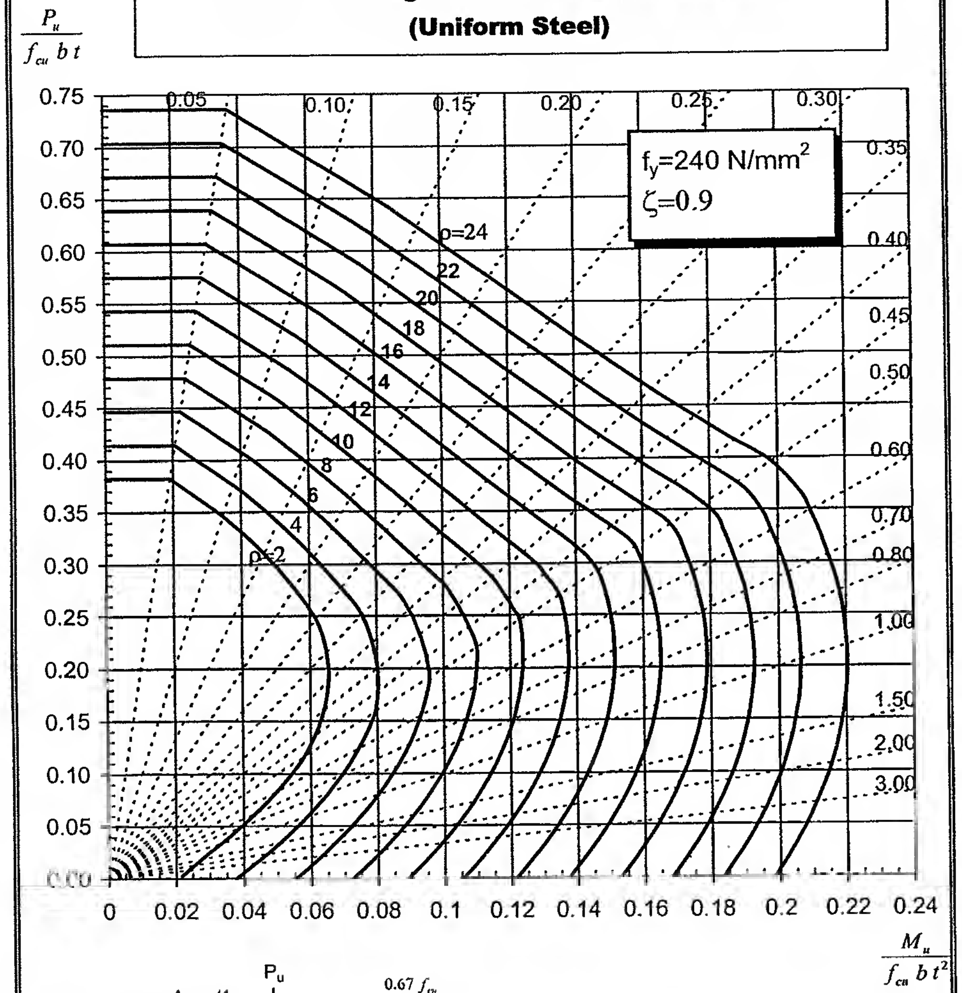
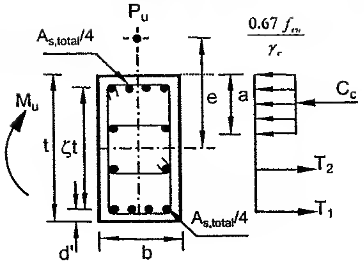
APPENDIX C

Interaction Diagrams (Uniform steel)

Interaction Diagram for Rectangular Sections (Uniform Steel)



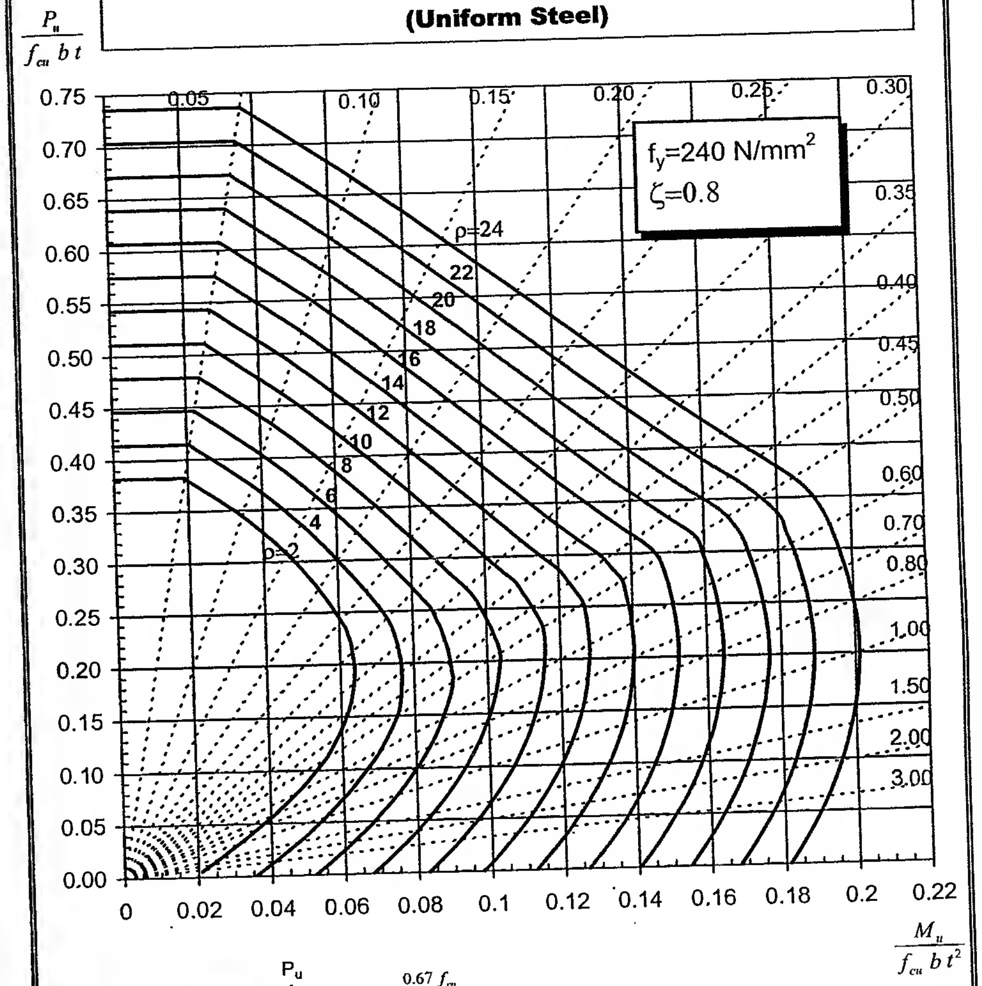


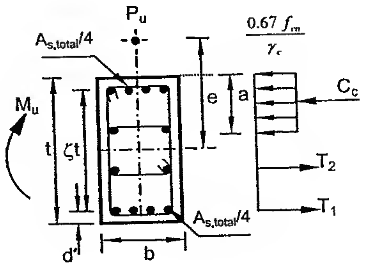
$\mu = \rho\, f_{cu} \times 10^{-4}$

$A_{s,total} = \mu\ b\ t$

$\zeta = \frac{d - d'}{t}$

Interaction Diagram for Rectangular Sections (Uniform Steel)



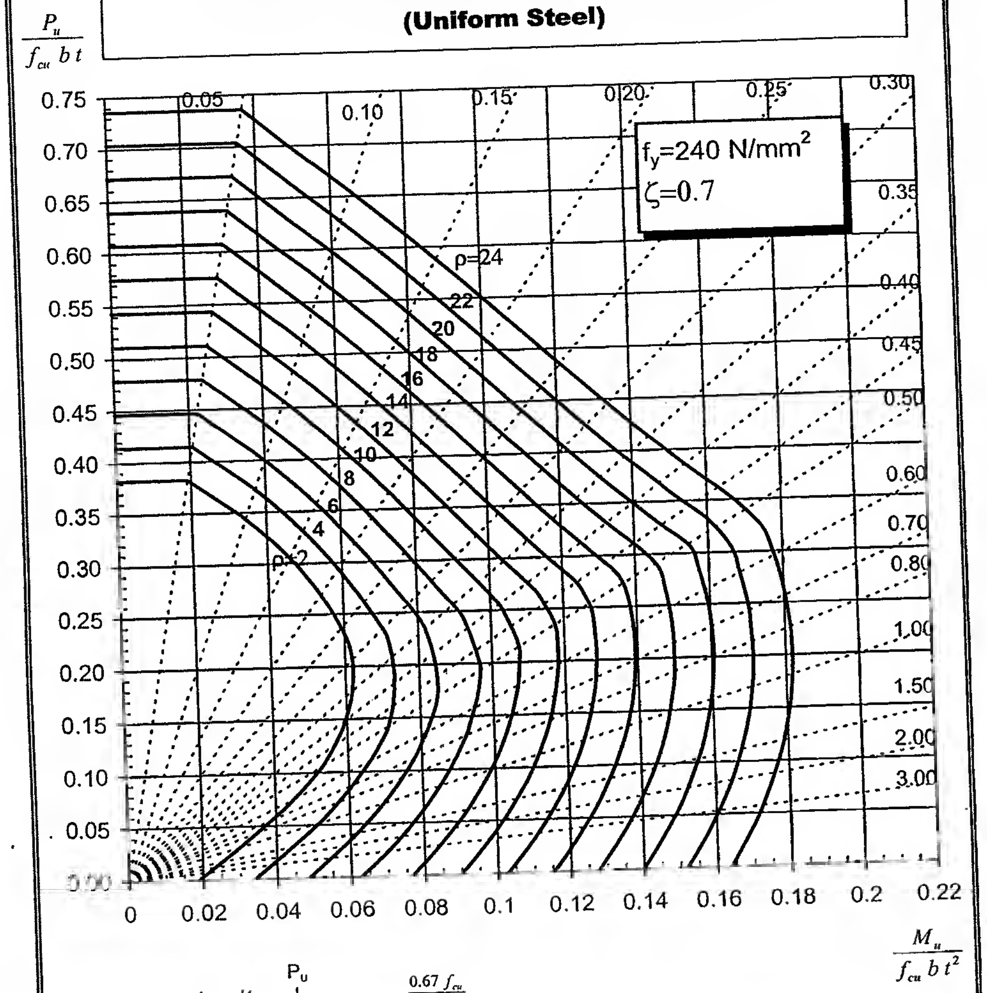


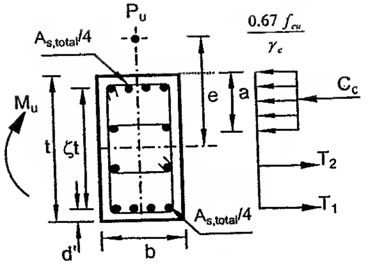
$\mu = \rho\, f_{cu} \times 10^{-4}$

$A_{s,total} = \mu\ b\ t$

$\zeta = \frac{d - d'}{t}$

Interaction Diagram for Rectangular Sections
(Uniform Steel)



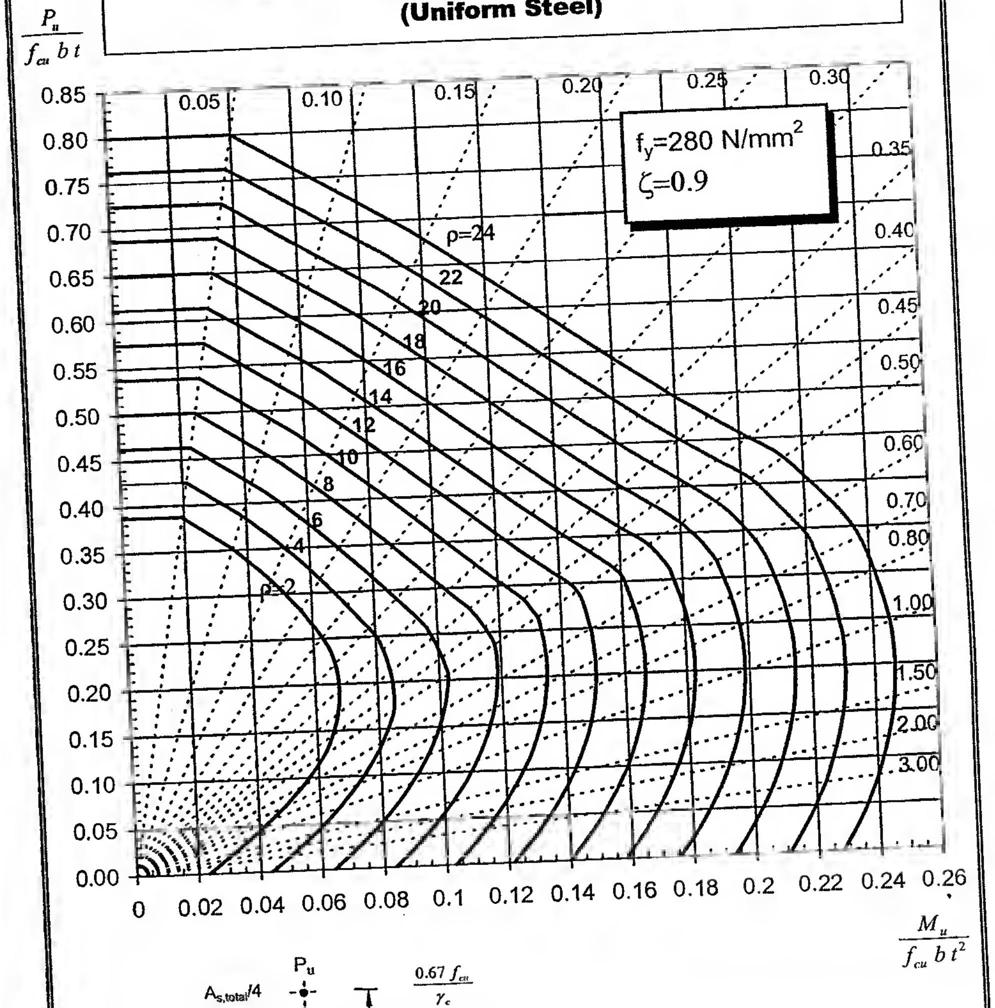


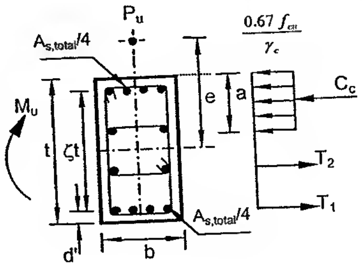
$\mu = \rho\, f_{cu} \times 10^{-4}$

$A_{s,total} = \mu\; b\; t$

$\zeta = \frac{d - d'}{t}$

Interaction Diagram for Rectangular Sections
(Uniform Steel)





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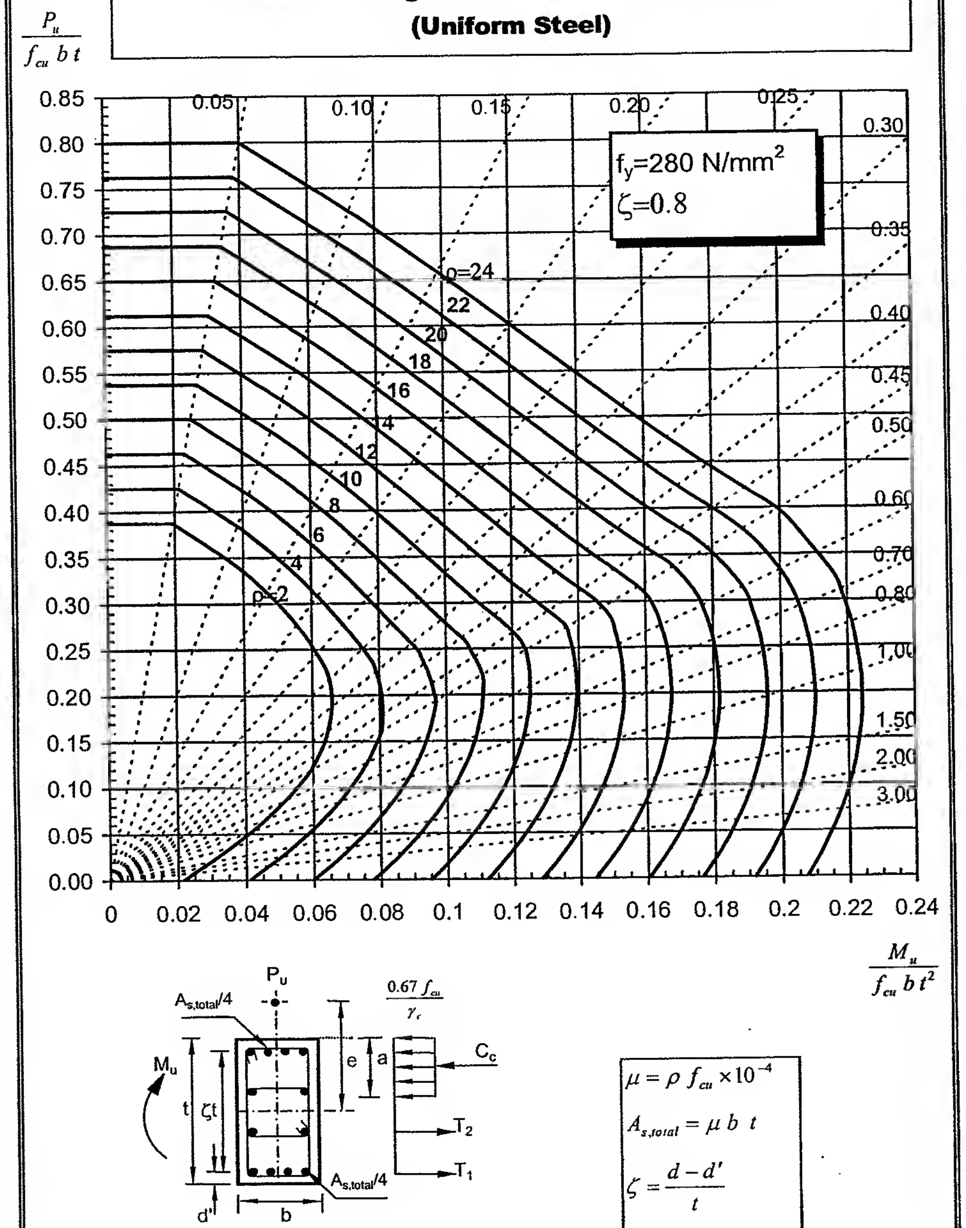
Interaction Diagram for Rectangular Sections
(Uniform Steel)



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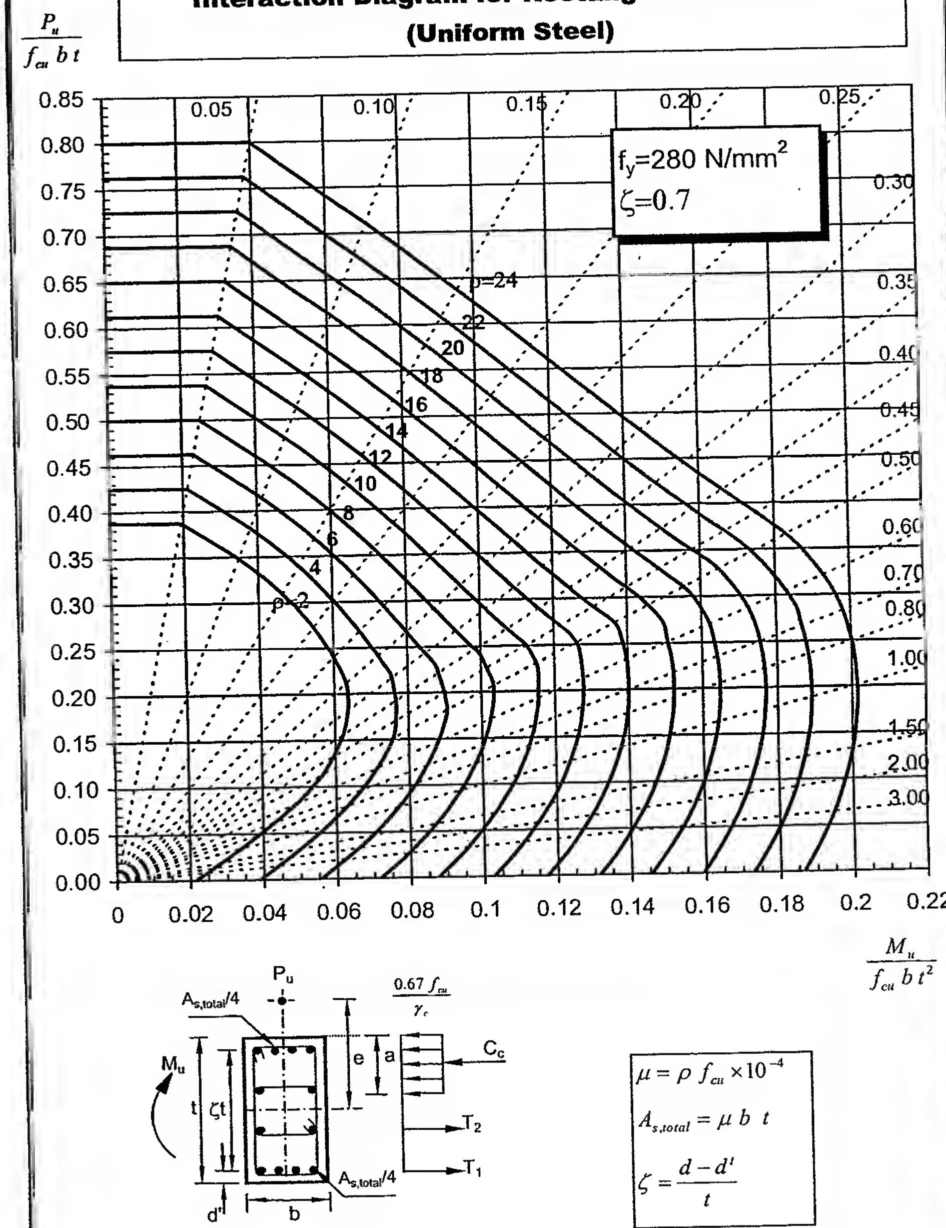
Interaction Diagram for Rectangular Sections
(Uniform Steel)



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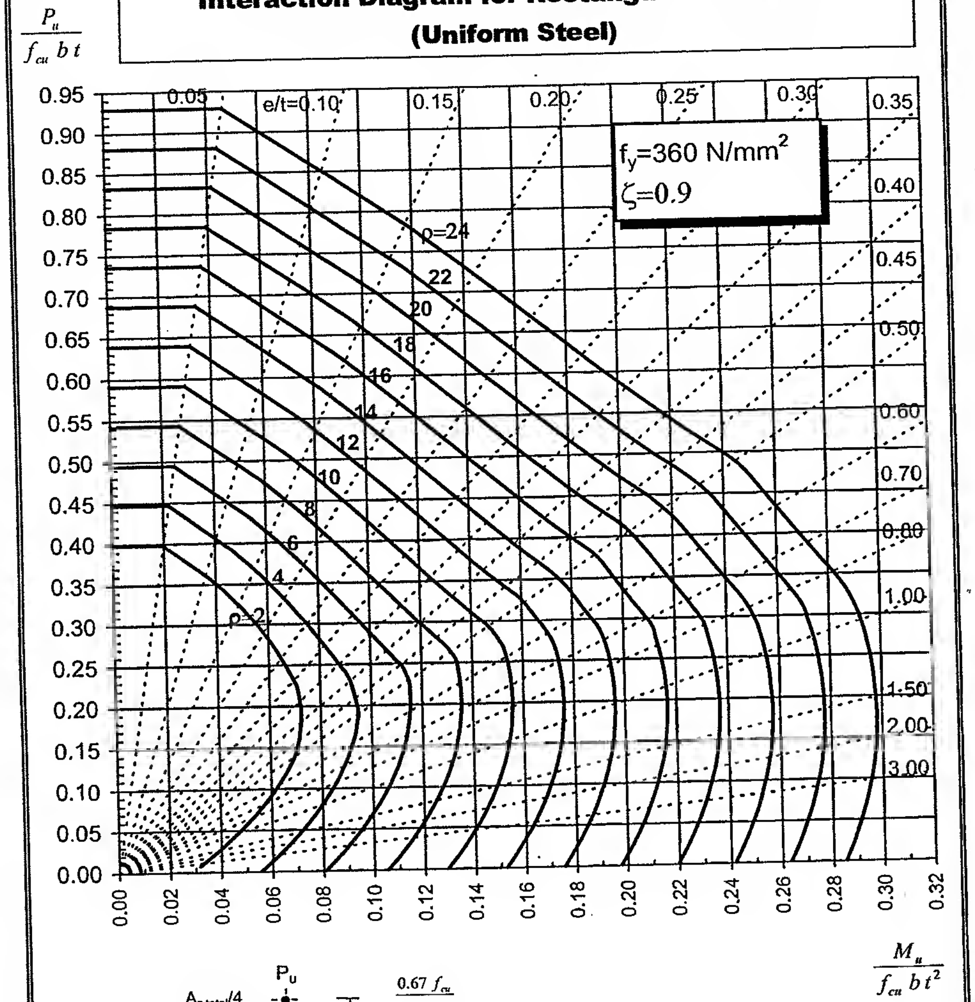
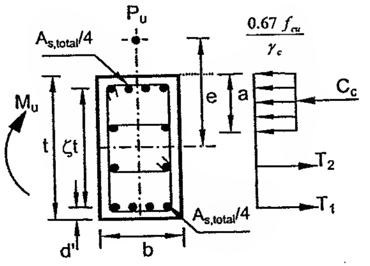
Interaction Diagram for Rectangular Sections
(Uniform Steel)





$\mu = \rho\ f_{cu} \times 10^{-4}$

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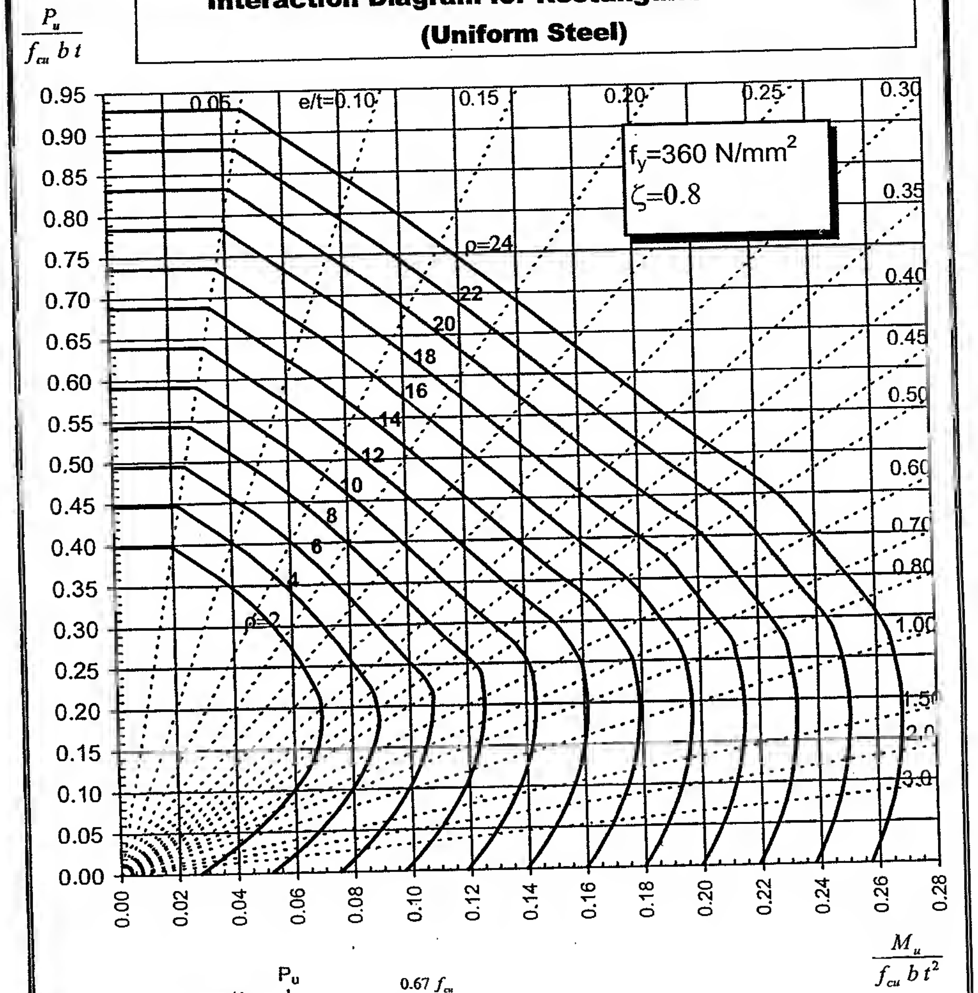
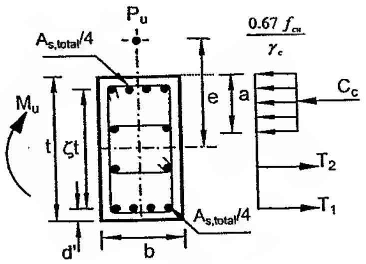
Interaction Diagram for Rectangular Sections
(Uniform Steel)





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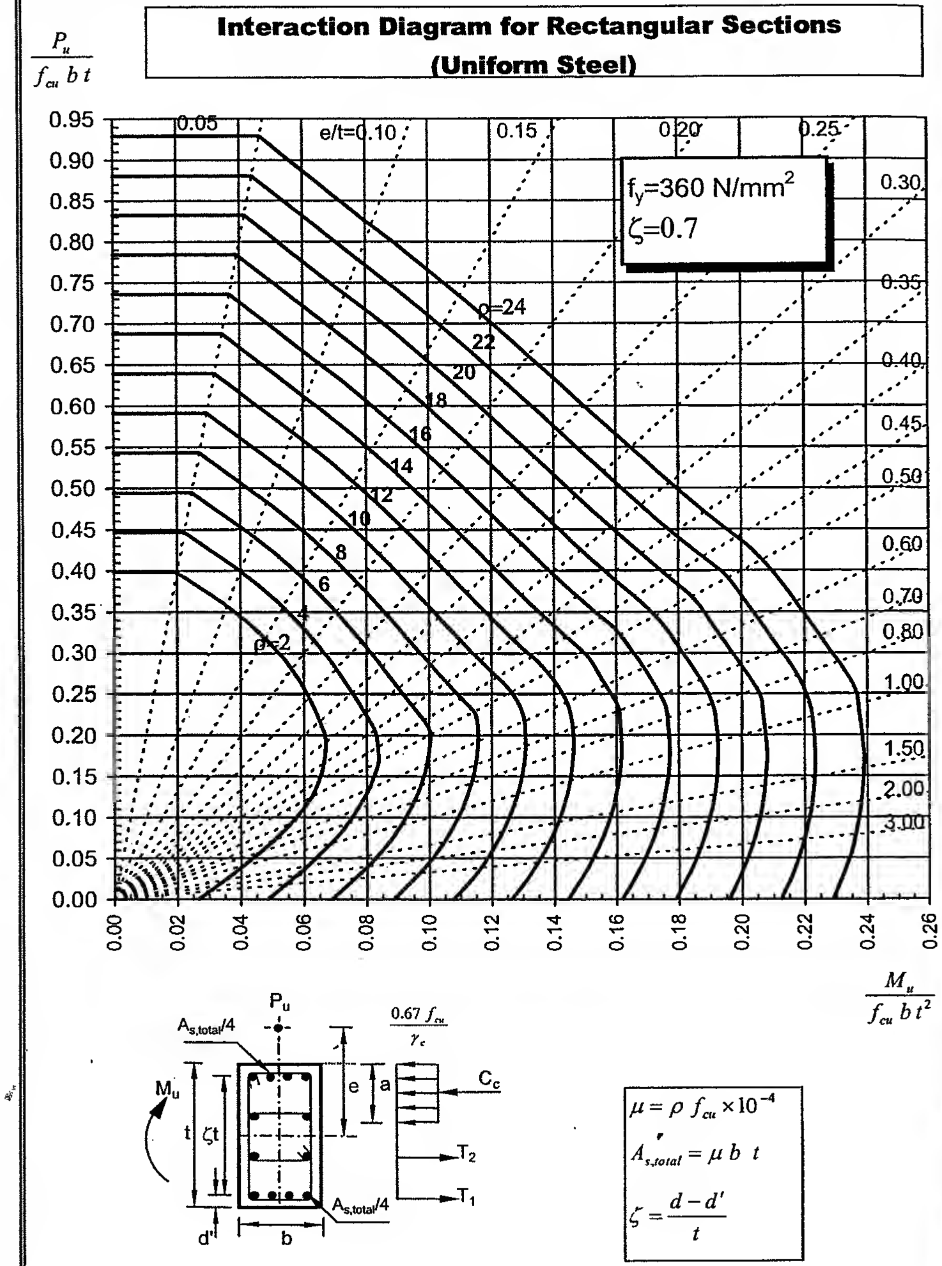
Interaction Diagram for Rectangular Sections (Uniform Steel)



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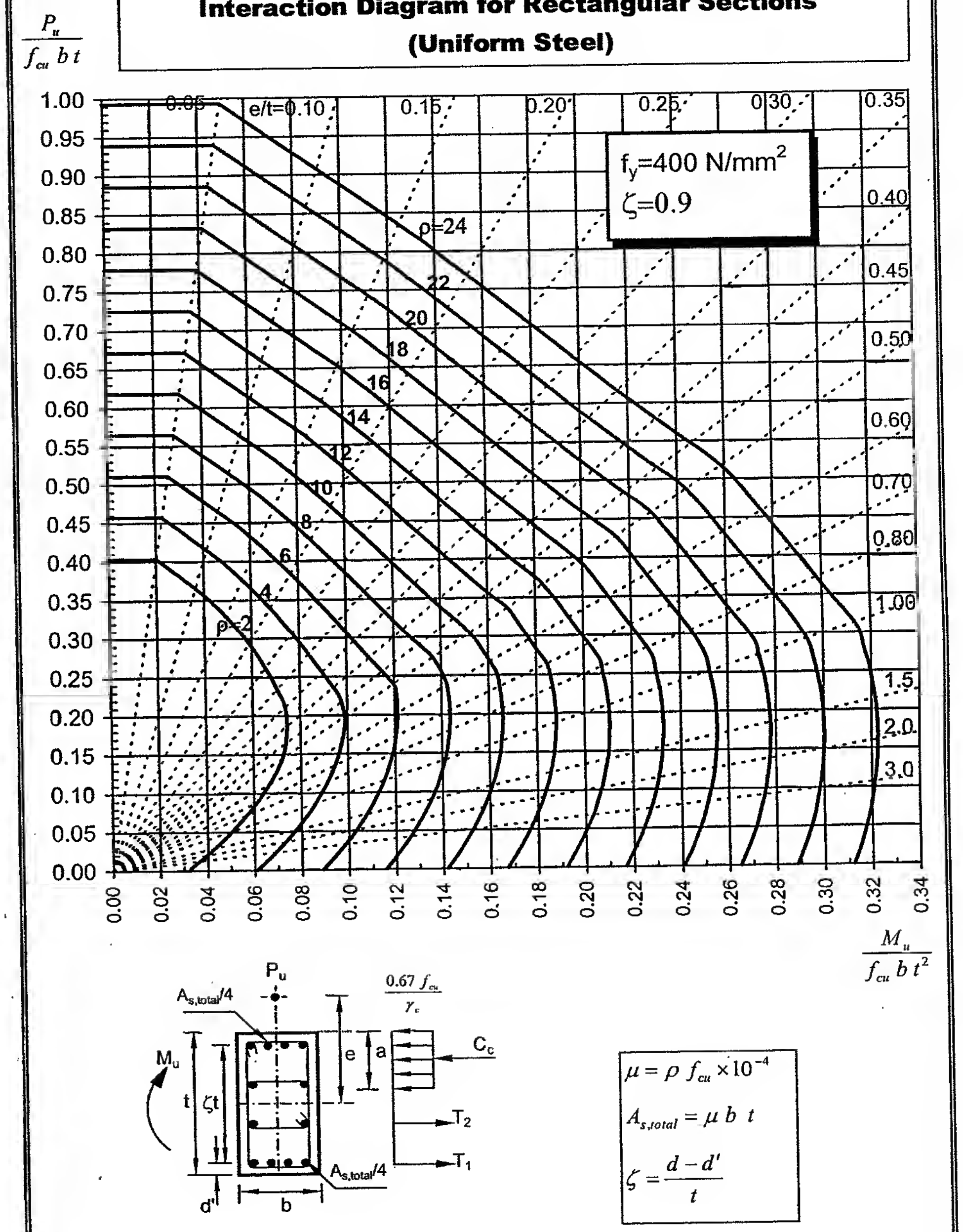

$\mu = \rho\; f_{cu} \times 10^{-4}$

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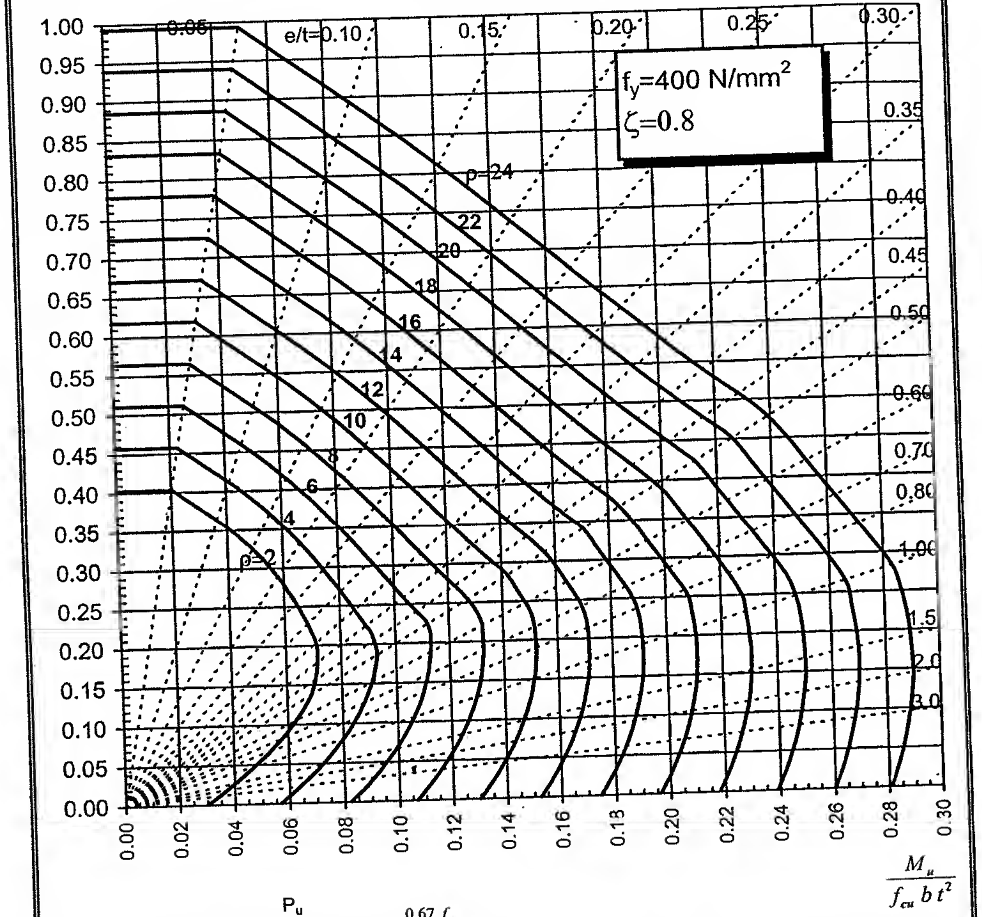
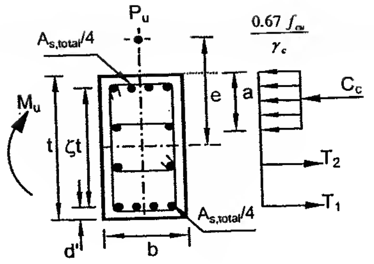
Interaction Diagram for Rectangular Sections (Uniform Steel)





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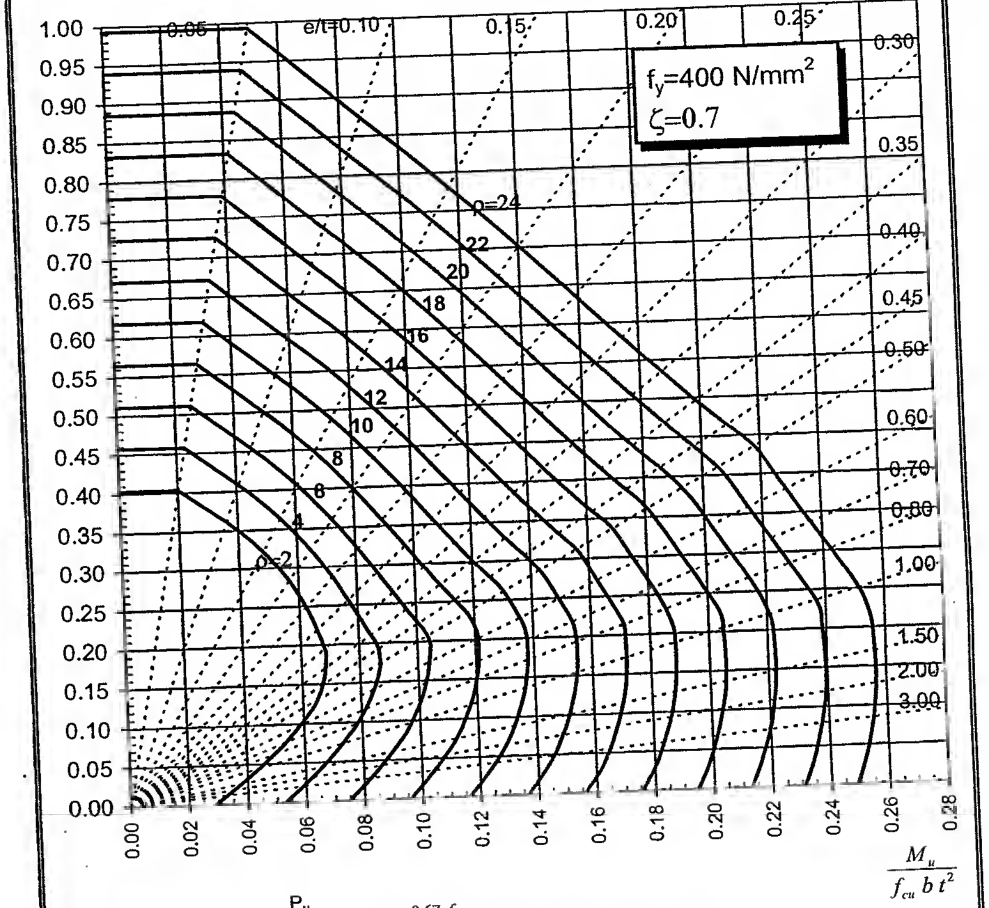

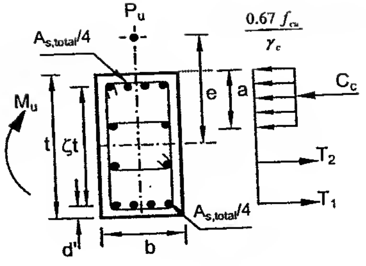

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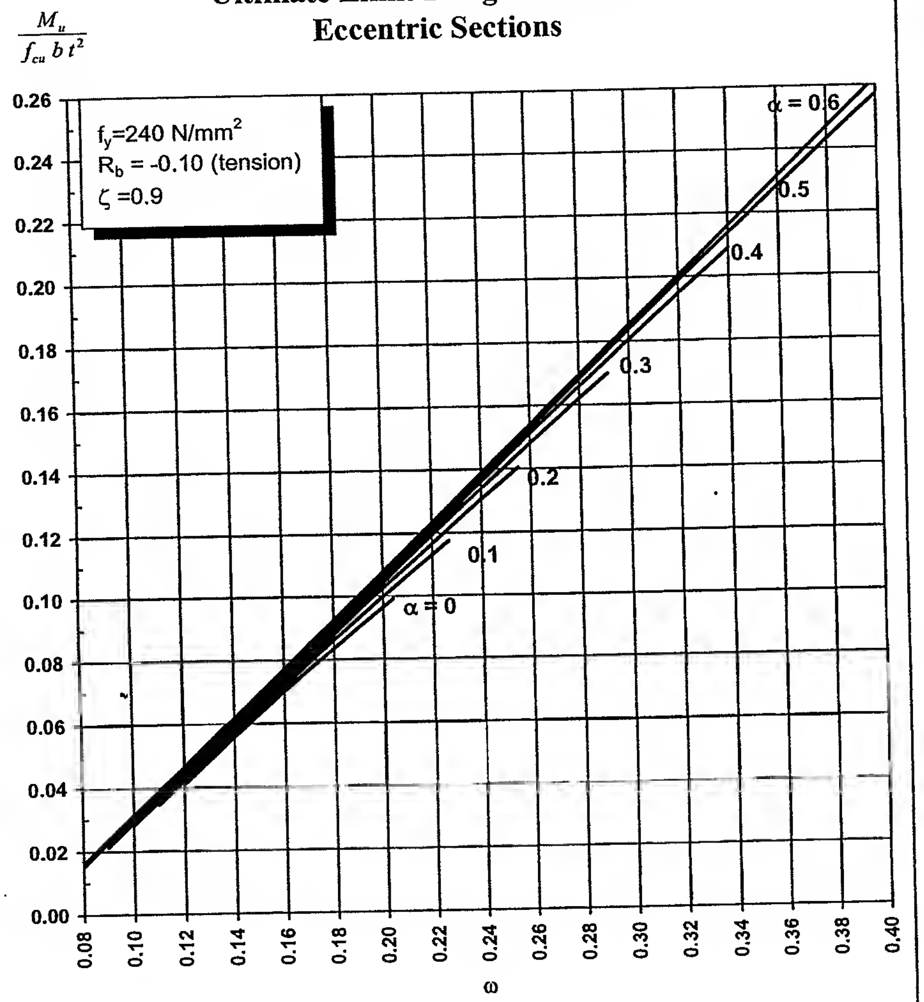
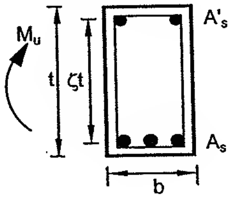
APPENDIX D

Design Charts for Sections Subjected to Eccentric Forces

Ultimate Limit Design Chart for Eccentric Sections



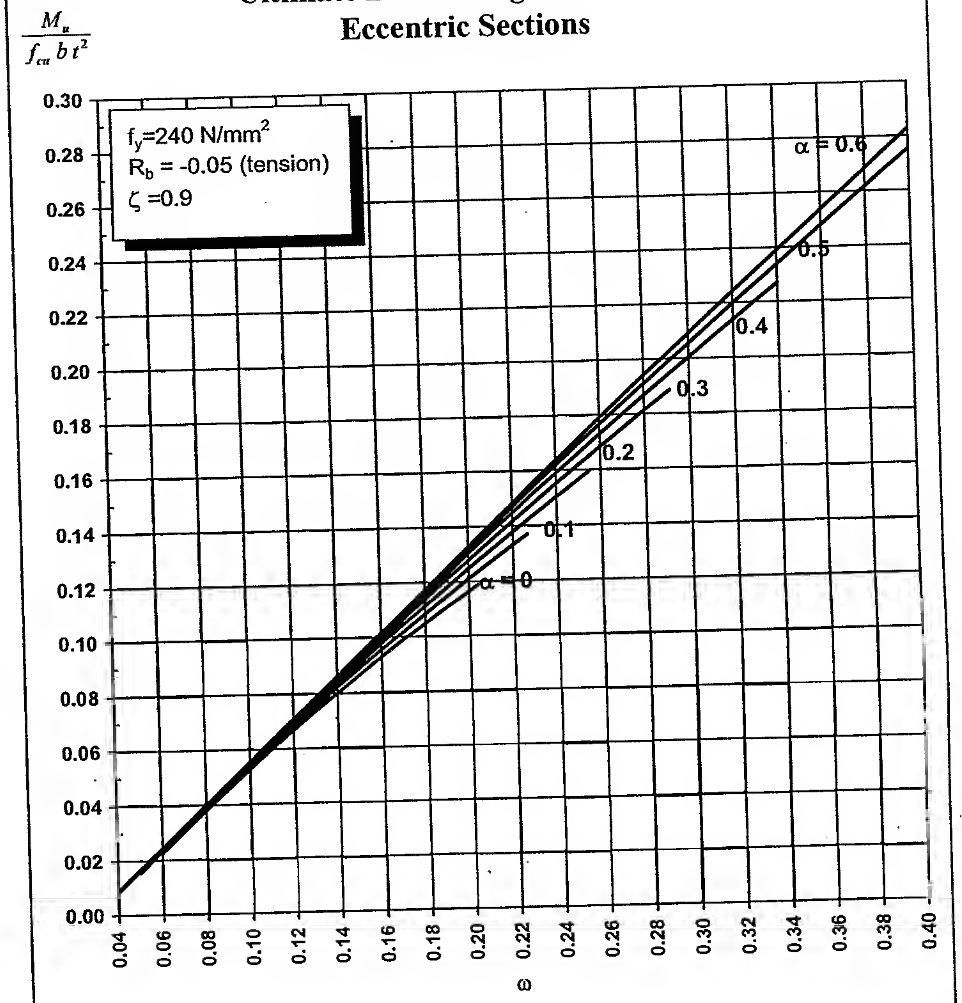


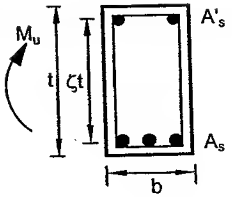
$$R_b = \frac{P_u}{f_{cu}\, b \times t}$$

$$A_s = \omega \times \frac{f_{cu}}{f_y} b \times t$$

$$A'_s = \alpha \quad A_s$$

Ultimate Limit Design Chart for Eccentric Sections





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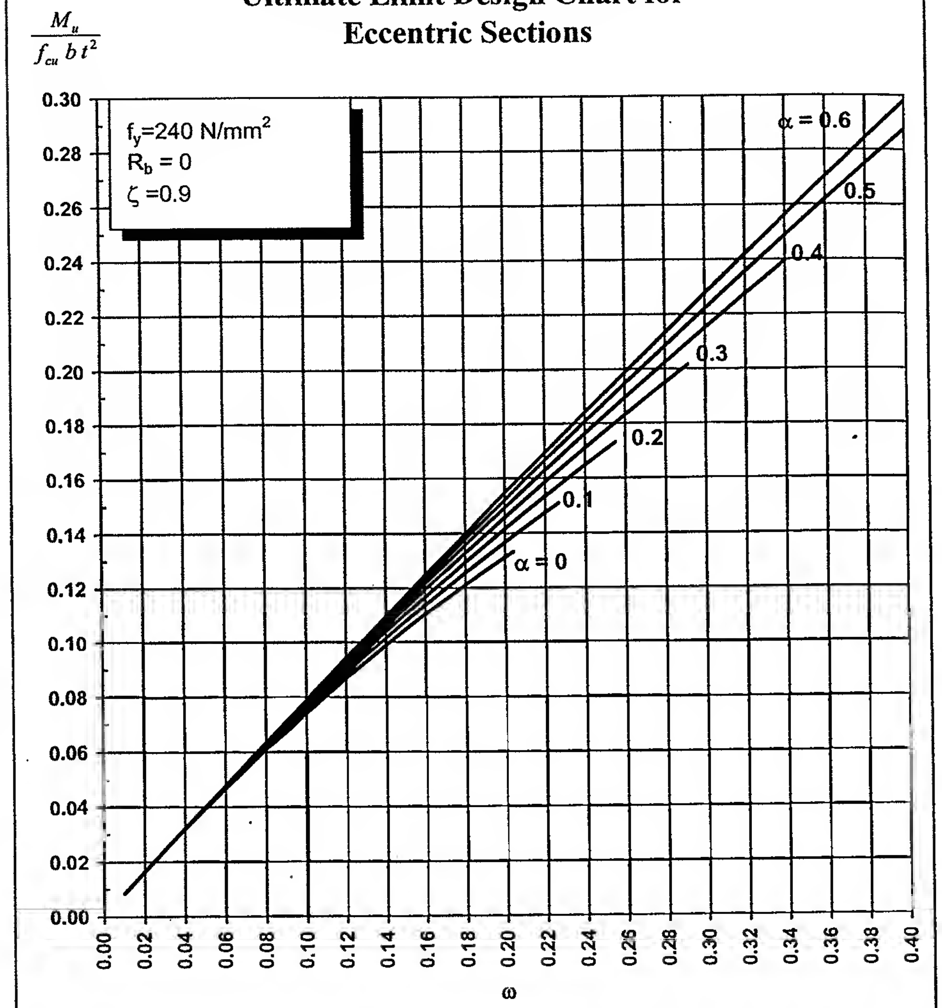
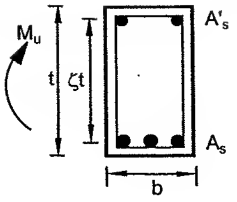
Ultimate Limit Design Chart for Eccentric Sections





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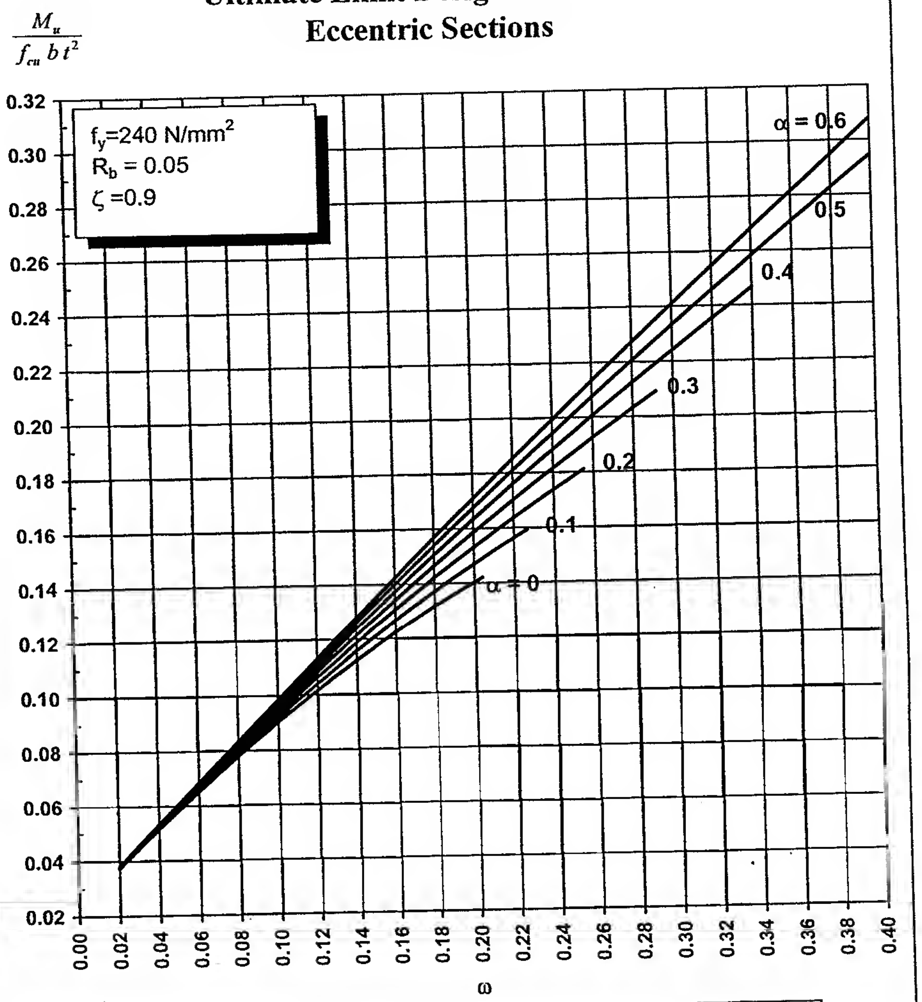

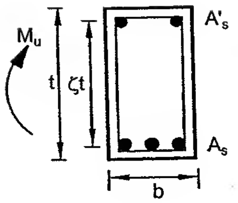

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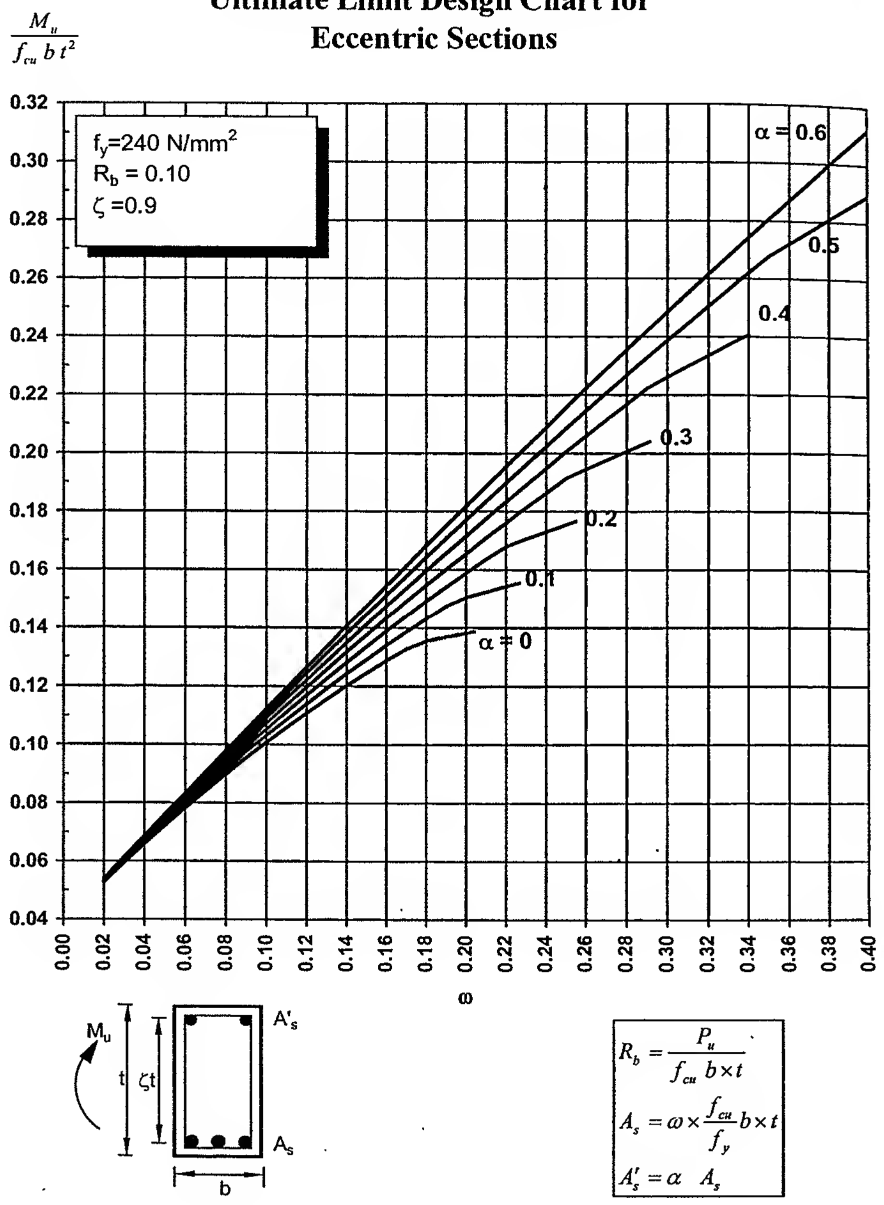

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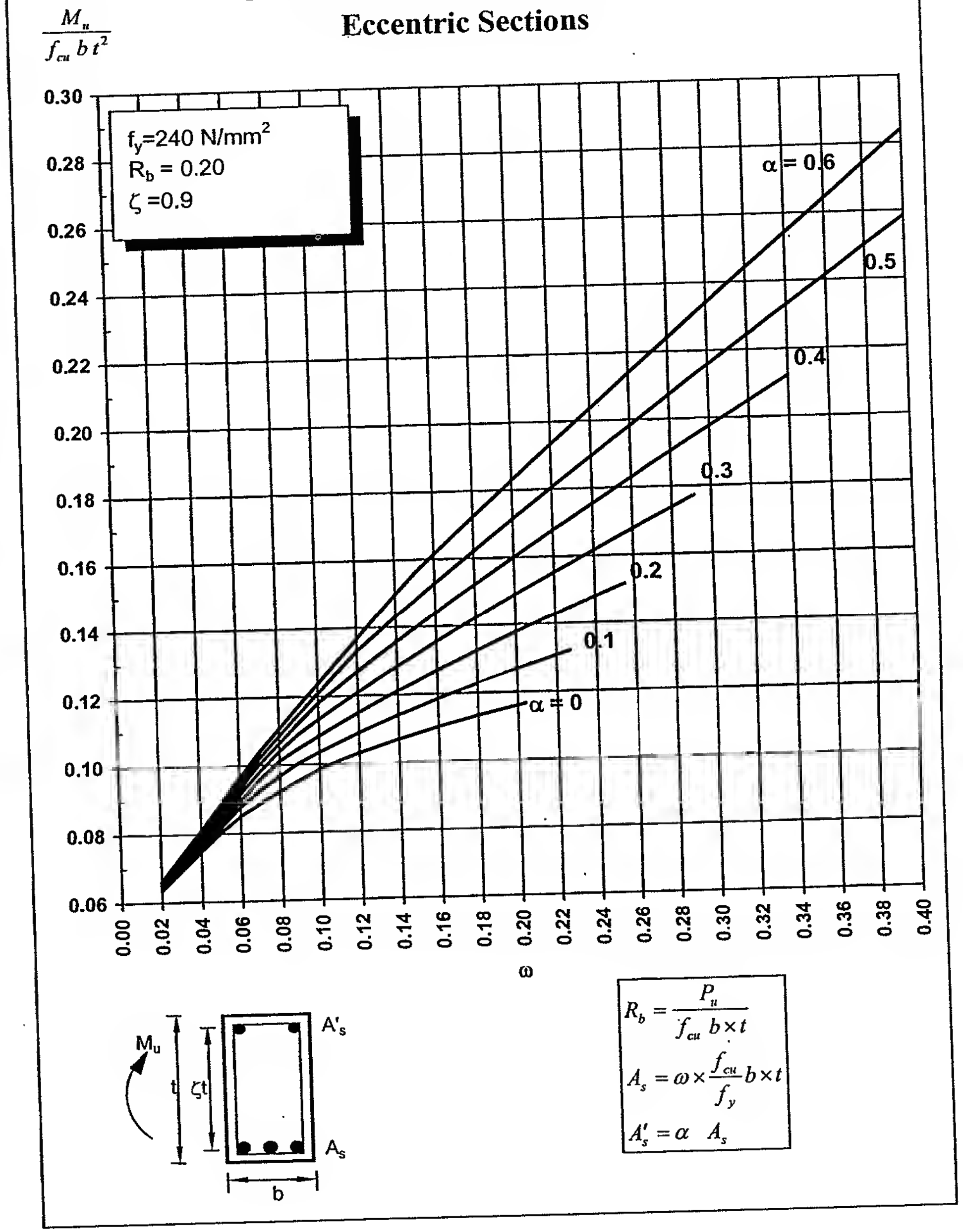

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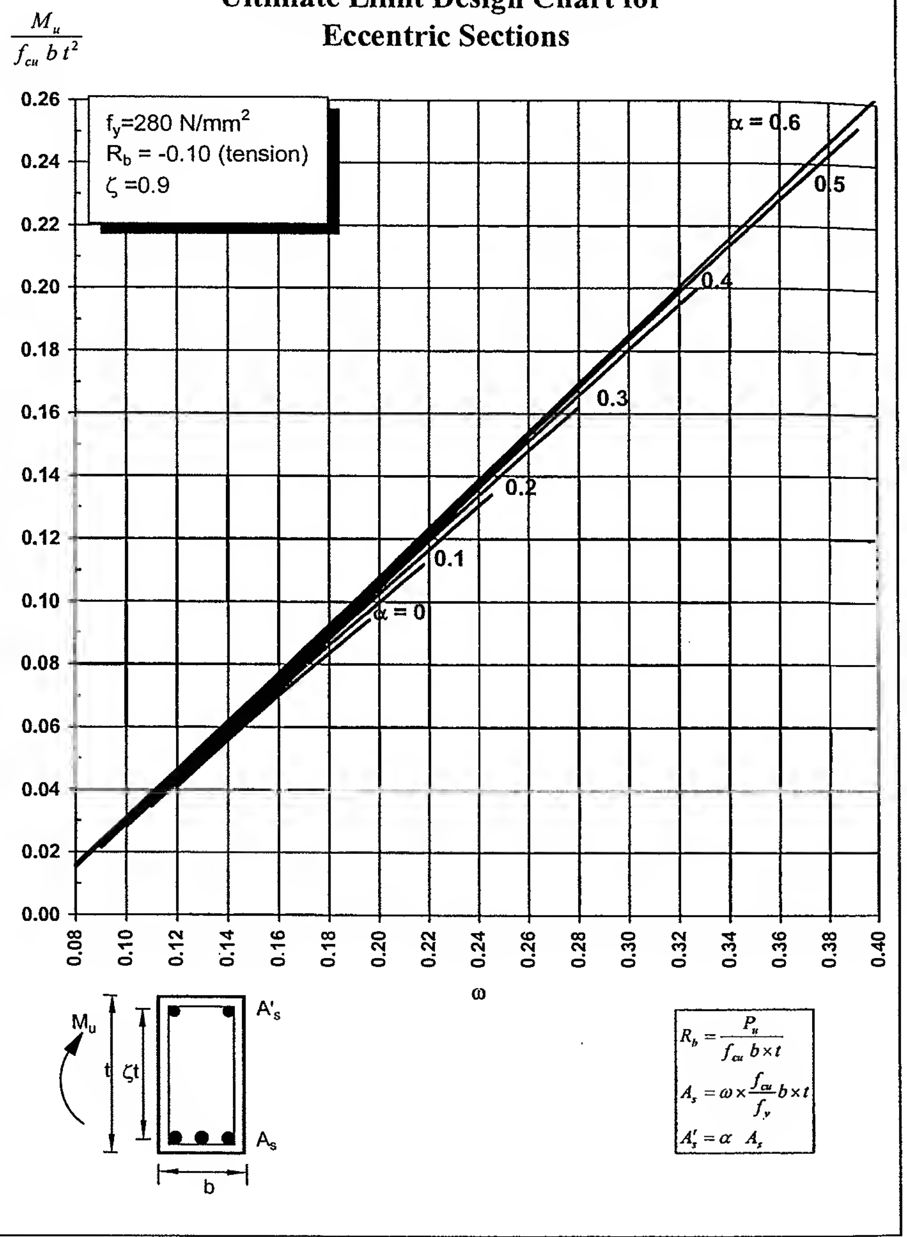

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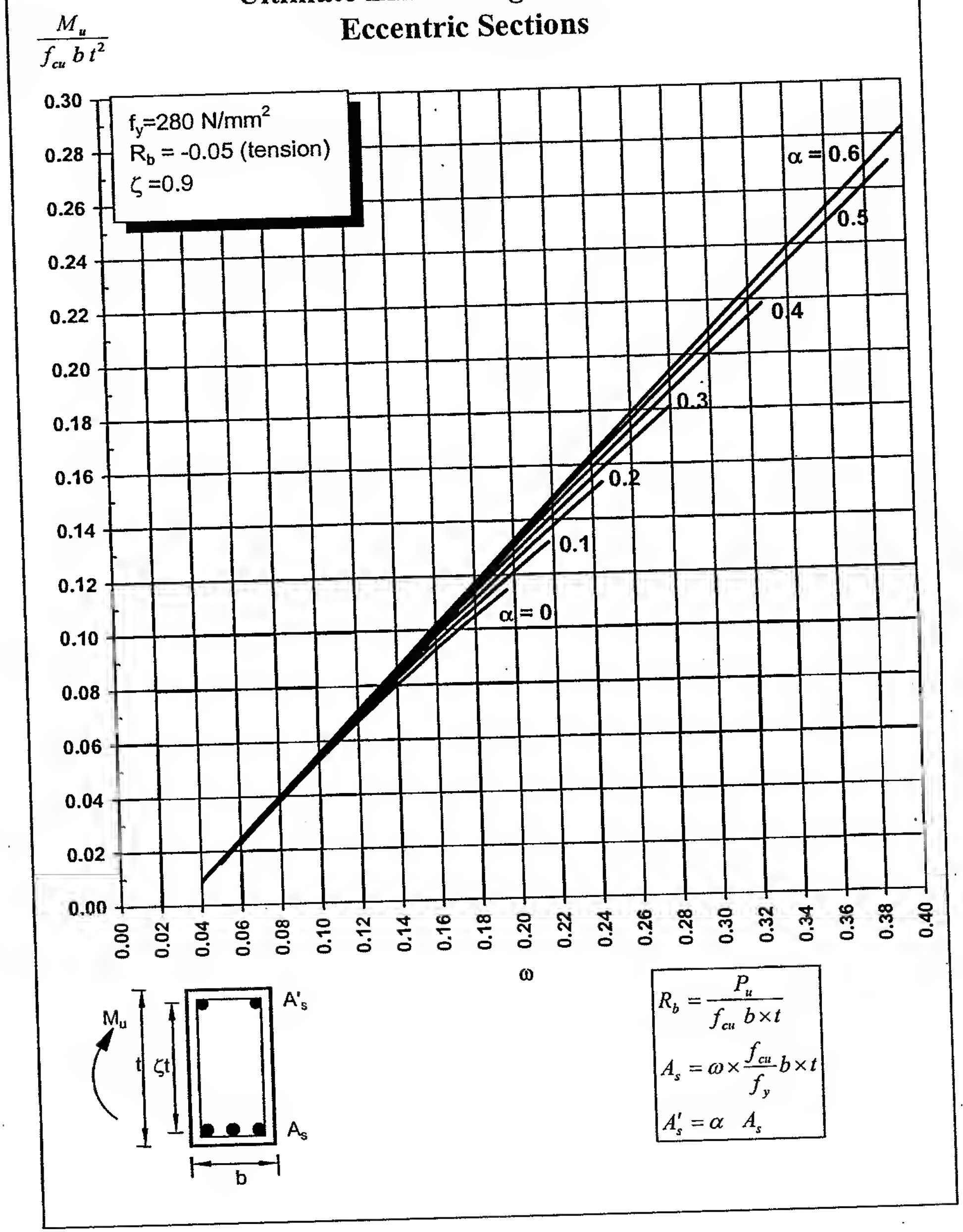

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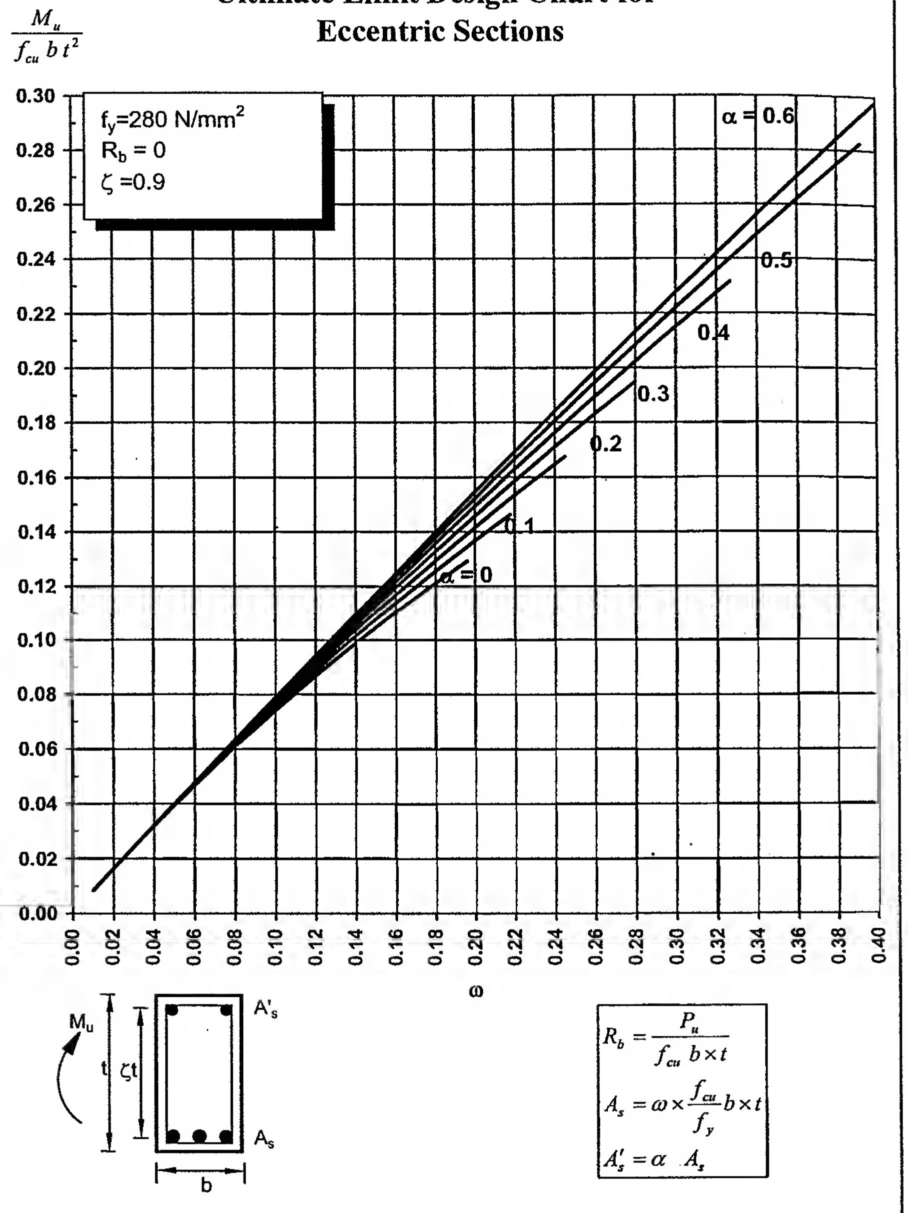

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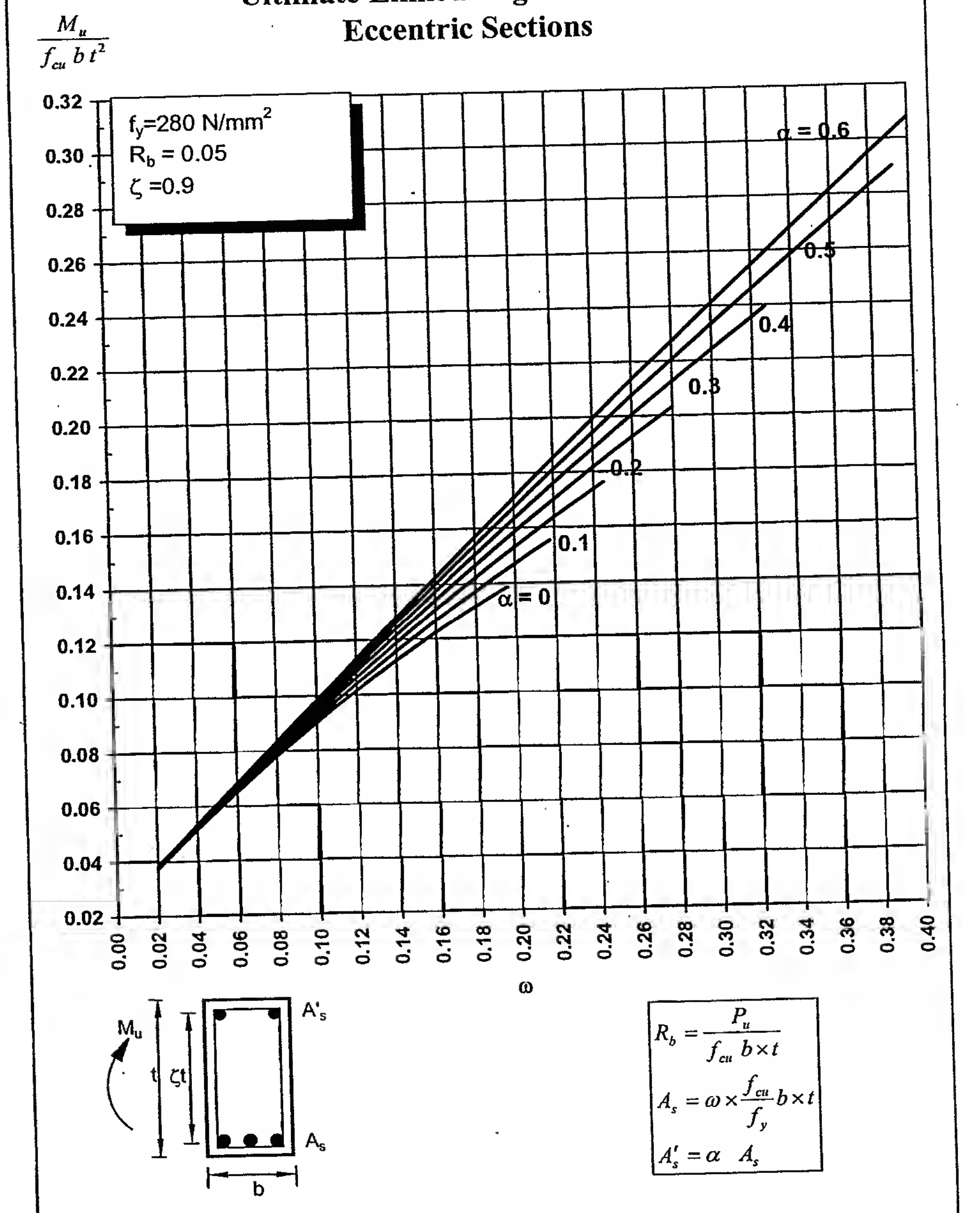

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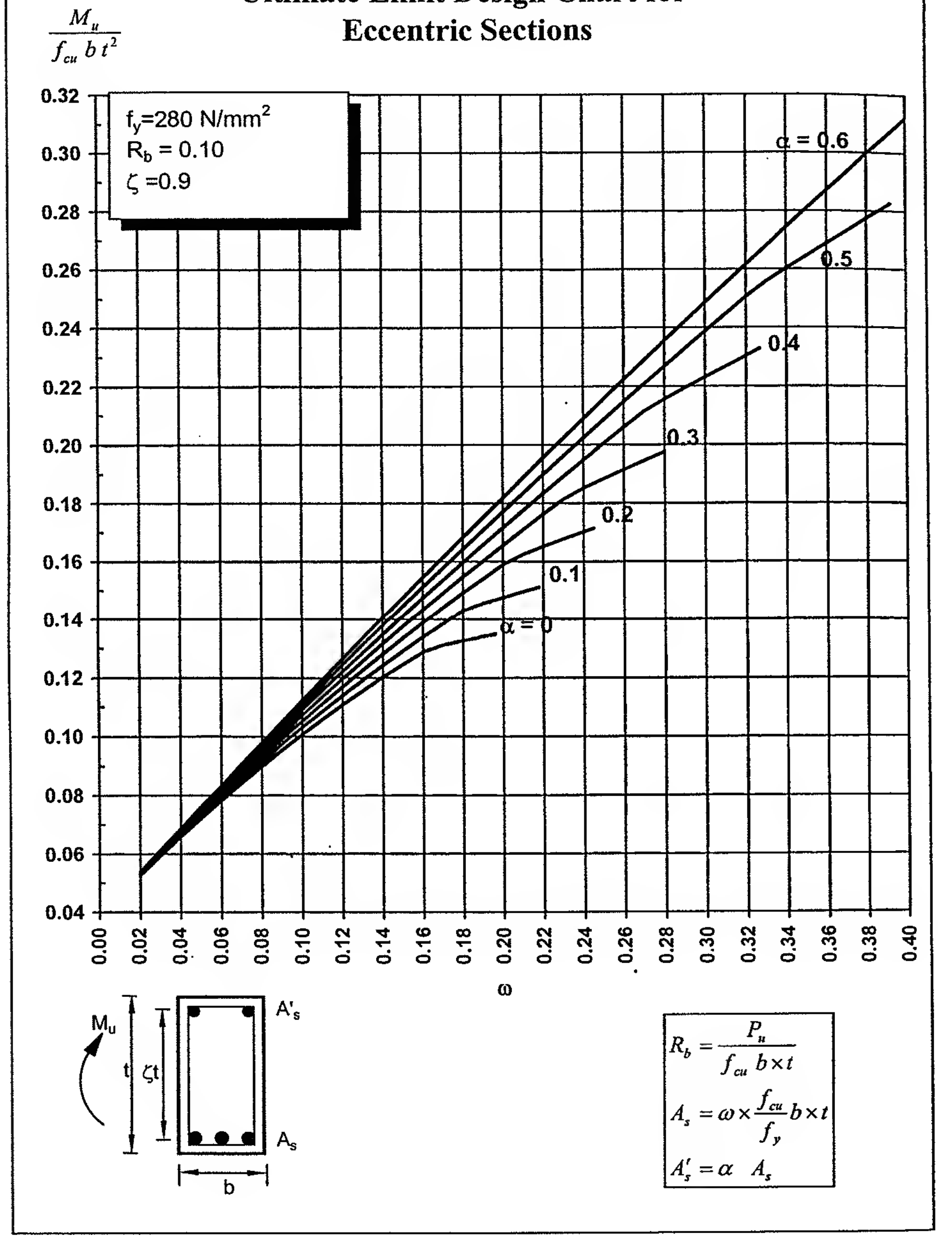

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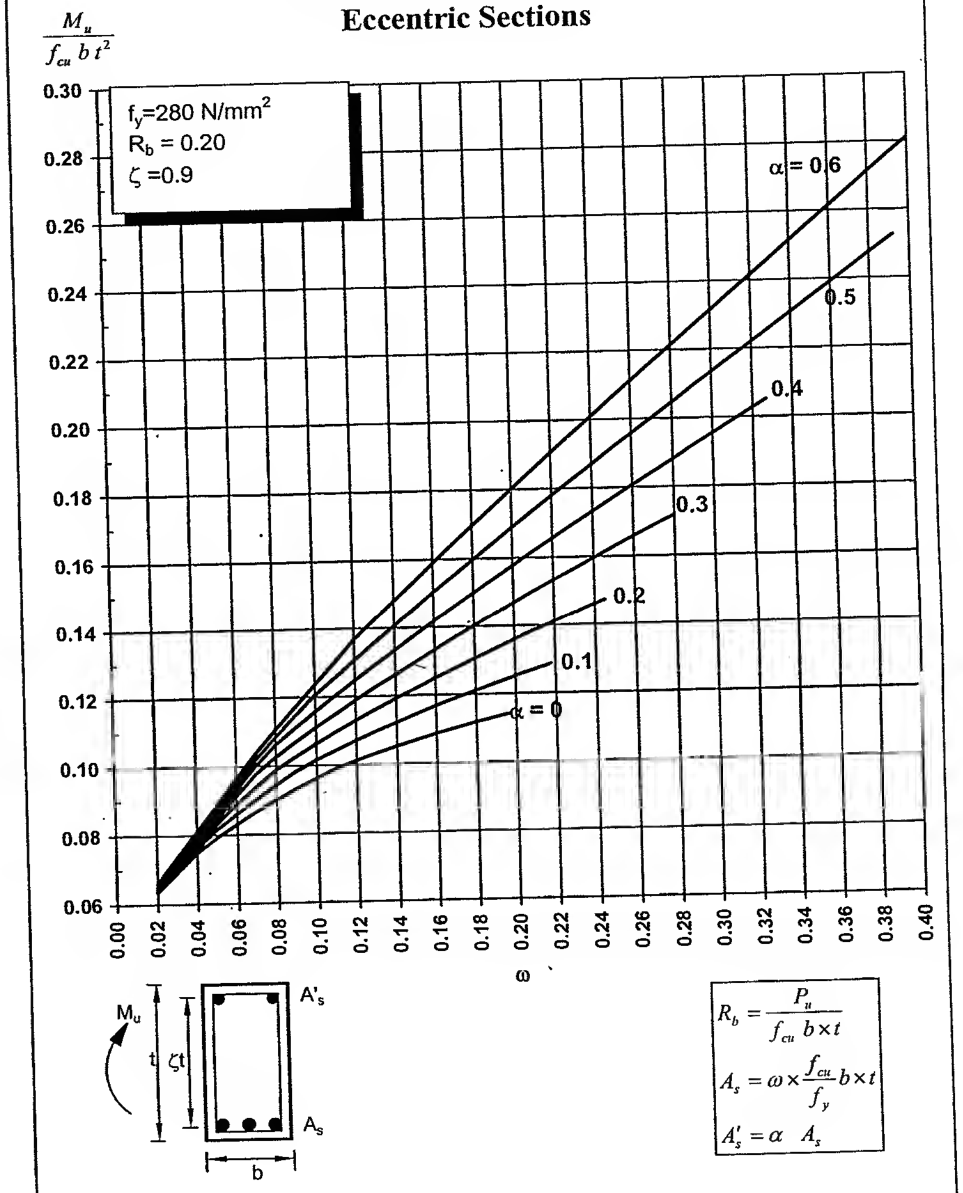

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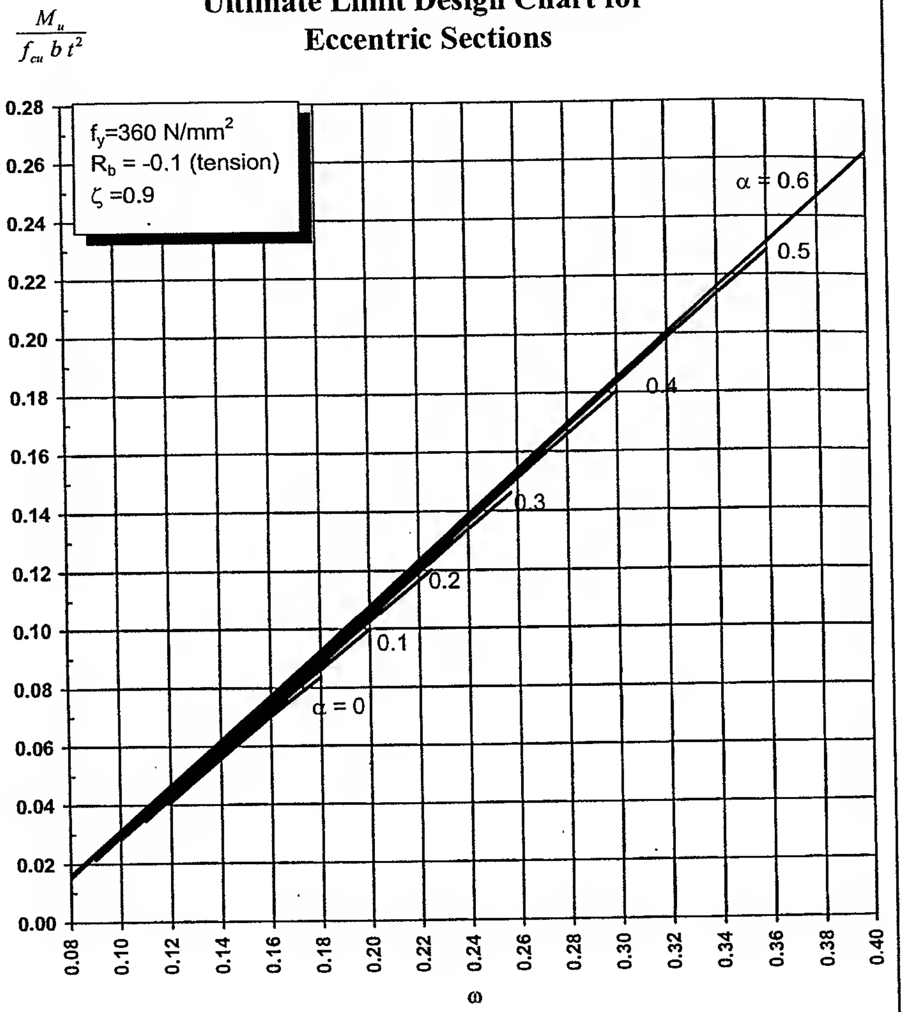

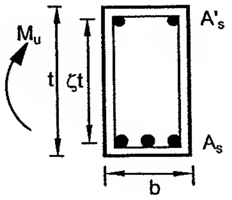

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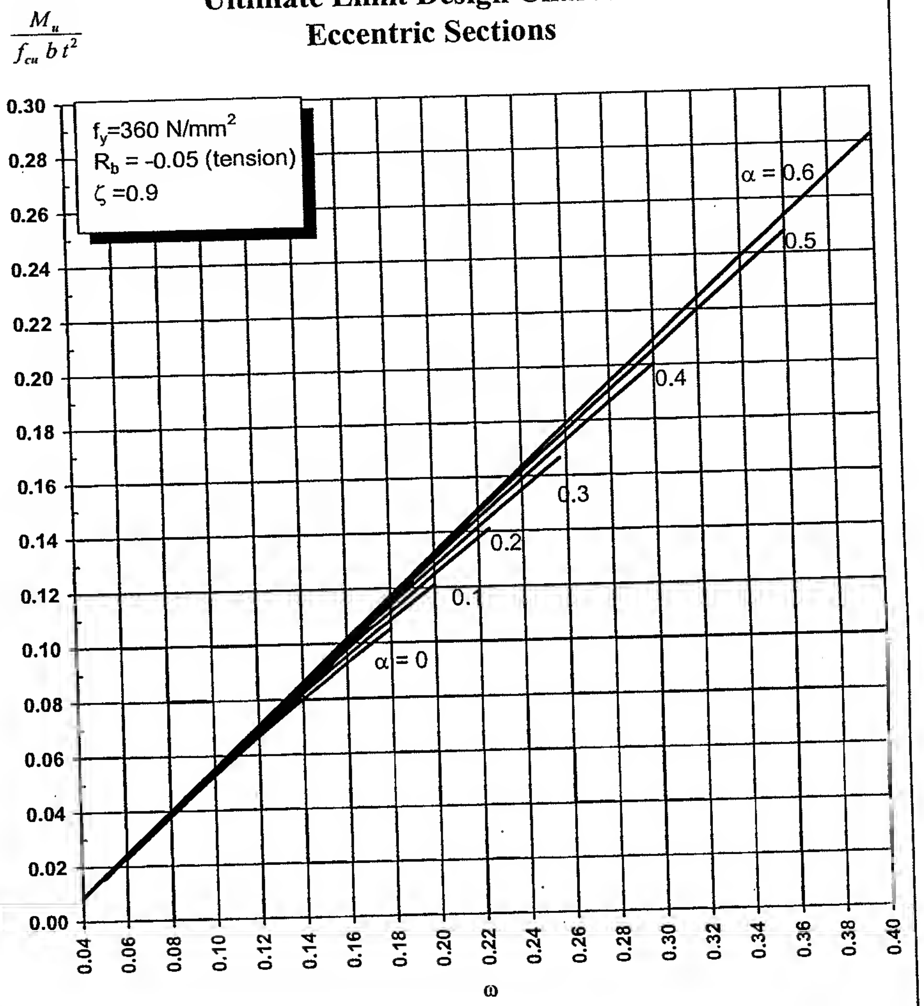

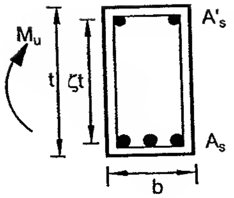

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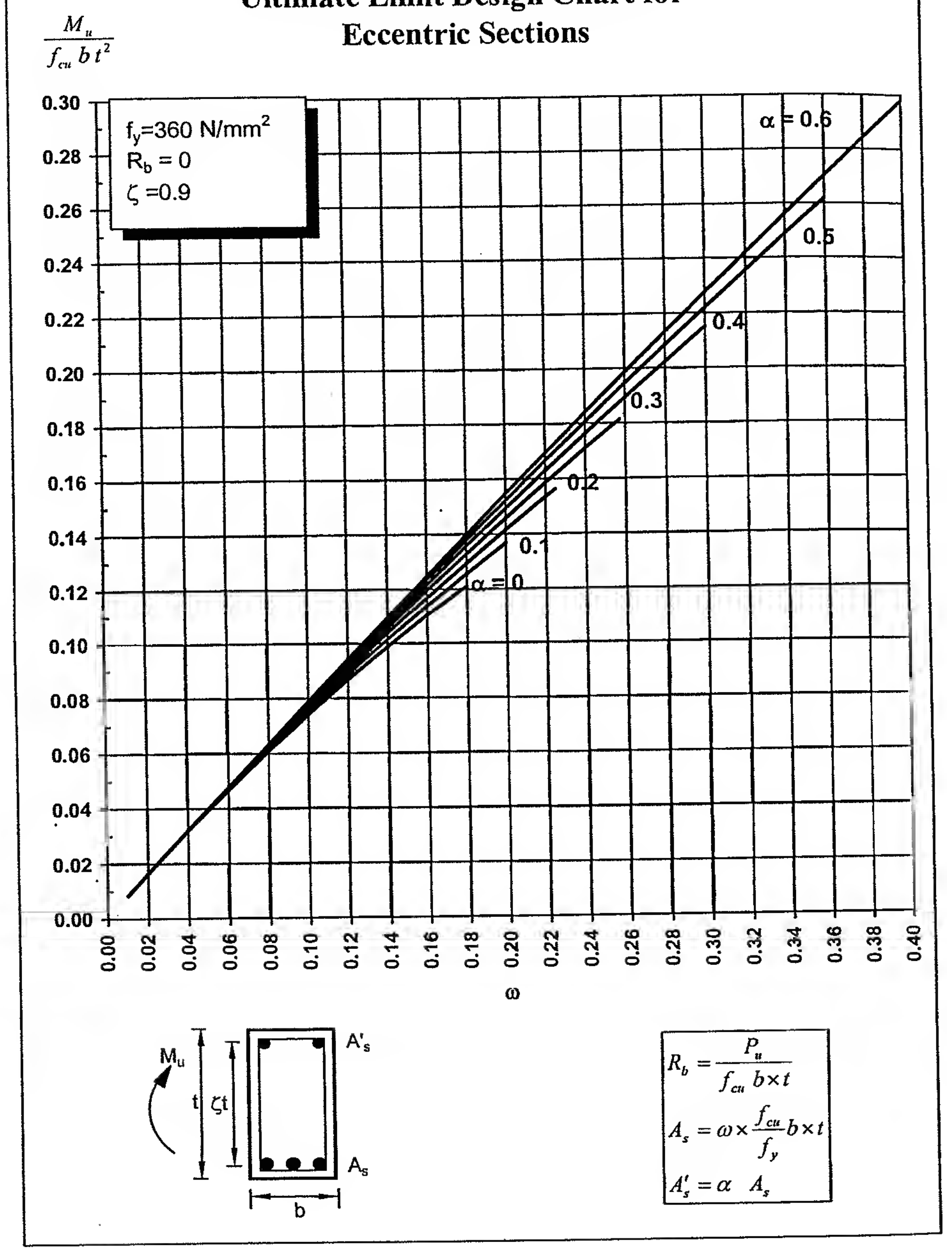

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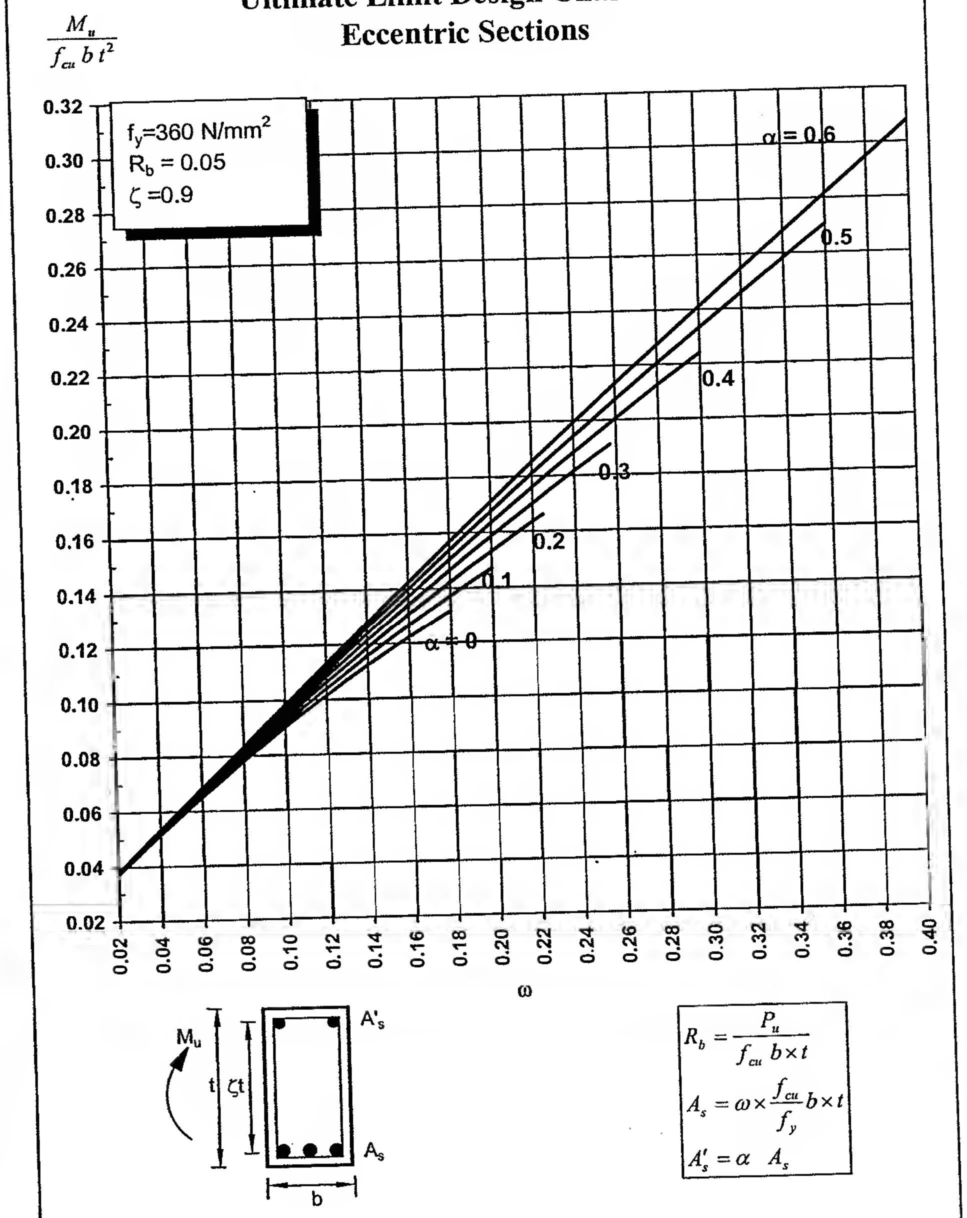

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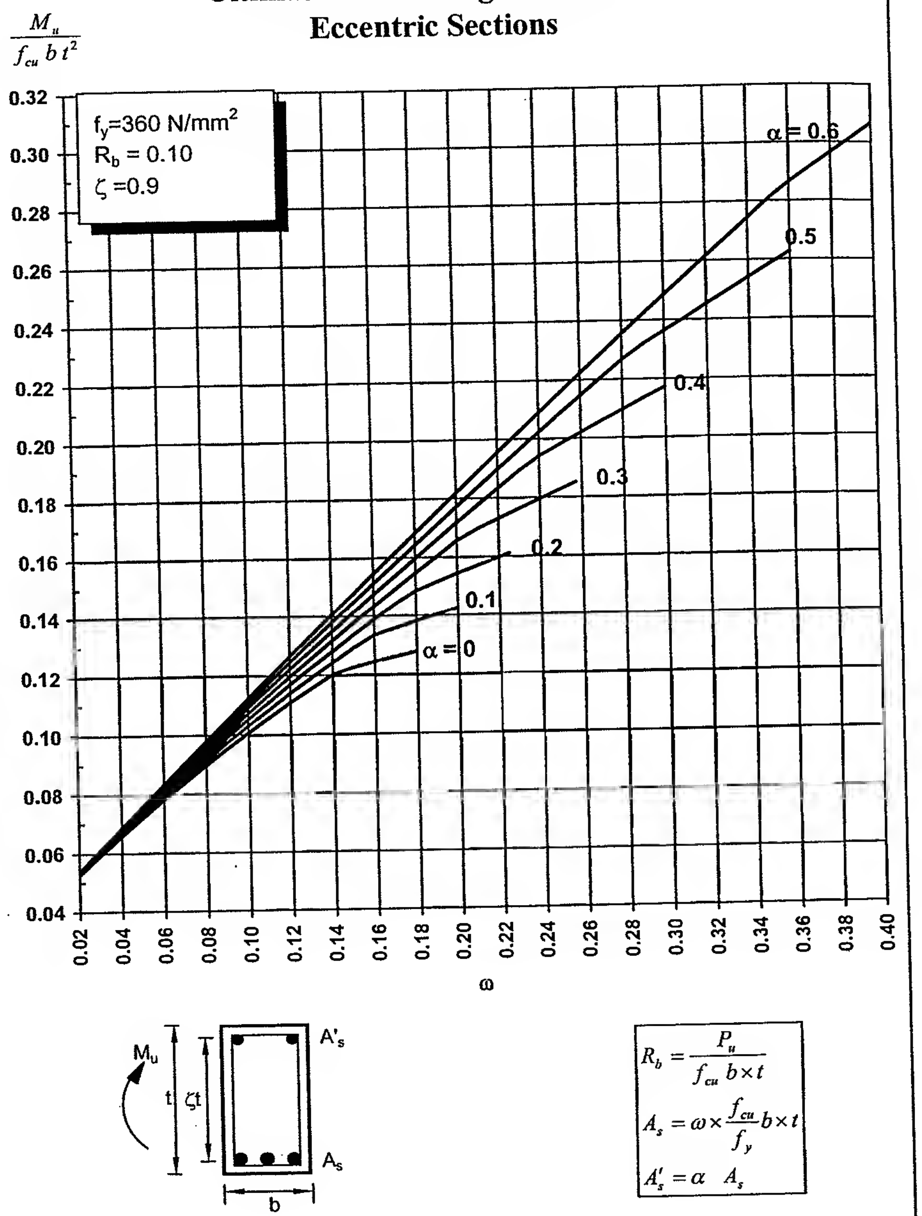

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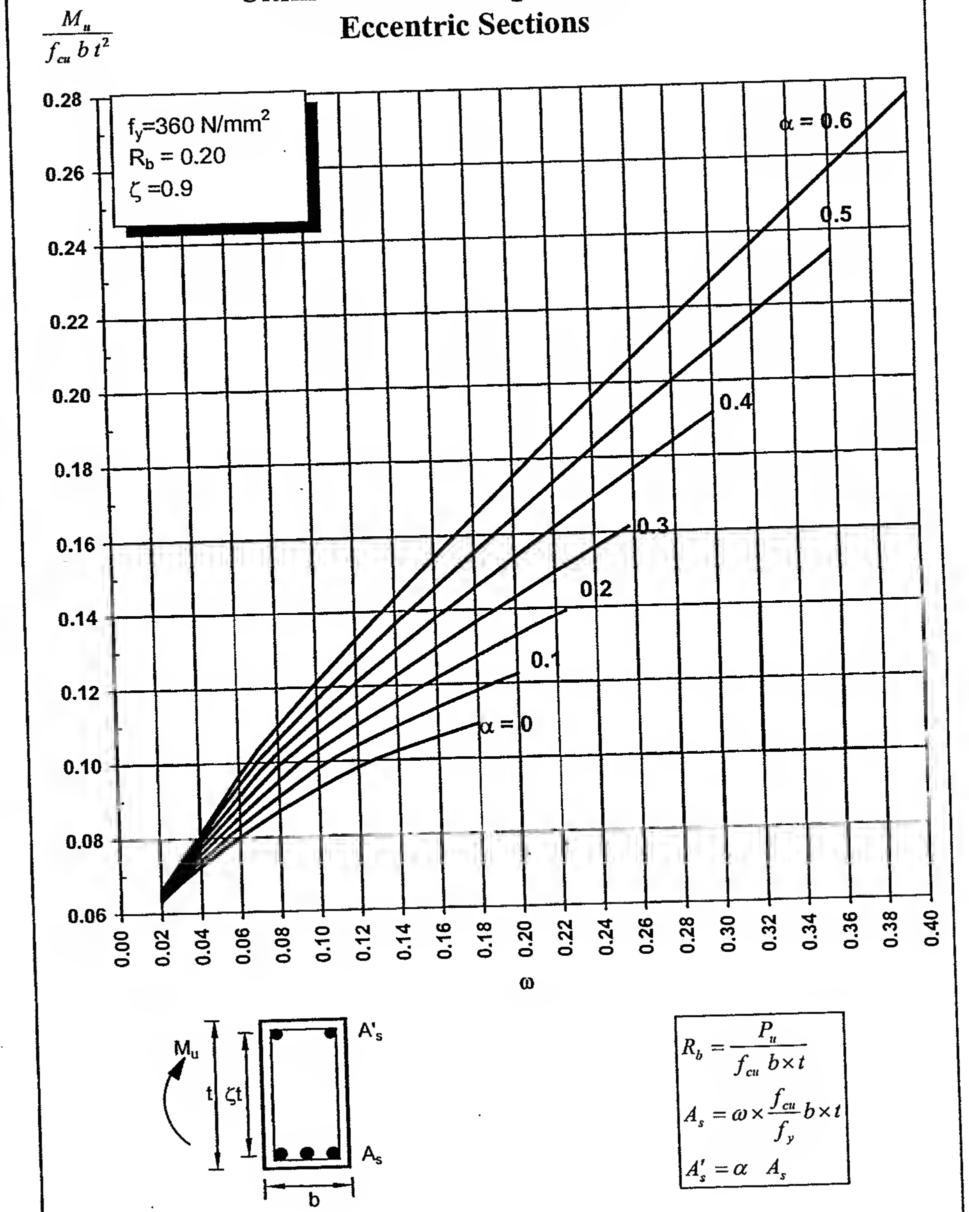

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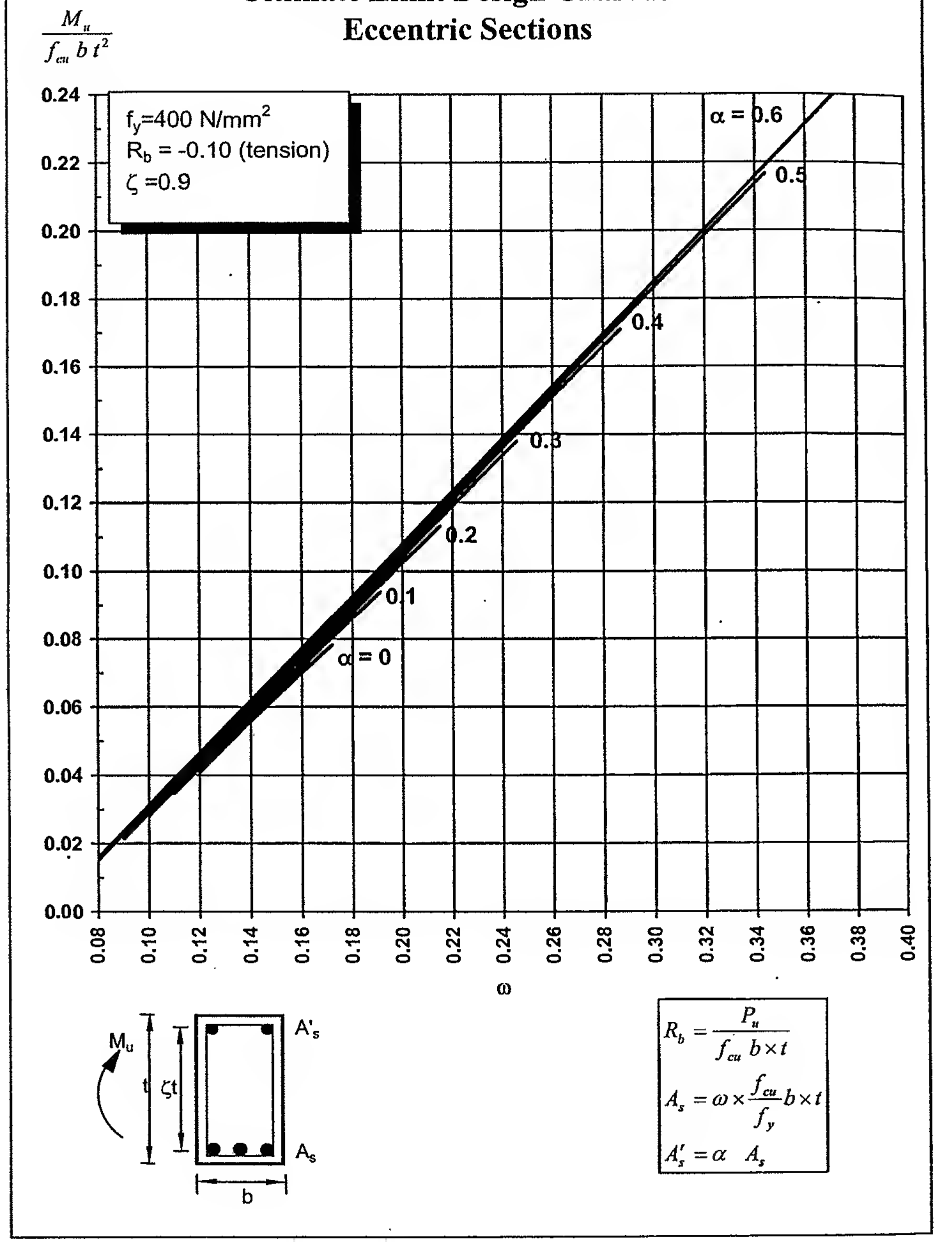
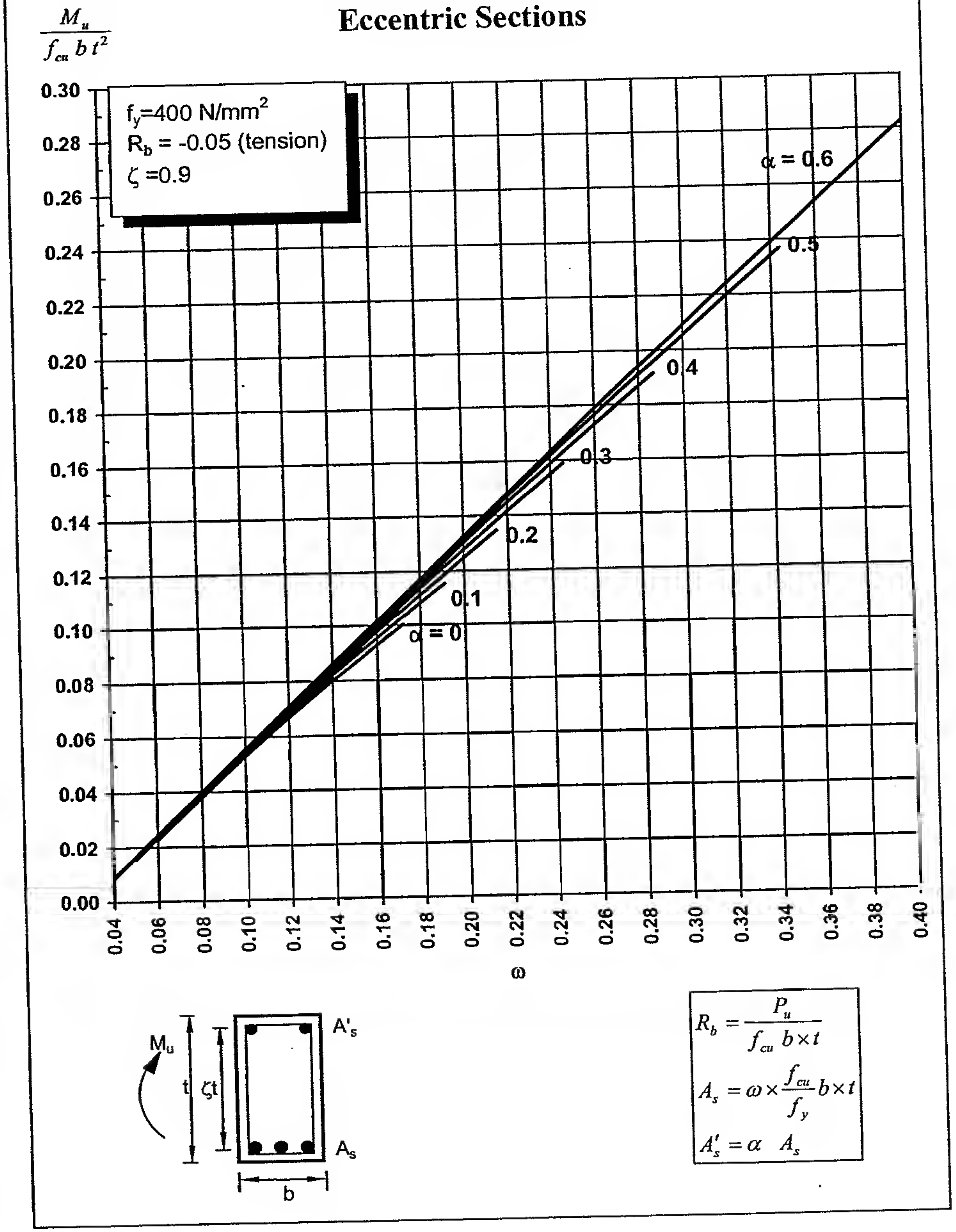
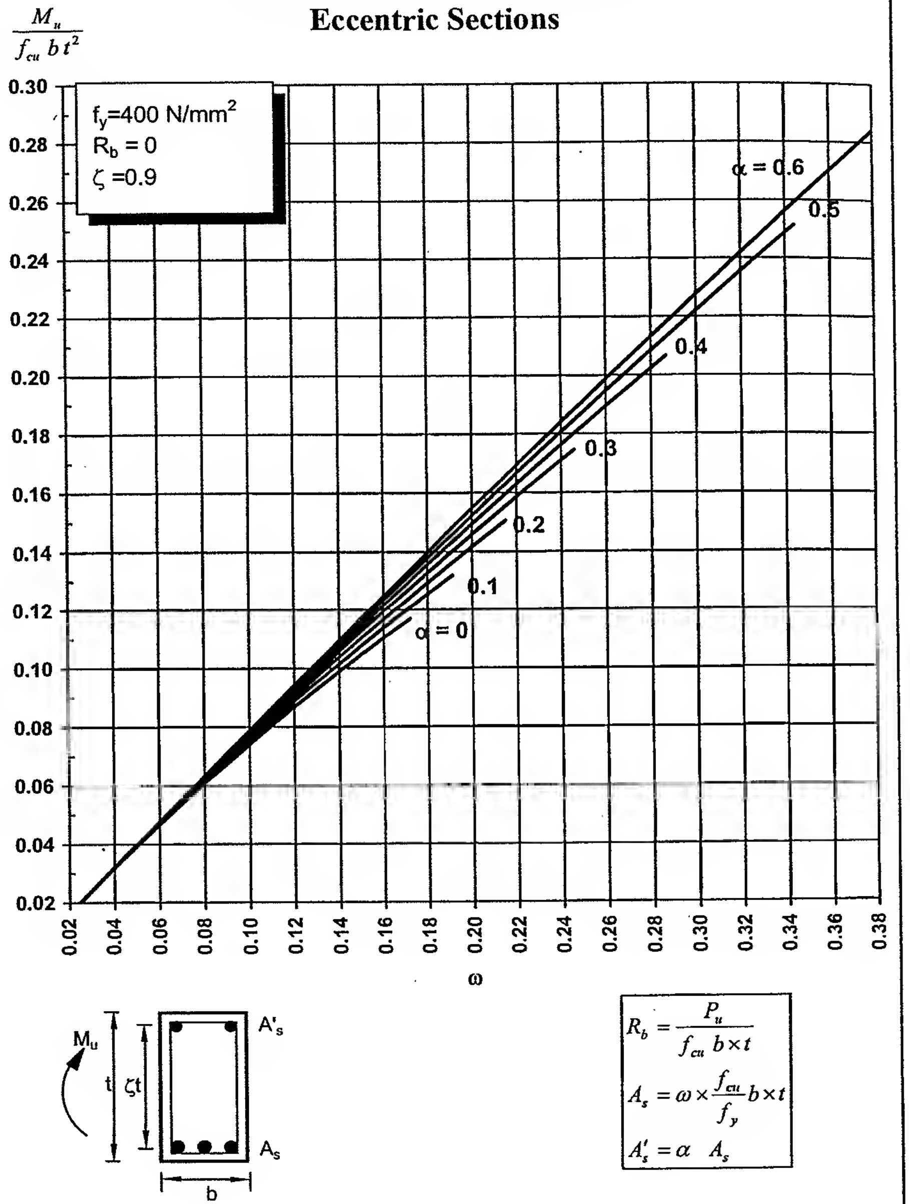


Ultimate Limit Design Chart for Eccentric Sections



Ultimate Limit Design Chart for Eccentric Sections



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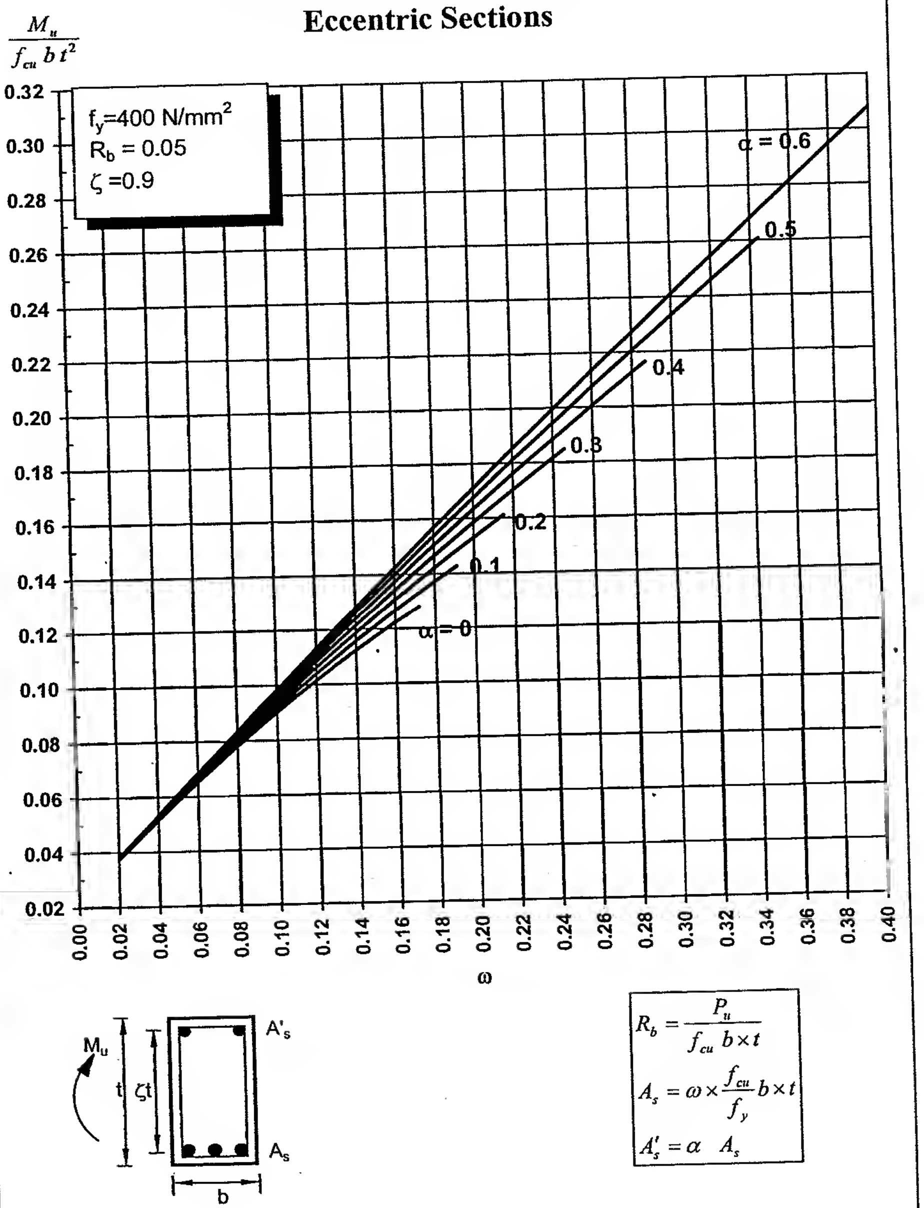

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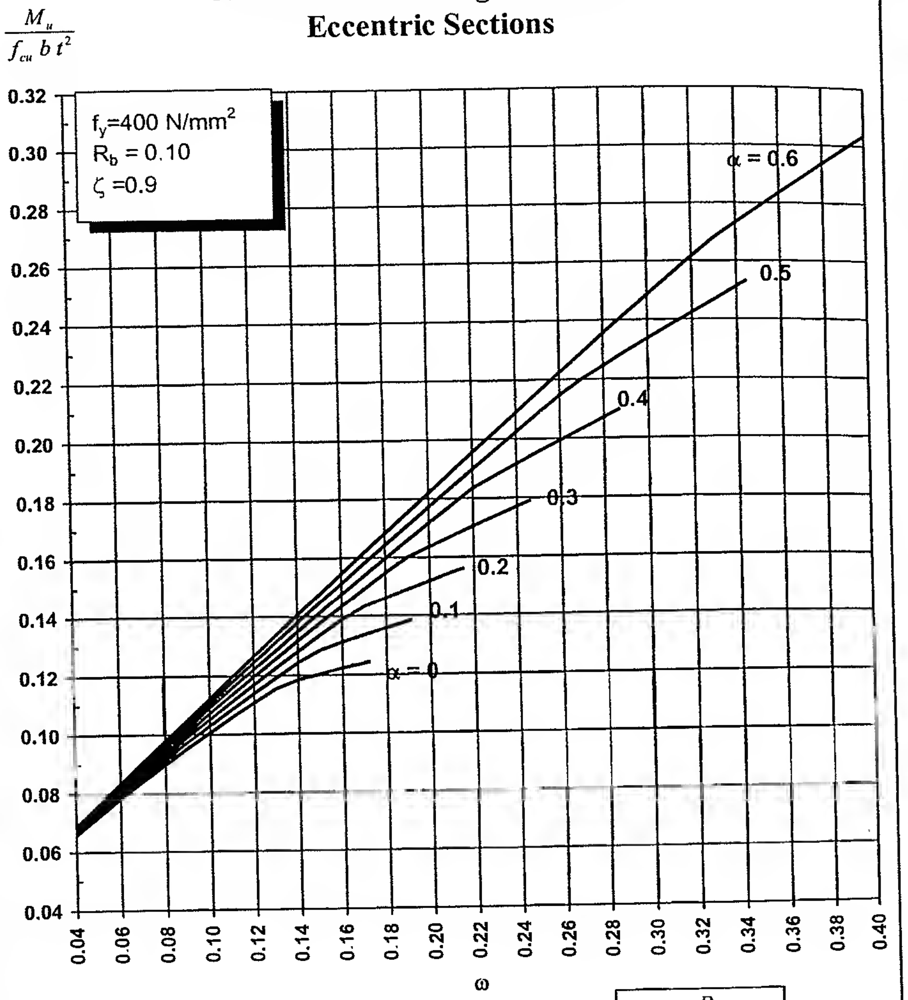

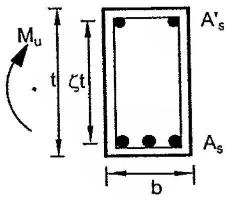

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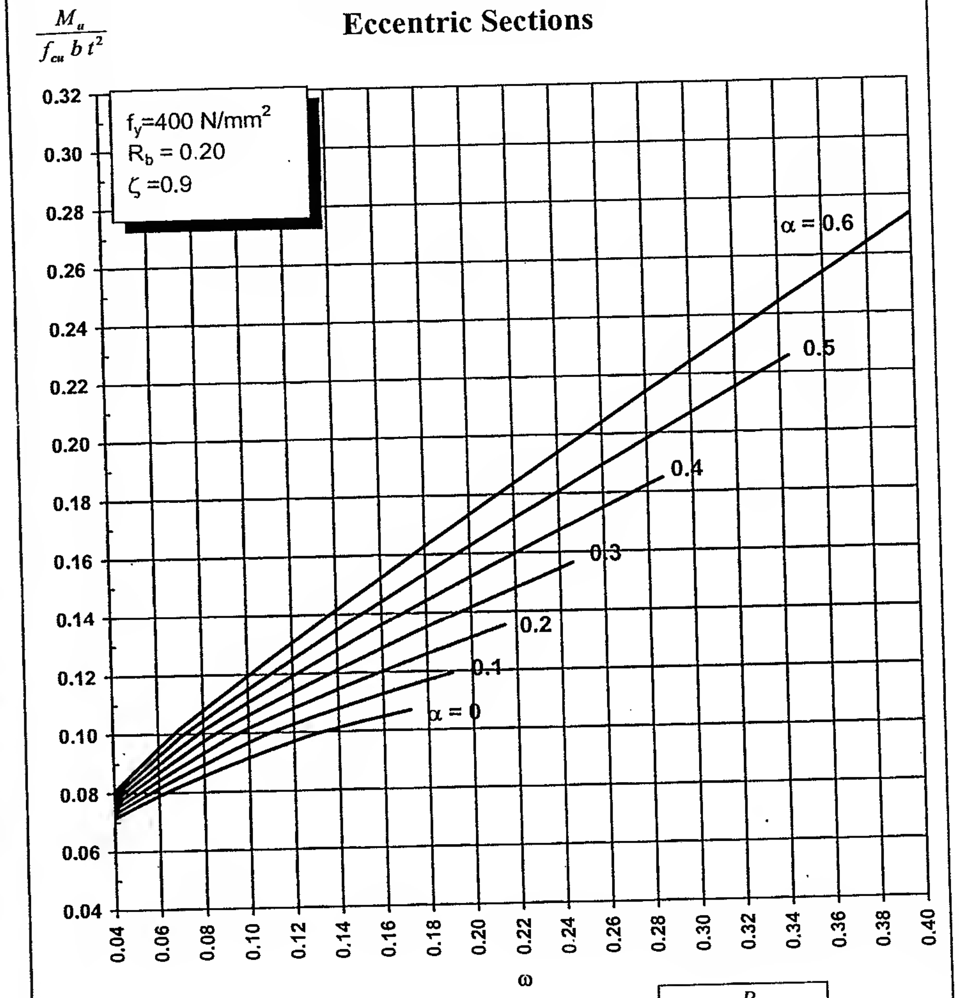

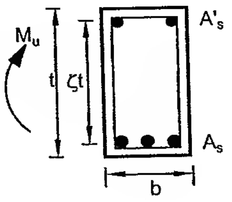

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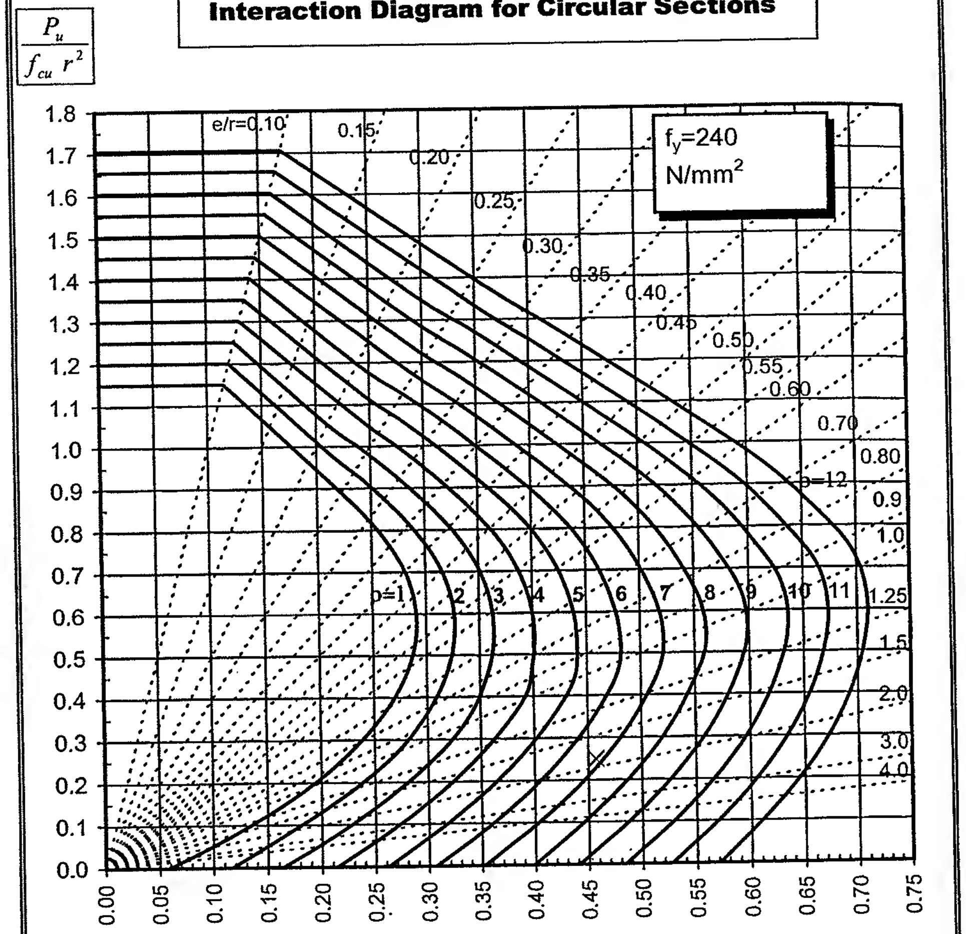
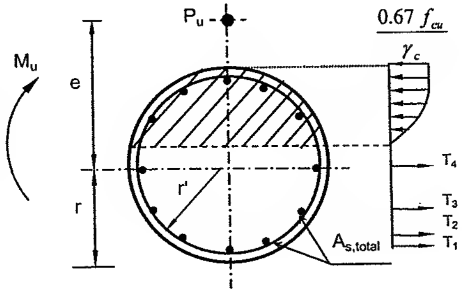
APPENDIX E

Interaction Diagrams for Circular Sections

Interaction Diagram for Circular Sections



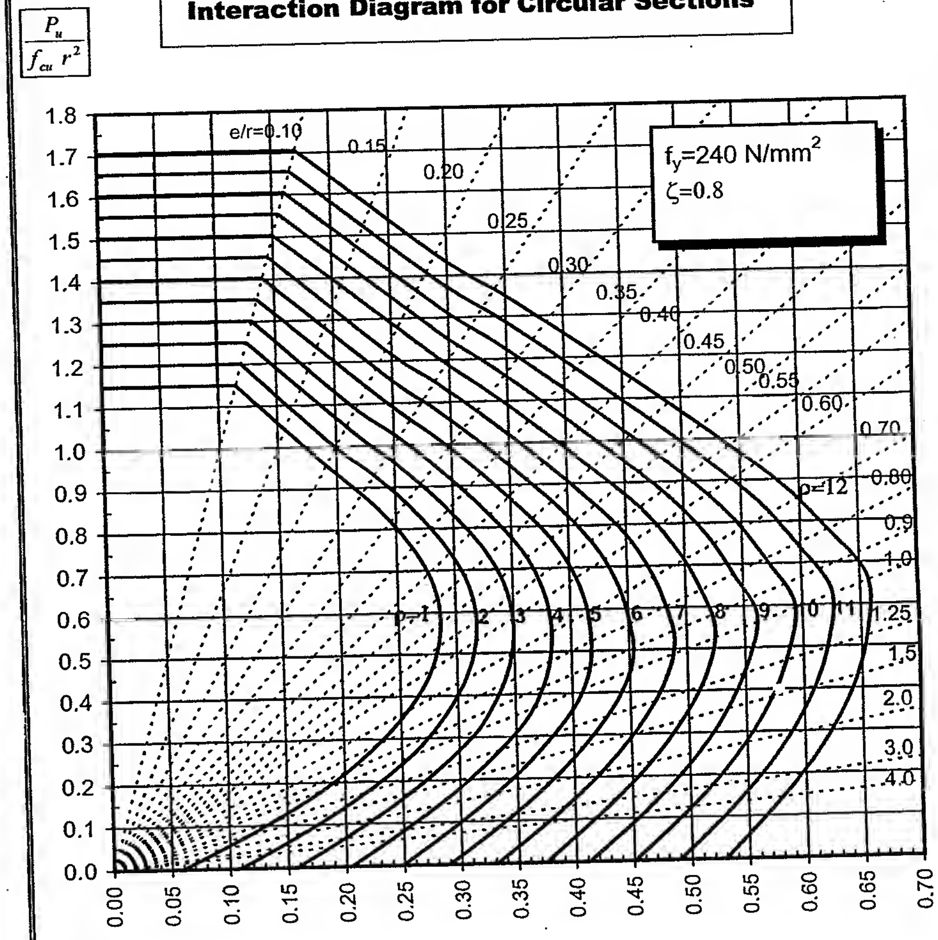


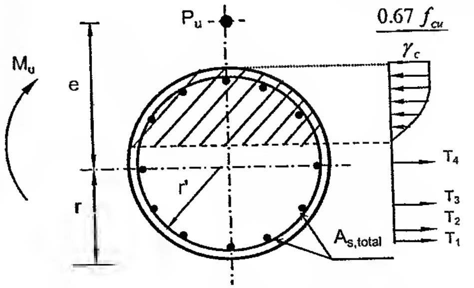
$\xi = \frac{r'}{r}$

$\mu = \rho \, f_{cu} \times 10^{-4}$

$A_{s,total} = \mu \, \pi \, r^2$

Interaction Diagram for Circular Sections





$\xi = \frac{r'}{r}$

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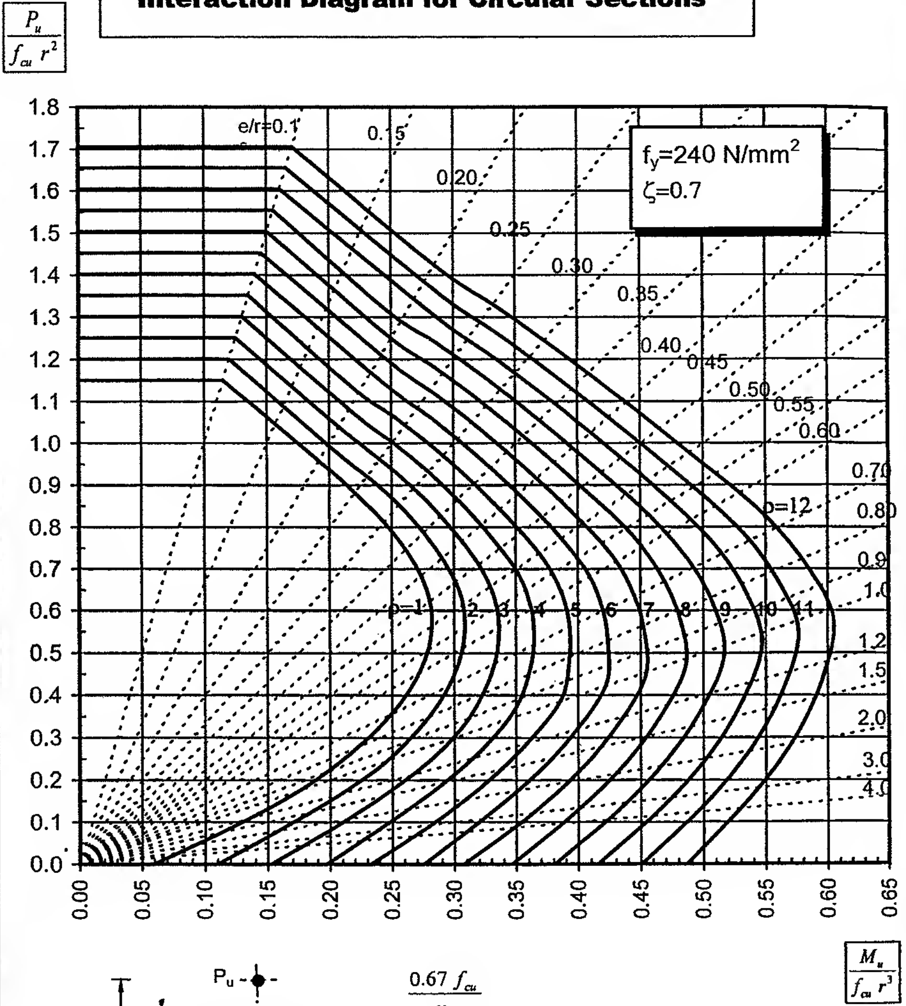
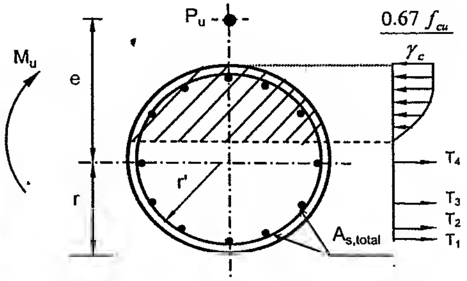


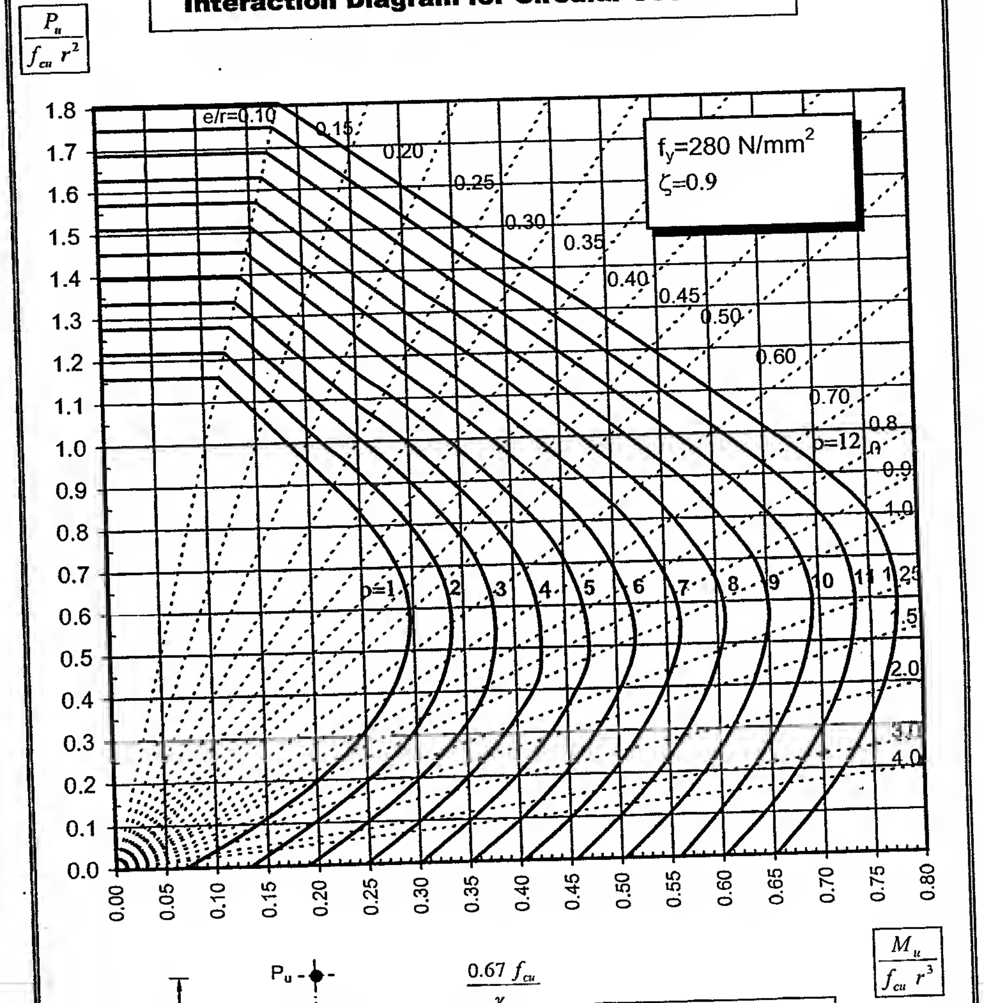


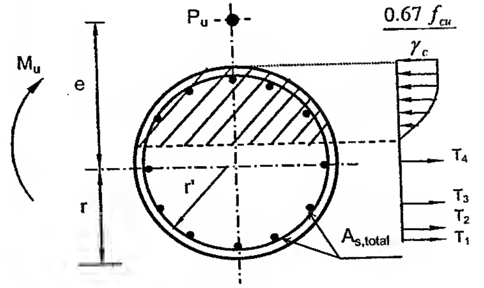
$$\xi = \frac{r'}{r}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\, \pi\, r^2$$

Interaction Diagram for Circular Sections





$$\xi = \frac{r'}{r}$$

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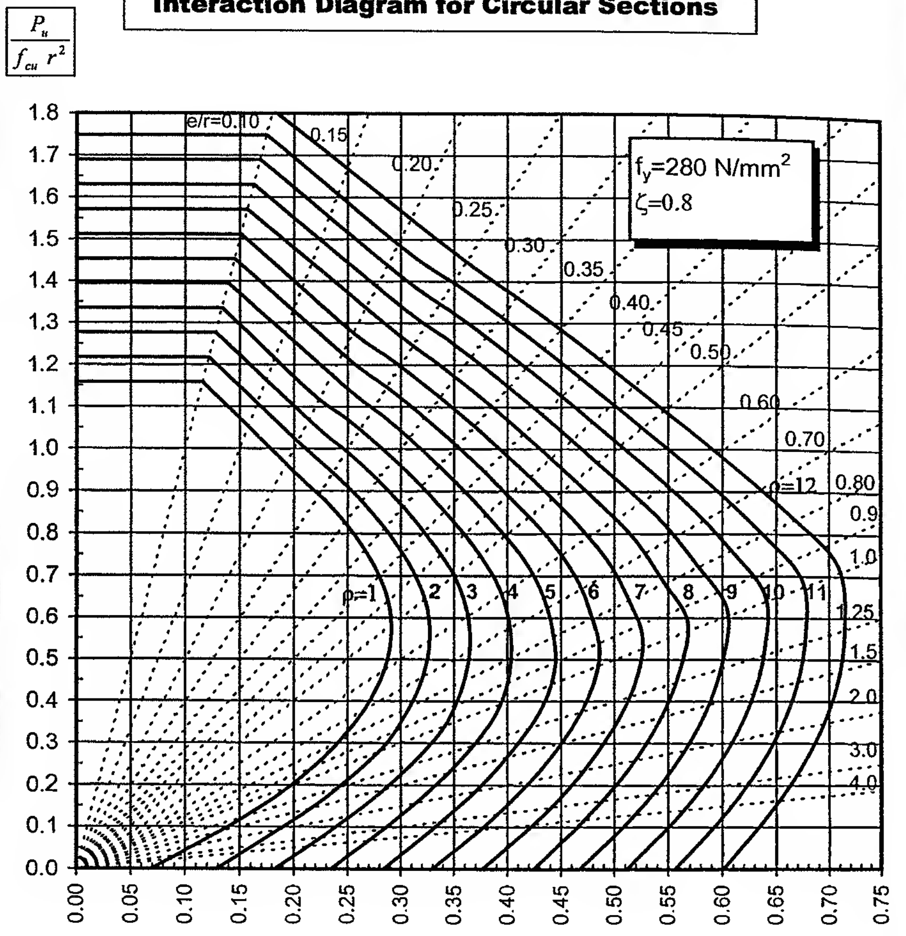
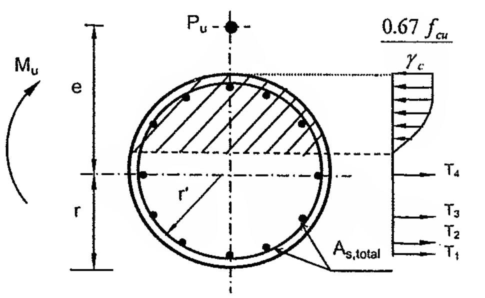
$$A_{s,total} = \mu\, \pi\, r^2$$

Interaction Diagram for Circular Sections





$$\xi = \frac{r'}{r}$$

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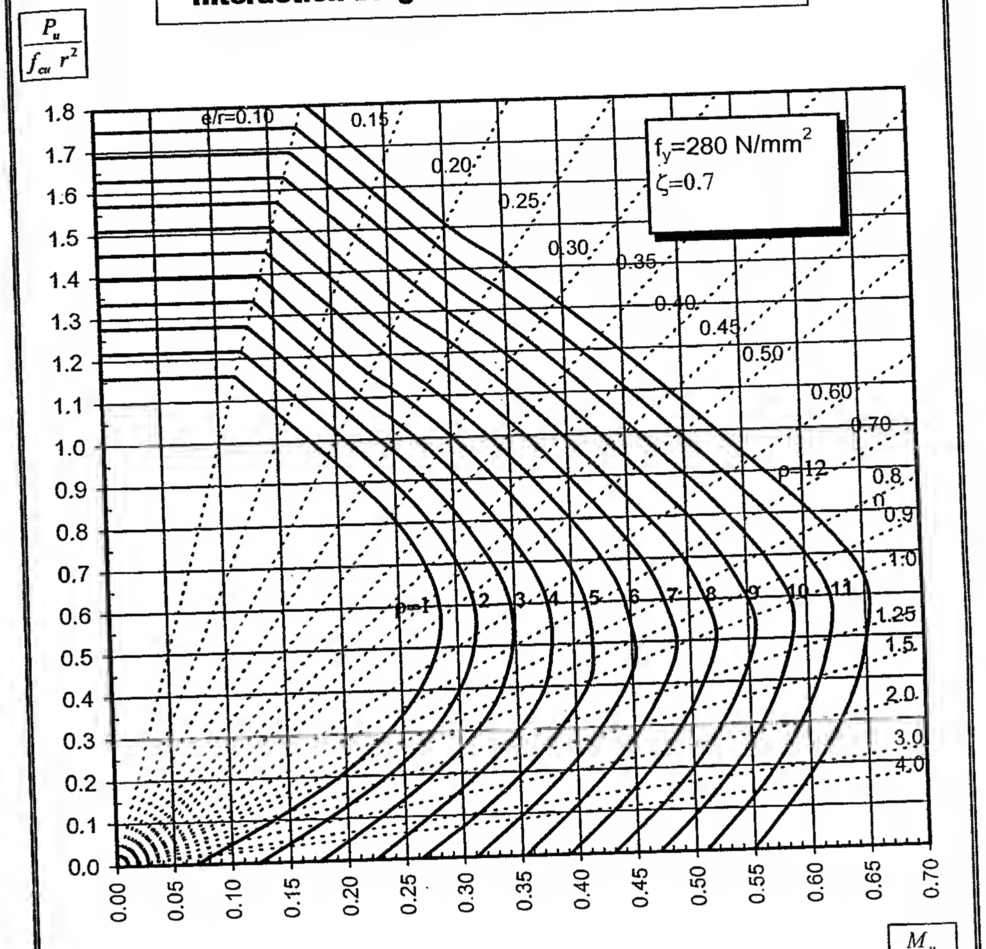
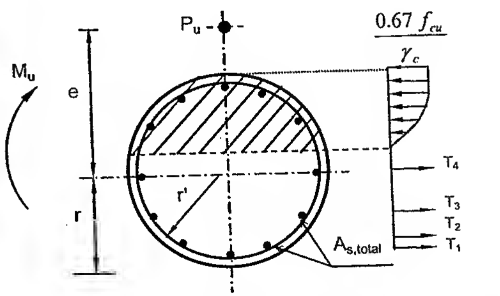
Interaction Diagram for Circular Sections



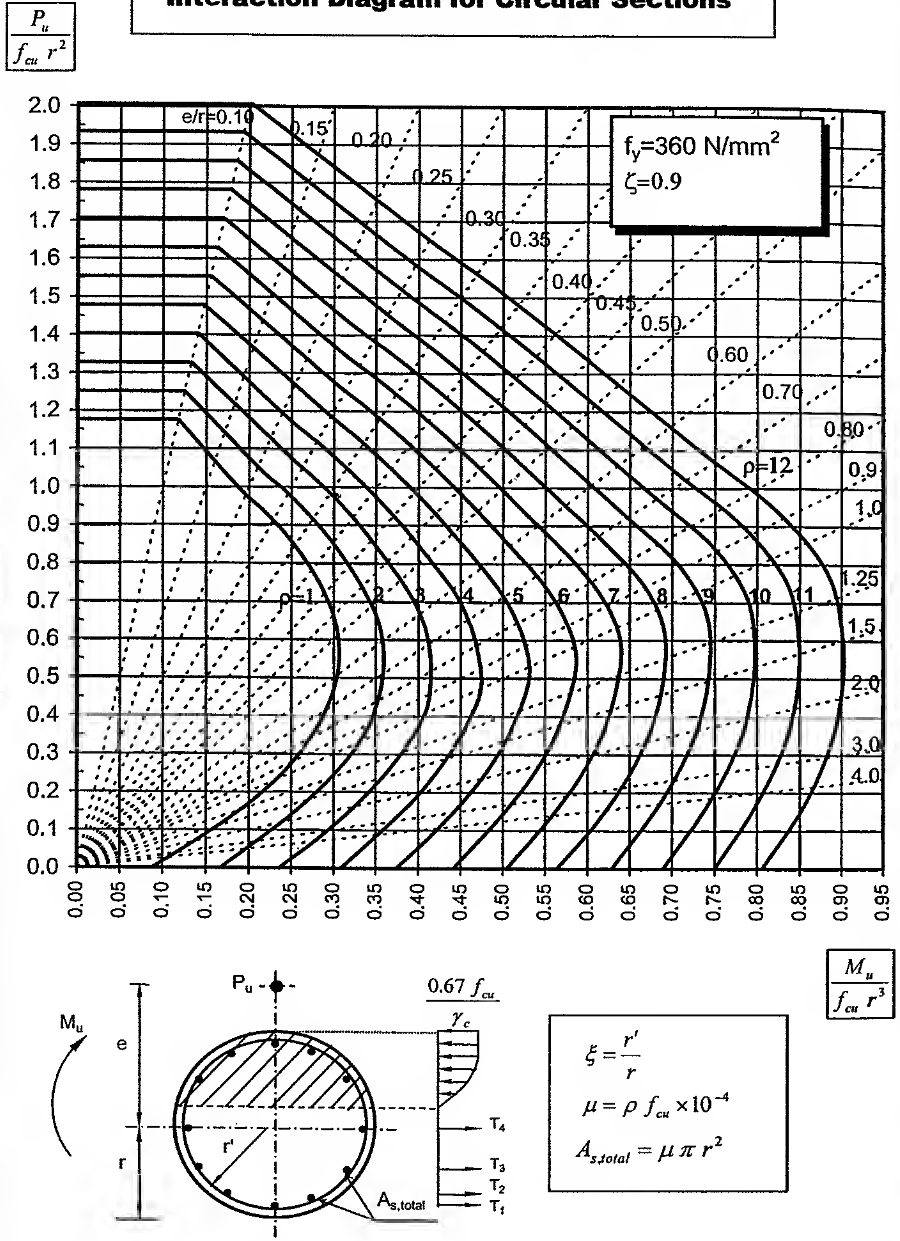


$$\xi = \frac{r'}{r}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\, \pi\, r^2$$

Interaction Diagram for Circular Sections

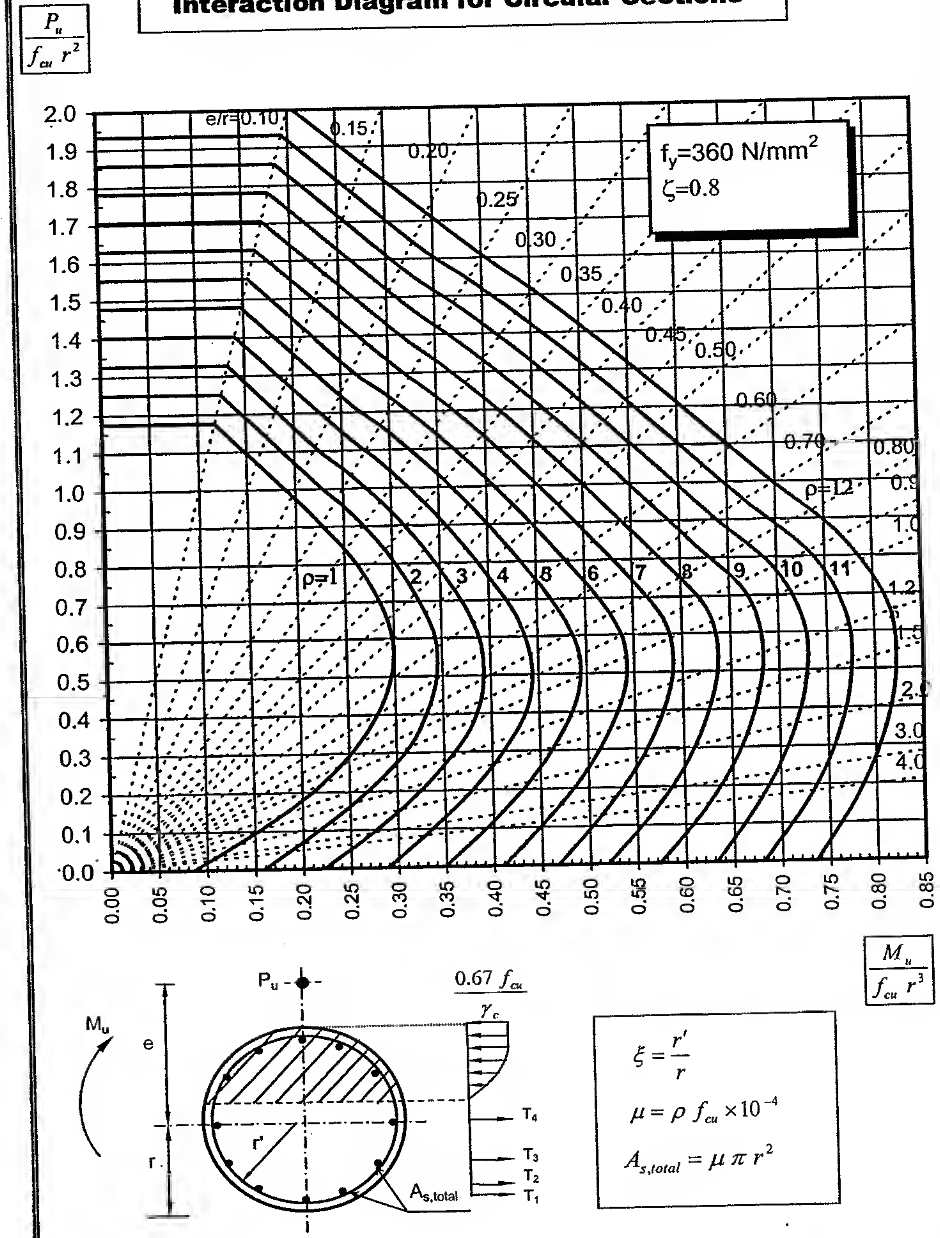


$\xi = \frac{r'}{r}$

$\mu = \rho\, f_{cu} \times 10^{-4}$

$A_{s,total} = \mu\,\pi\,r^2$

Interaction Diagram for Circular Sections

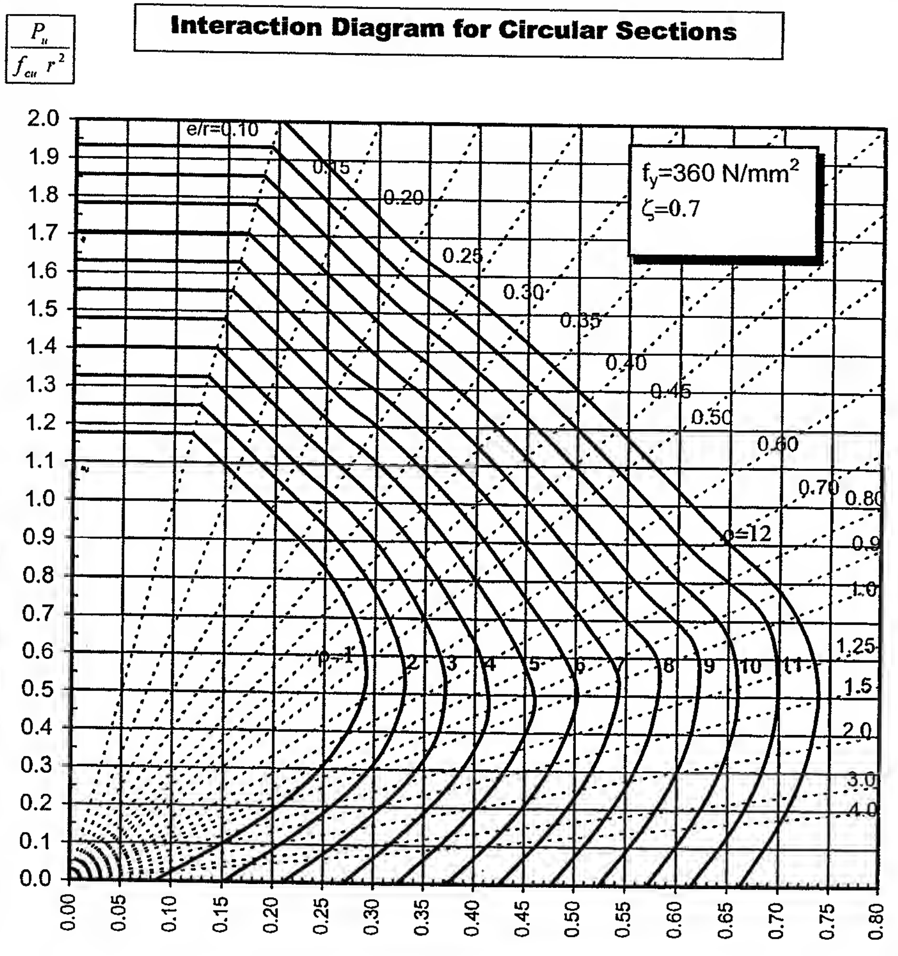


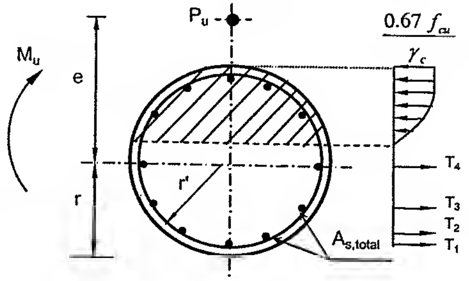
$\xi = \frac{r'}{r}$

$\mu = \rho\, f_{cu} \times 10^{-4}$

$A_{s,total} = \mu\,\pi\,r^2$

Interaction Diagram for Circular Sections



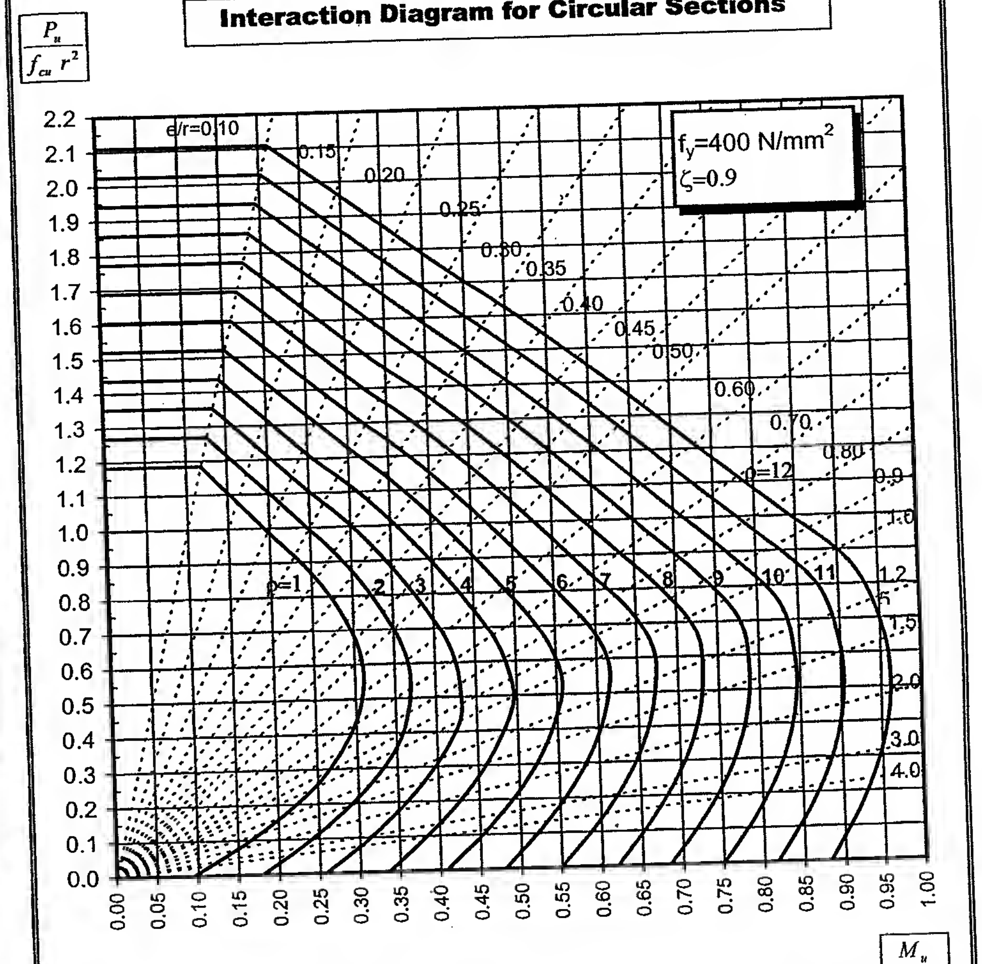


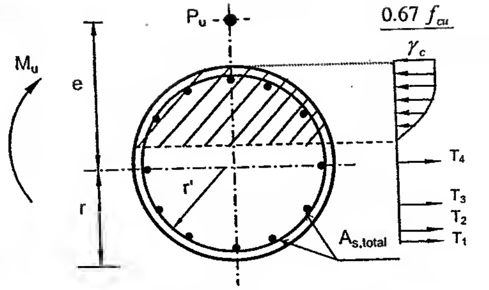
$$\xi = \frac{r'}{r}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\, \pi\, r^2$$

Interaction Diagram for Circular Sections



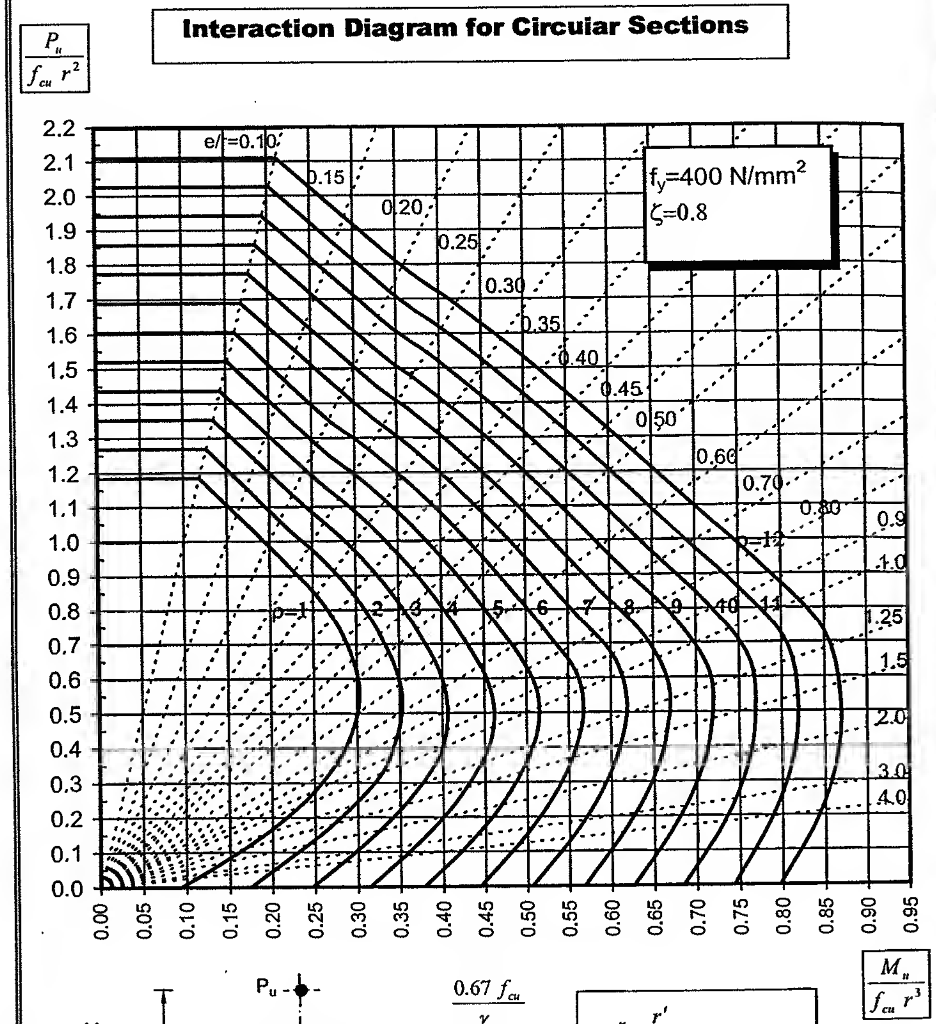


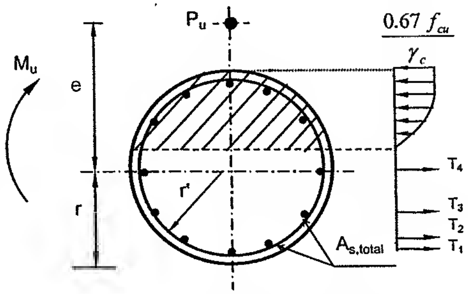
$$\xi = \frac{r'}{r}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\, \pi\, r^2$$

Interaction Diagram for Circular Sections



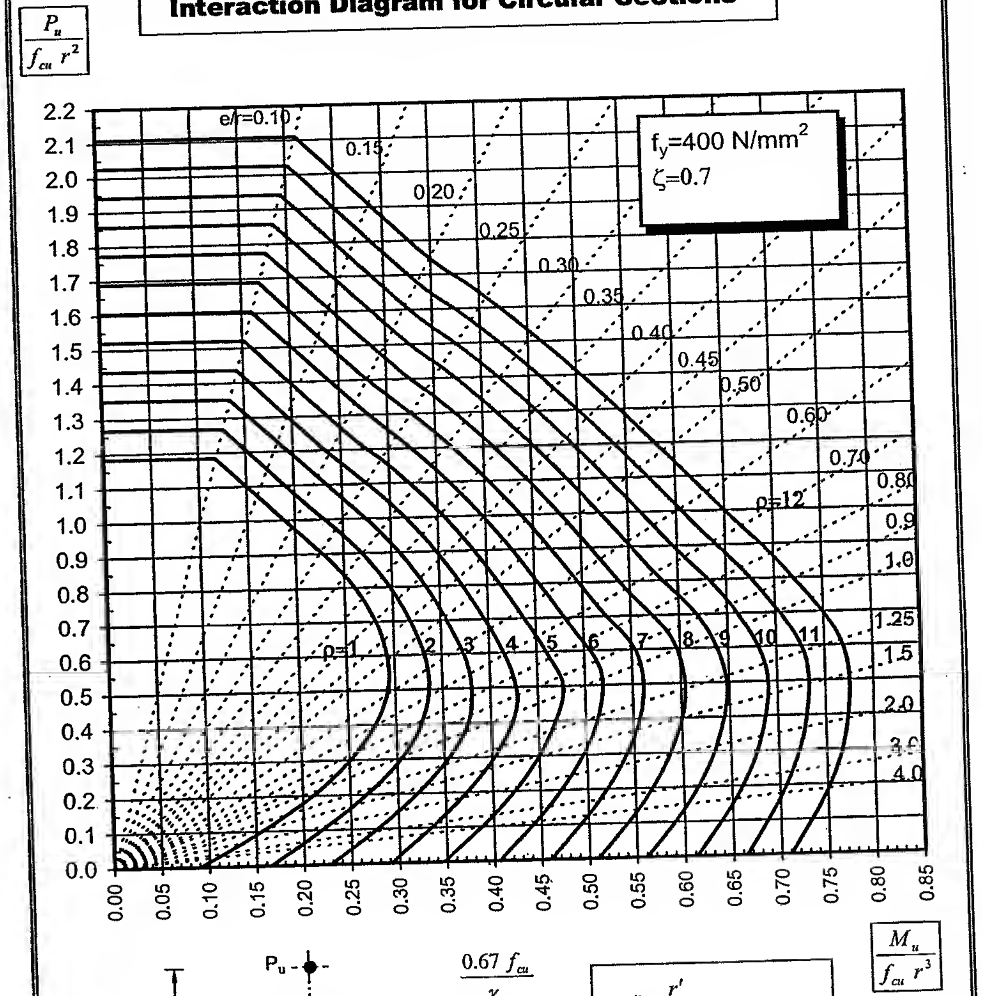


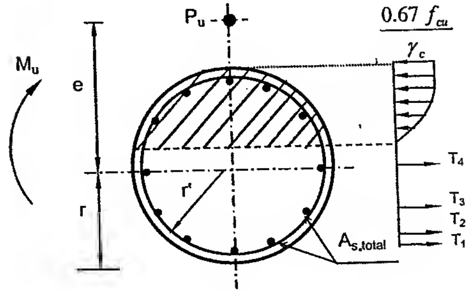
$$\xi = \frac{r'}{r}$$

$$\mu = \rho\ f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\ \pi\ r^2$$

Interaction Diagram for Circular Sections





$$\xi = \frac{r'}{r}$$

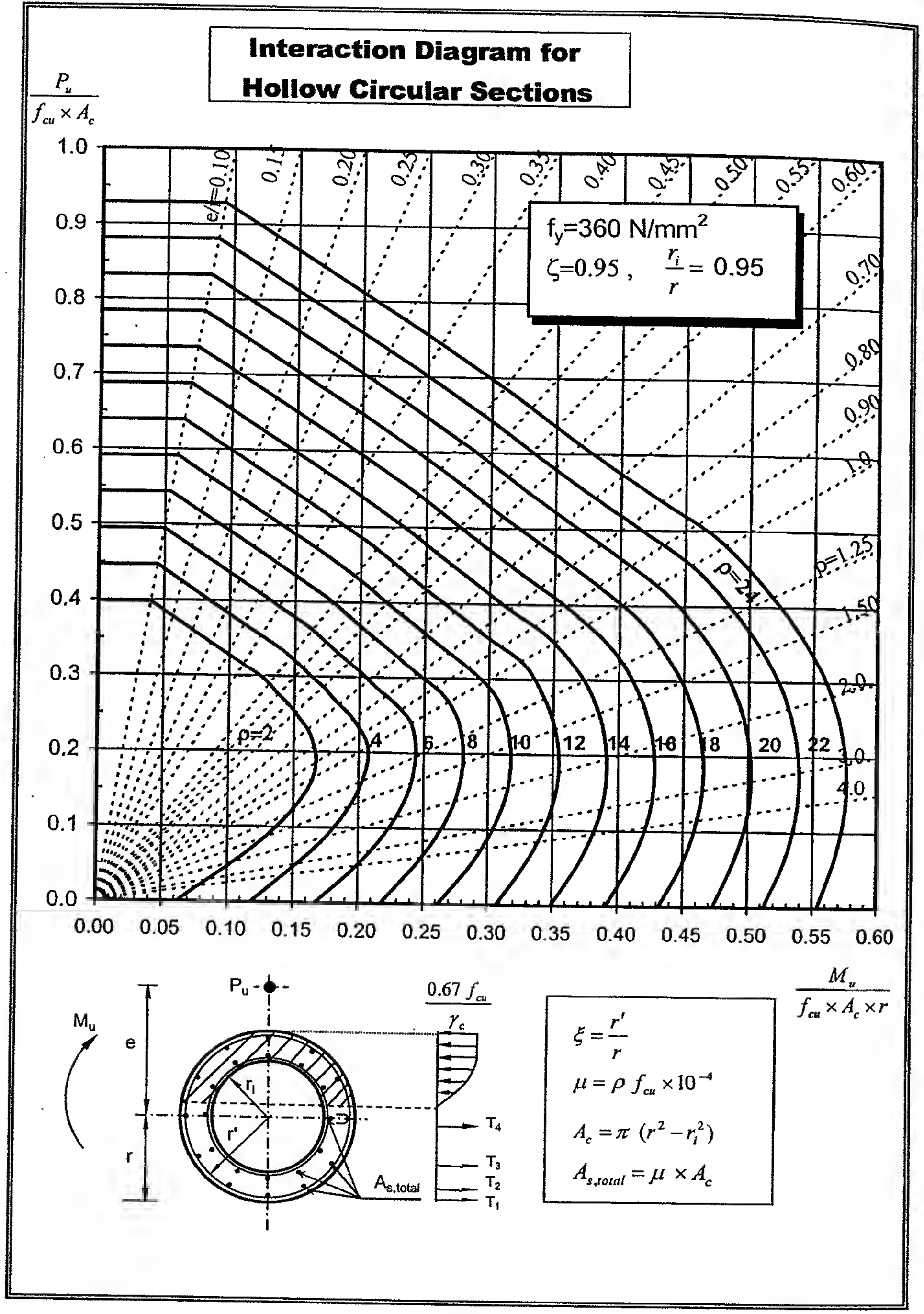
$$\mu = \rho\ f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\ \pi\ r^2$$

APPENDIX F

Interaction Diagrams for Hollow Circular Sections

Interaction Diagram for Hollow Circular Sections



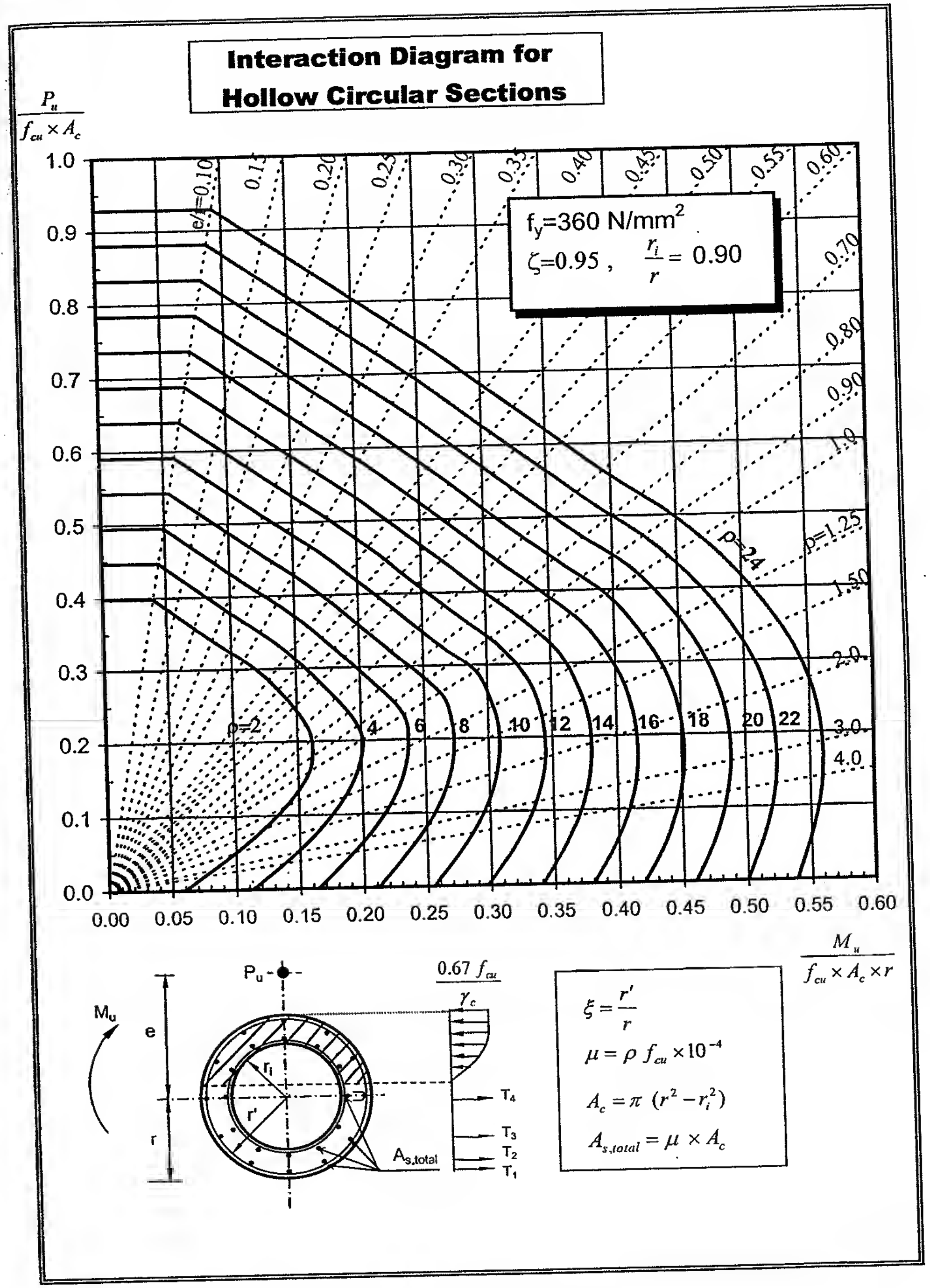
$$\xi = \frac{r'}{r}$$

$$\mu = \rho\ f_{cu} \times 10^{-4}$$

$$A_c = \pi\ (r^2 - r_i^2)$$

$$A_{s,total} = \mu \times A_c$$

Interaction Diagram for Hollow Circular Sections



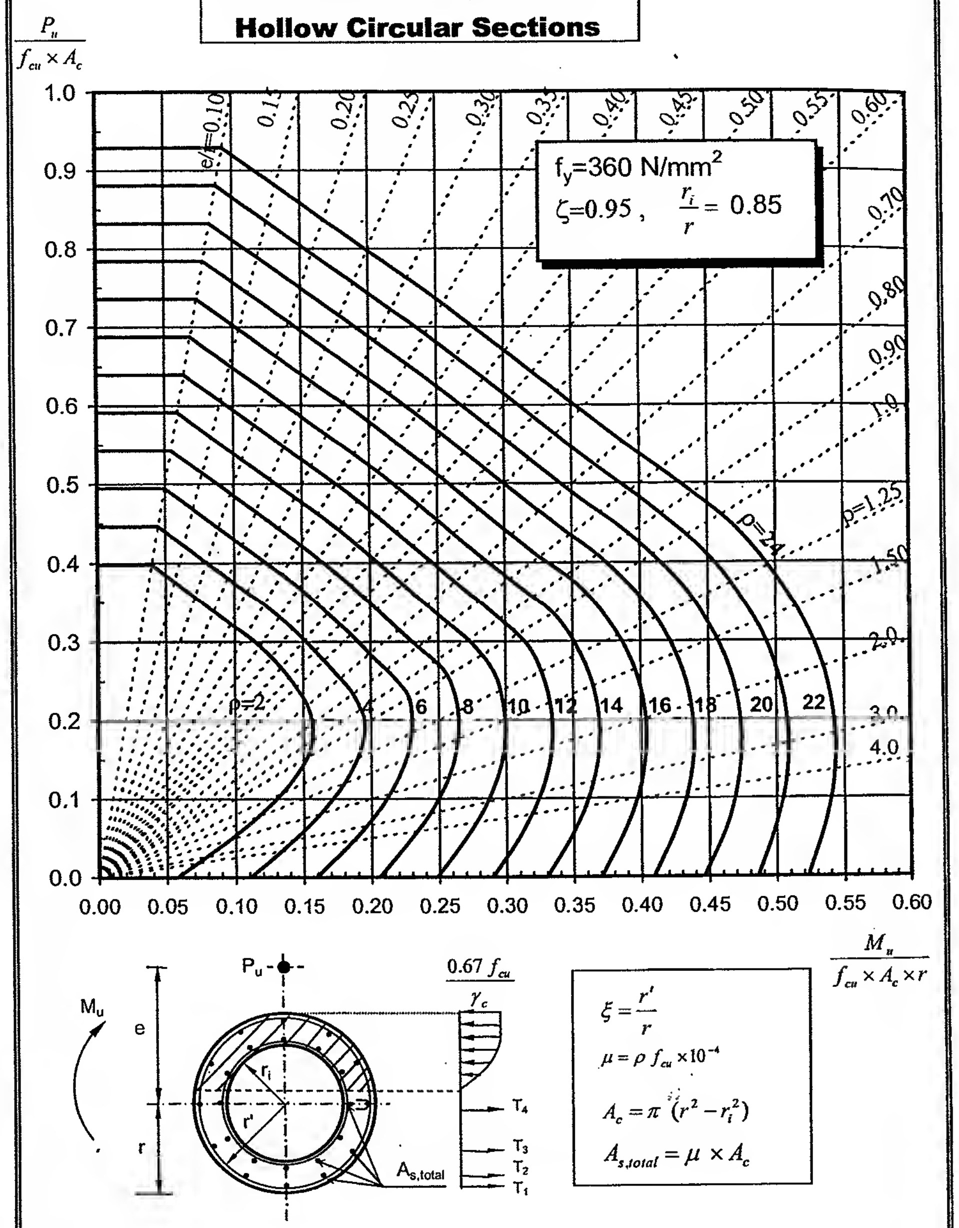
$$\xi = \frac{r'}{r}$$

$$\mu = \rho\ f_{cu} \times 10^{-4}$$

$$A_c = \pi\ (r^2 - r_i^2)$$

$$A_{s,total} = \mu \times A_c$$

Interaction Diagram for Hollow Circular Sections



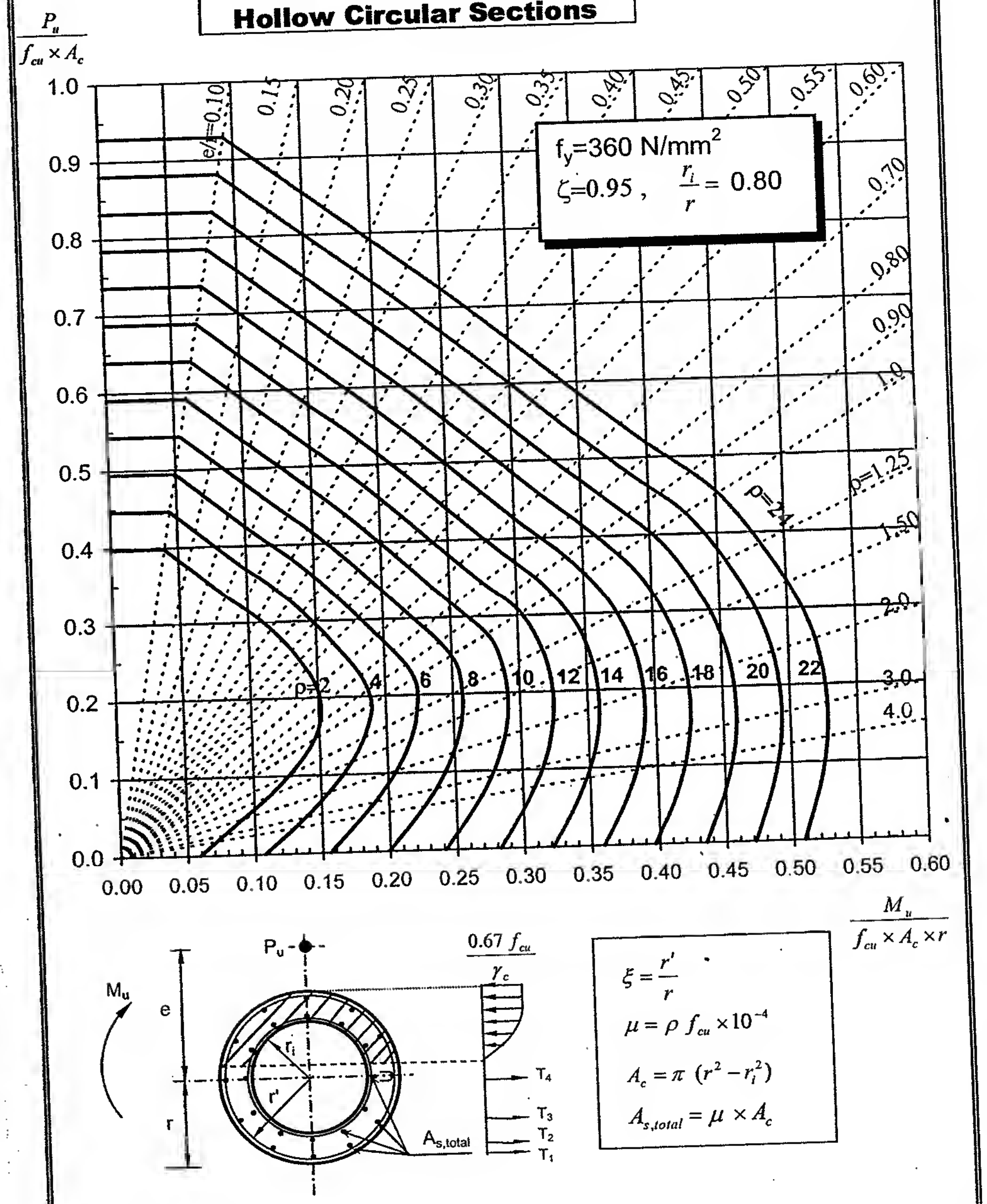
$\xi = \frac{r'}{r}$

$\mu = \rho\, f_{cu} \times 10^{-4}$

$A_c = \pi\,(r^2 - r_i^2)$

$A_{s,total} = \mu \times A_c$

Interaction Diagram for Hollow Circular Sections



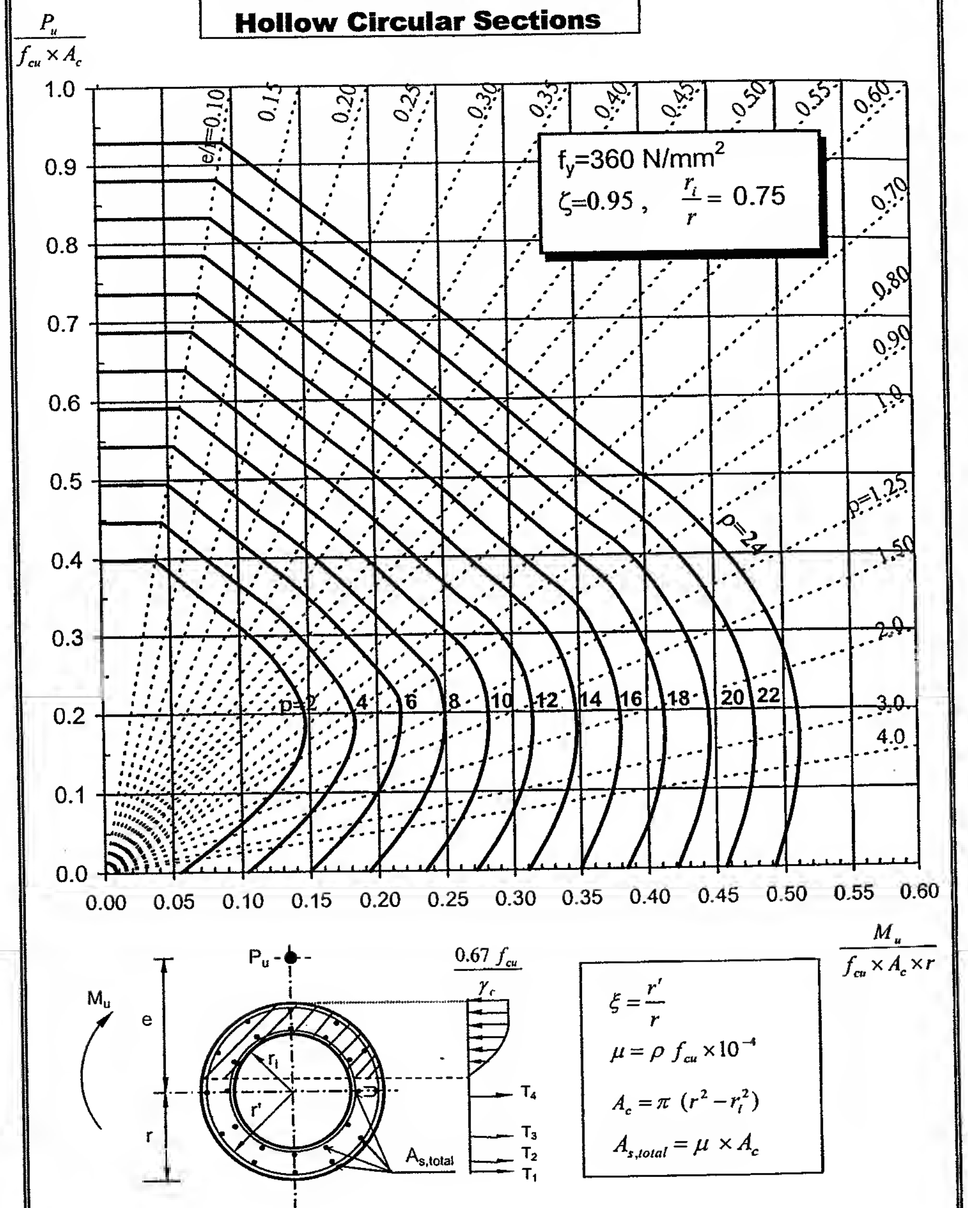
$\xi = \frac{r'}{r}$

$\mu = \rho\, f_{cu} \times 10^{-4}$

$A_c = \pi\,(r^2 - r_i^2)$

$A_{s,total} = \mu \times A_c$

Interaction Diagram for Hollow Circular Sections



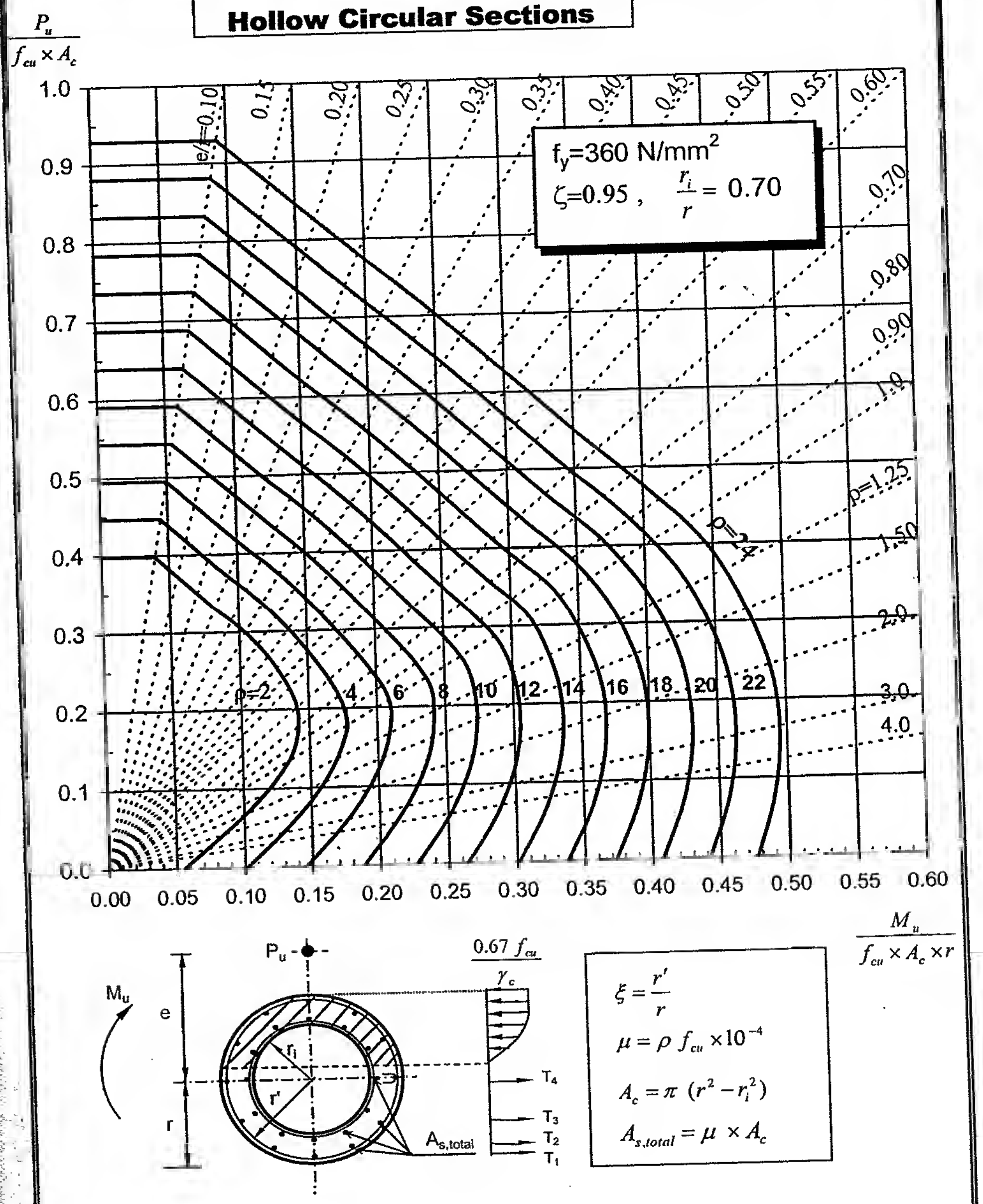
$\xi = \frac{r'}{r}$

$\mu = \rho\ f_{cu} \times 10^{-4}$

$A_c = \pi\ (r^2 - r_i^2)$

$A_{s,total} = \mu\ \times A_c$

Interaction Diagram for Hollow Circular Sections



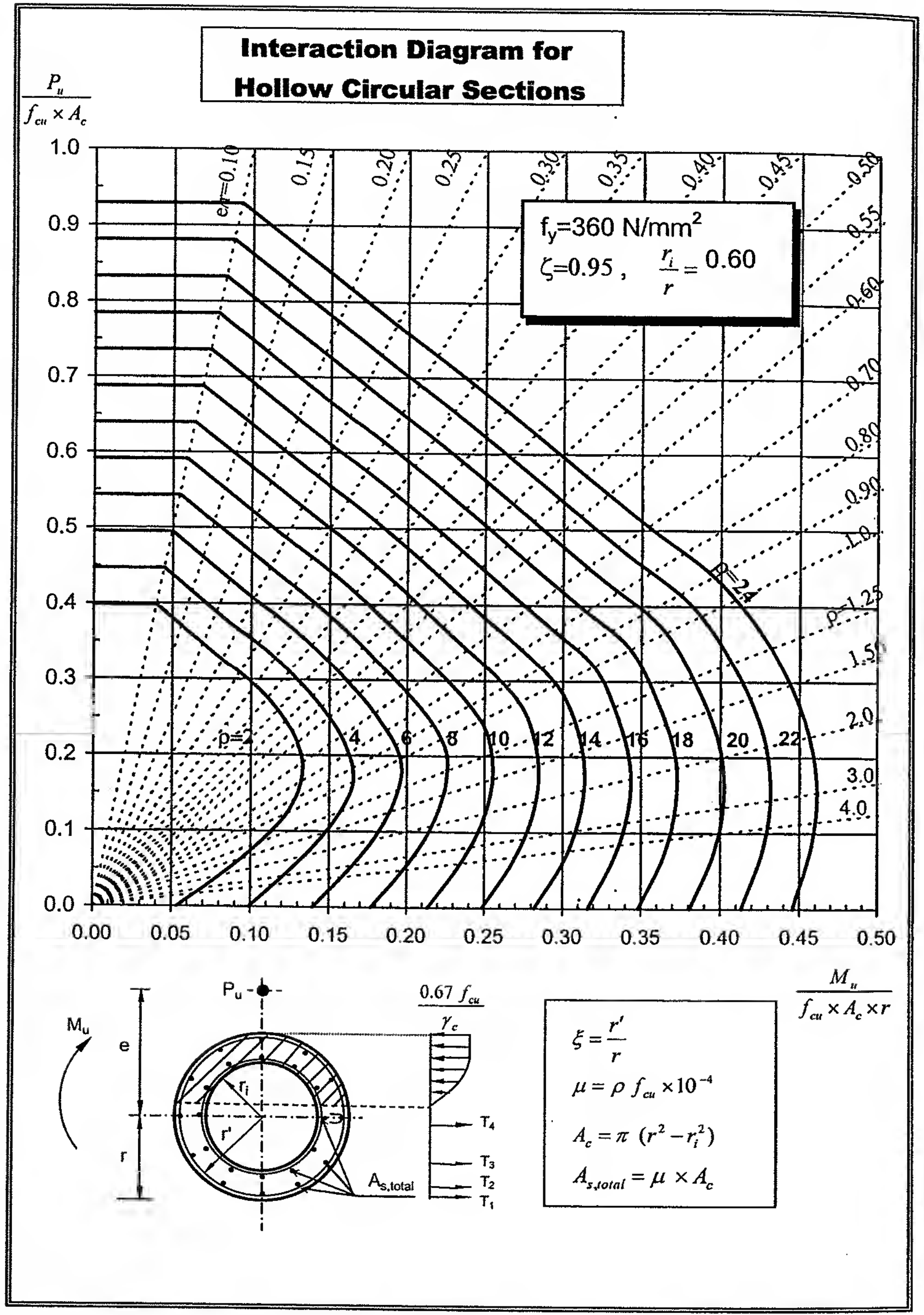
$\xi = \frac{r'}{r}$

$\mu = \rho\ f_{cu} \times 10^{-4}$

$A_c = \pi\ (r^2 - r_i^2)$

$A_{s,total} = \mu\ \times A_c$

Interaction Diagram for Hollow Circular Sections



$$\xi = \frac{r'}{r}$$

$$\mu = \rho \, f_{cu} \times 10^{-4}$$

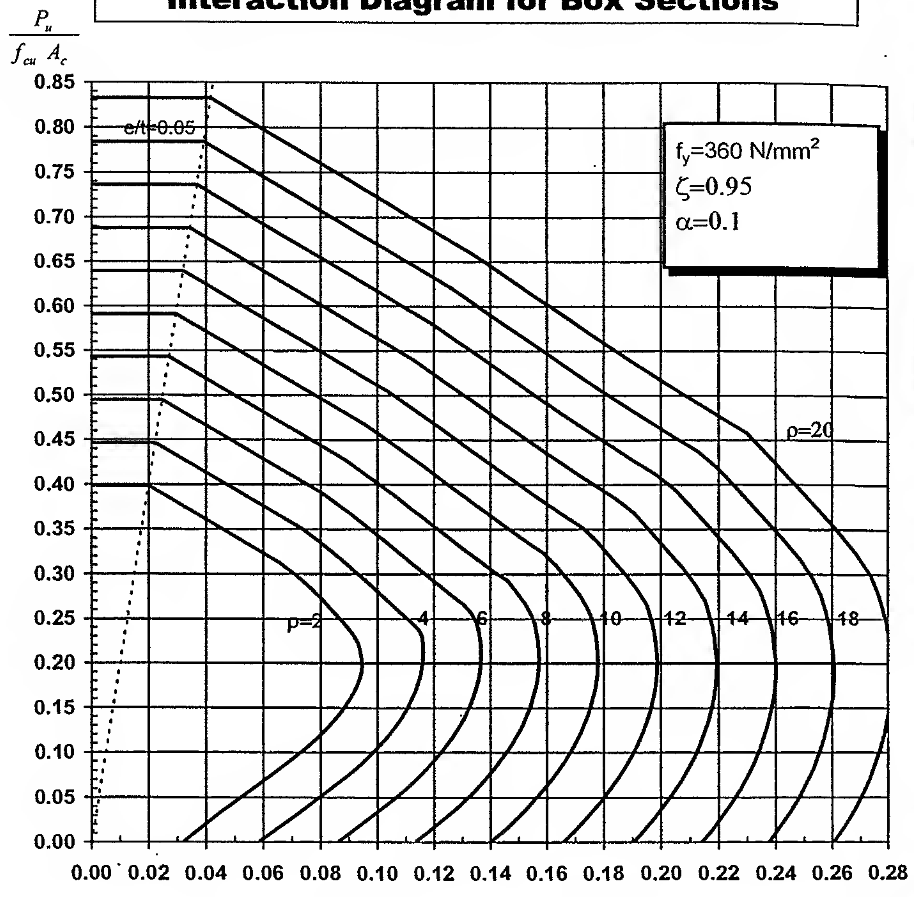
$$A_c = \pi \, (r^2 - r_i^2)$$

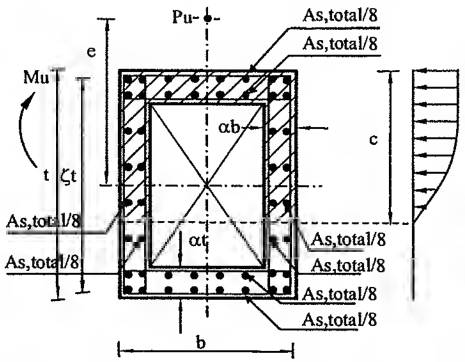
$$A_{s,total} = \mu \times A_c$$

APPENDIX G

Interaction Diagrams for Box Sections

Interaction Diagram for Box Sections





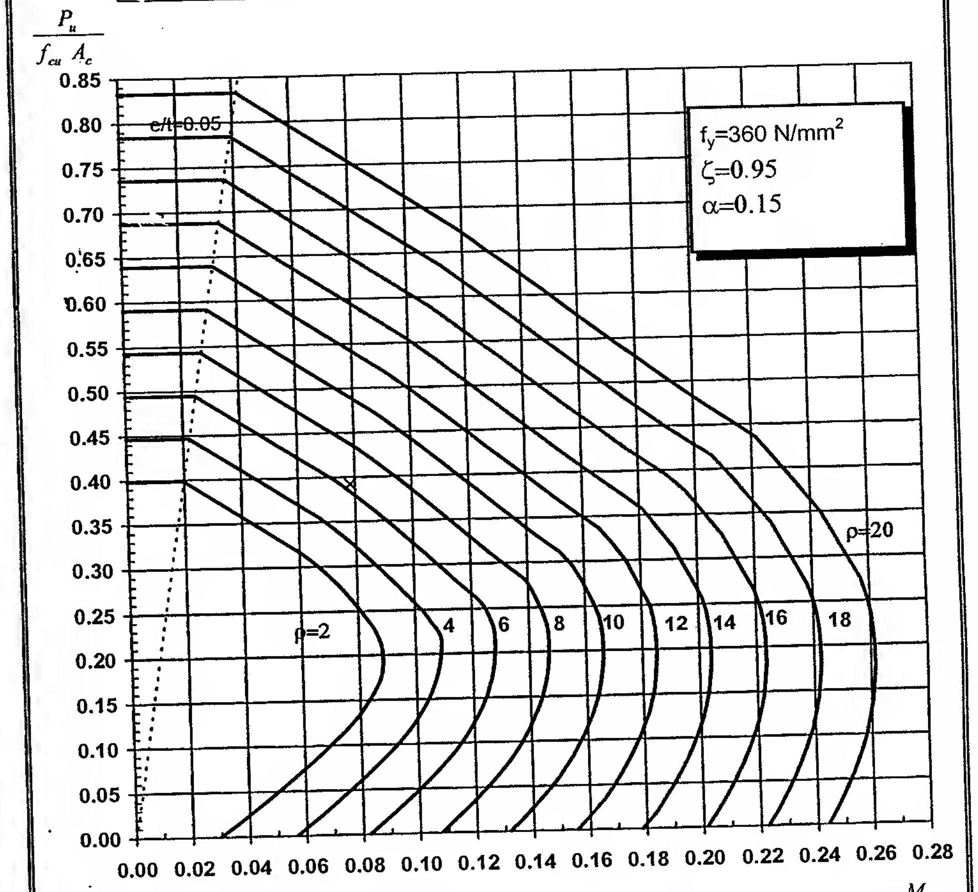
Ac = net area of the concrete

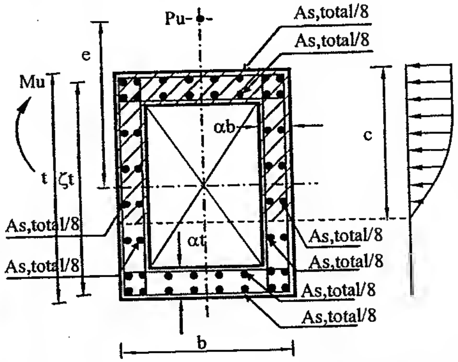
$$\zeta = (d - d')/t$$

$$\mu = \rho \times f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\ A_c$$

Interaction Diagram for Box Sections





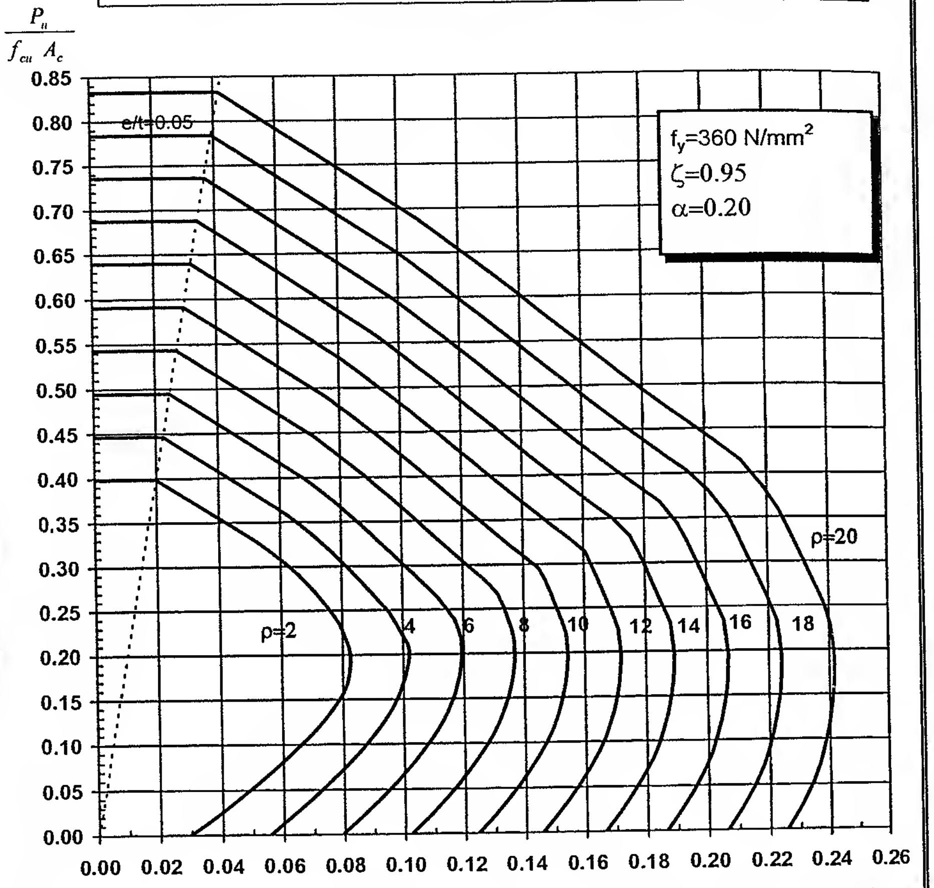
Ac = net area of the concrete

$$\zeta = (d - d')/t$$

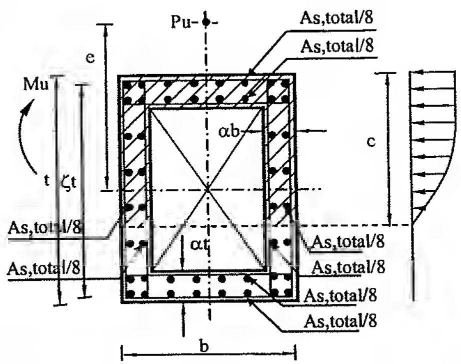
$$\mu = \rho \times f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\ A_c$$

Interaction Diagram for Box Sections



$\frac{M_u}{f_{cu} A_c t}$



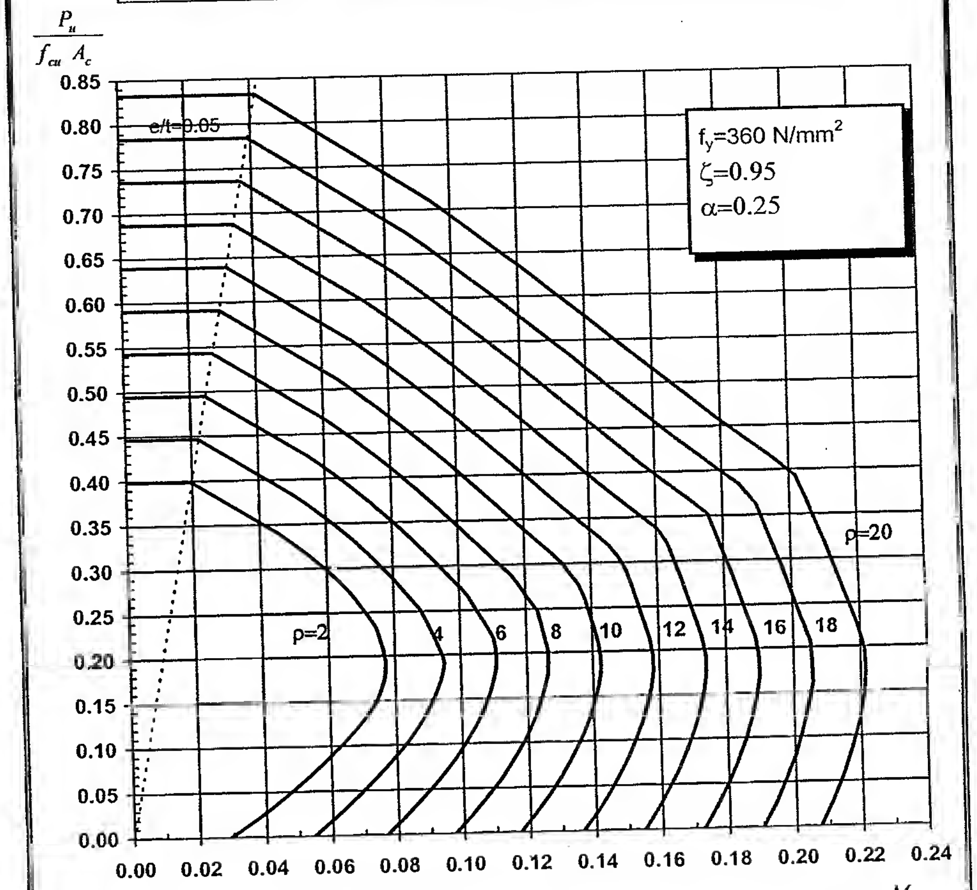
Ac = net area of the concrete

$$\zeta = (d - d')/t$$

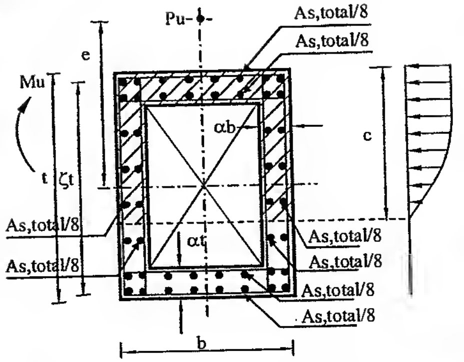
$$\mu = \rho \times f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu A_c$$

Interaction Diagram for Box Sections



$\frac{M_u}{f_{cu} A_c t}$



Ac = net area of the concrete

$$\zeta = (d - d')/t$$

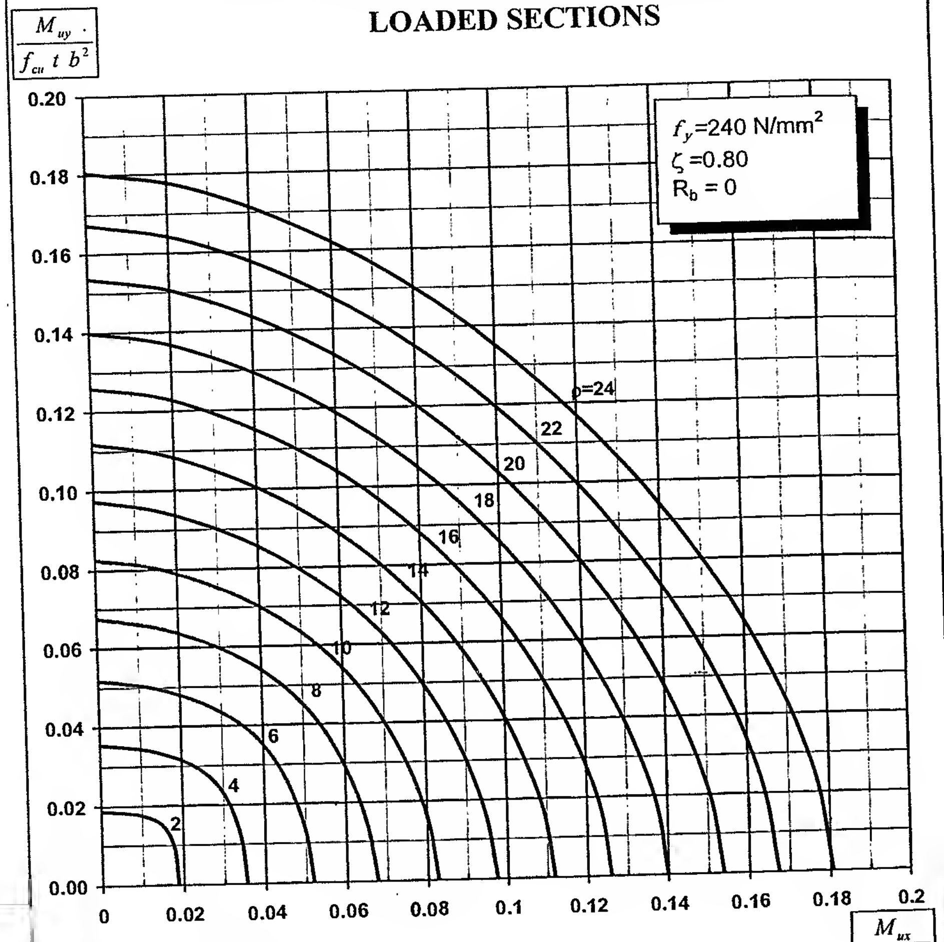
$$\mu = \rho \times f_{cu} \times 10^{-4}$$

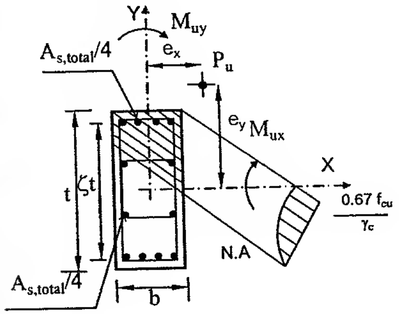
$$A_{s,total} = \mu A_c$$

APPENDIX H

Interaction Diagrams for Biaxially Loaded Sections

INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



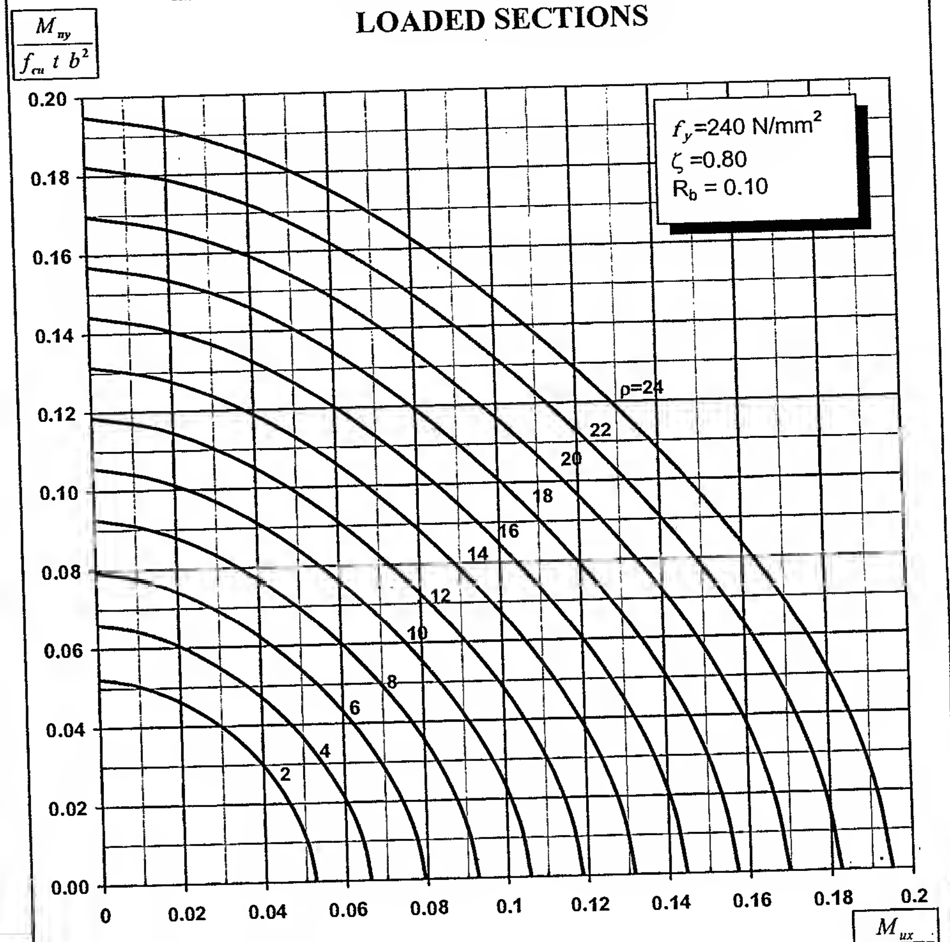


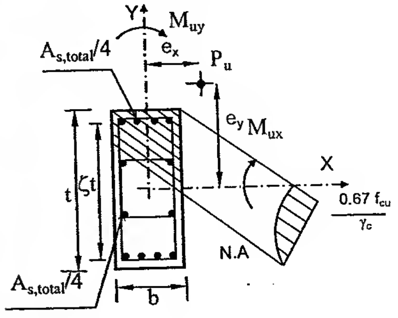
$$R_b = \frac{P_u}{f_{cu}\, b\, t}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



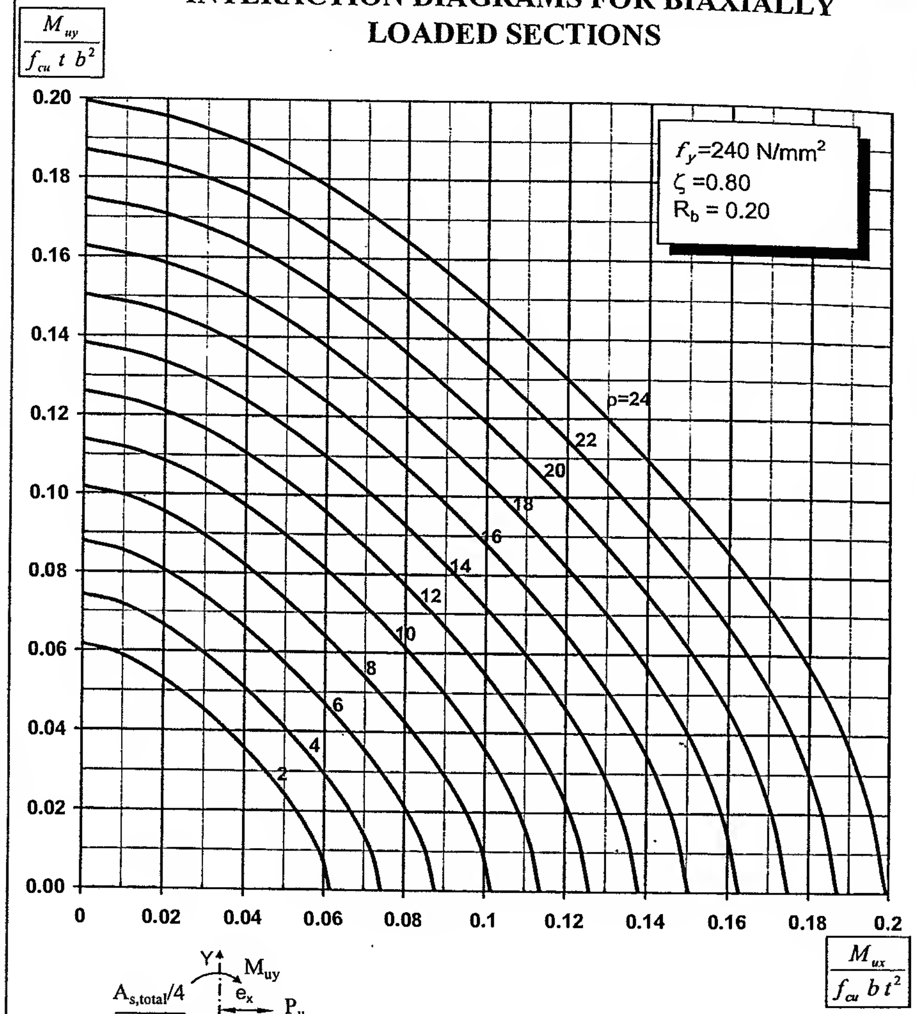


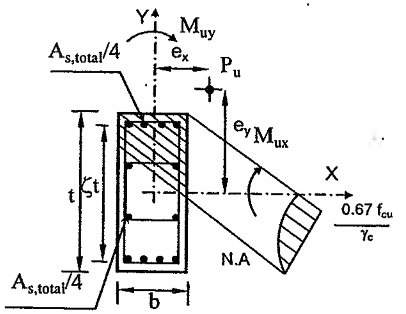
$$R_b = \frac{P_u}{f_{cu}\, b\, t}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



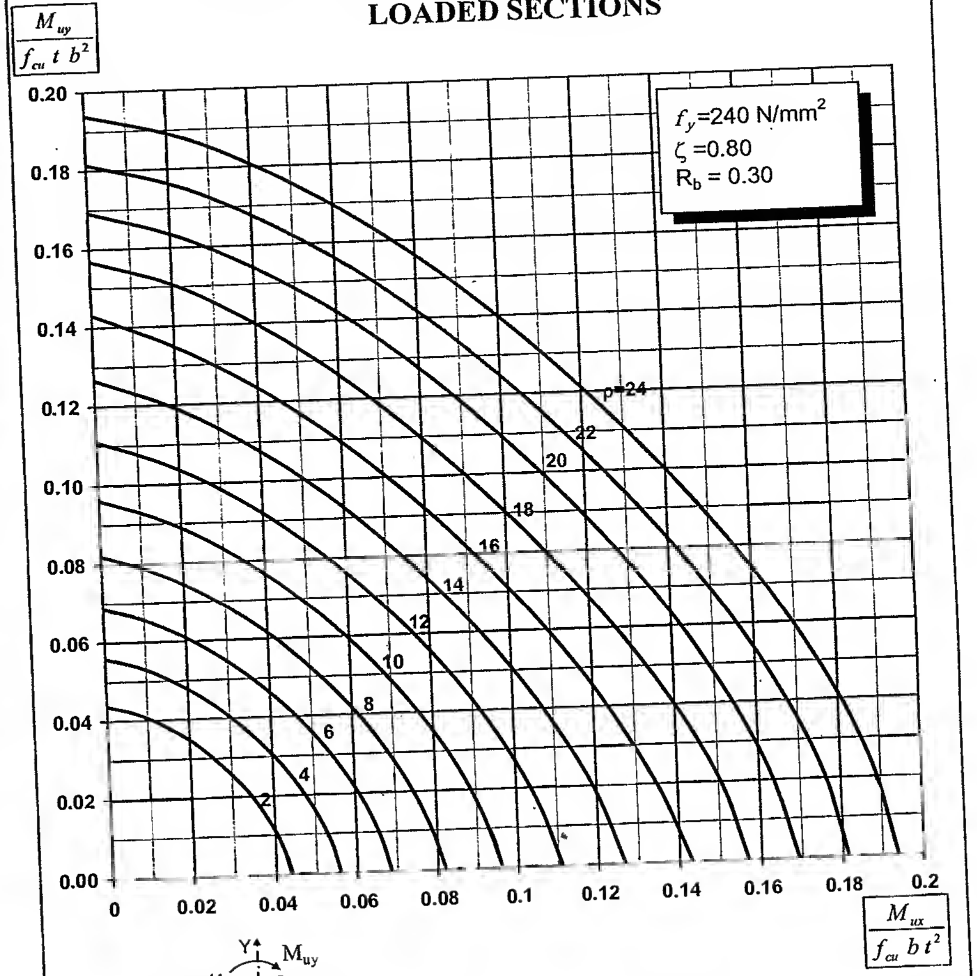


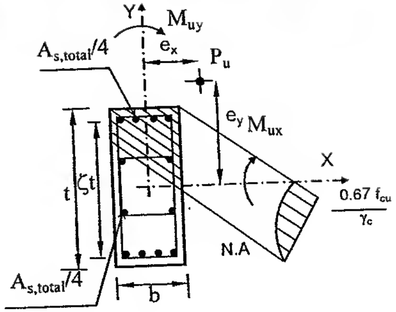
$$R_b = \frac{P_u}{f_{cu}\, b\, t}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\, b\, t$$

INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



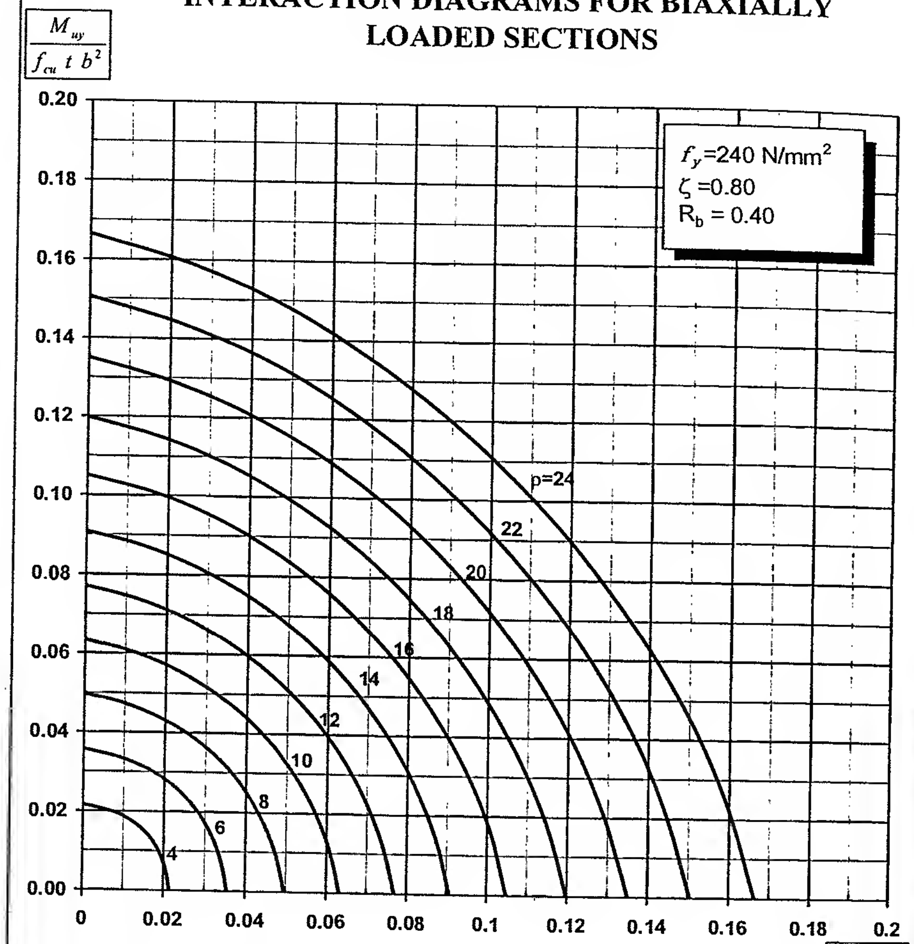


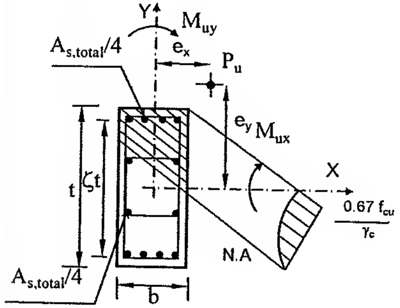
$$R_b = \frac{P_u}{f_{cu}\, b\, t}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\, b\, t$$

INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



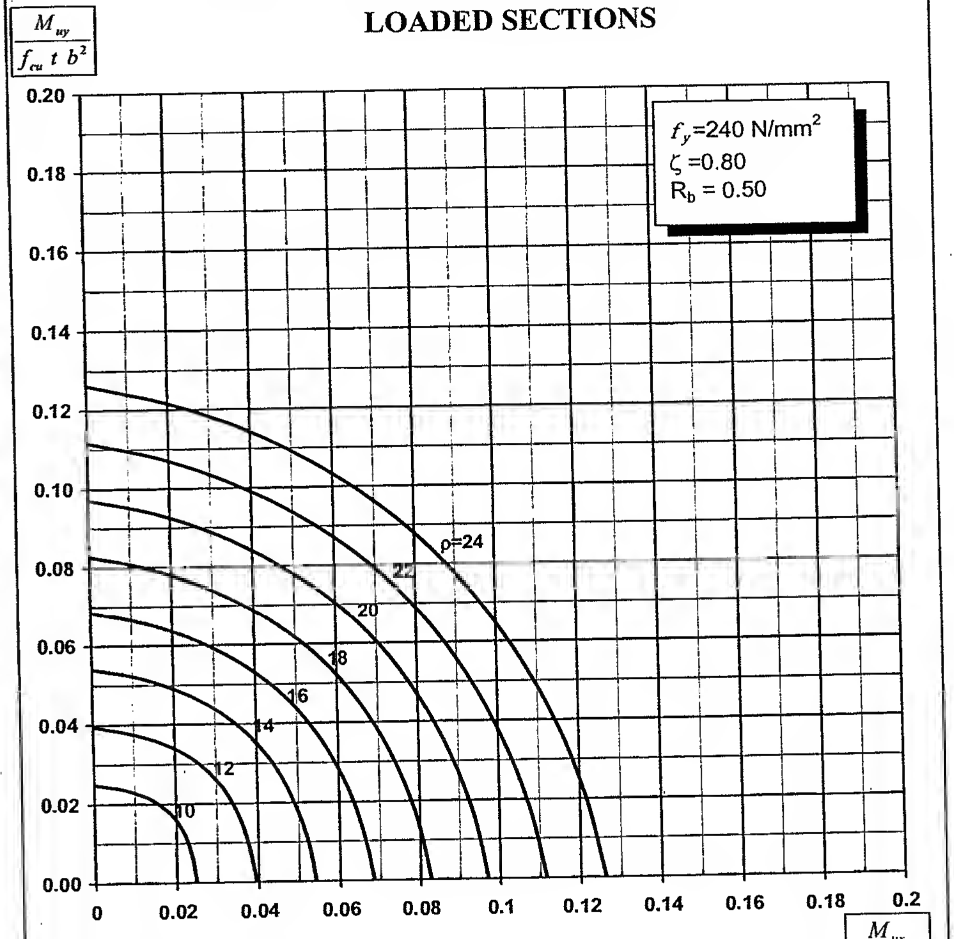


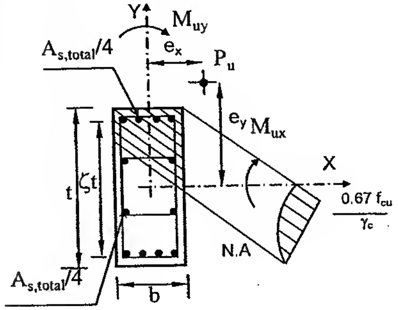
$$R_b = \frac{P_u}{f_{cu}\ b\ t}$$

$$\mu = \rho\ f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



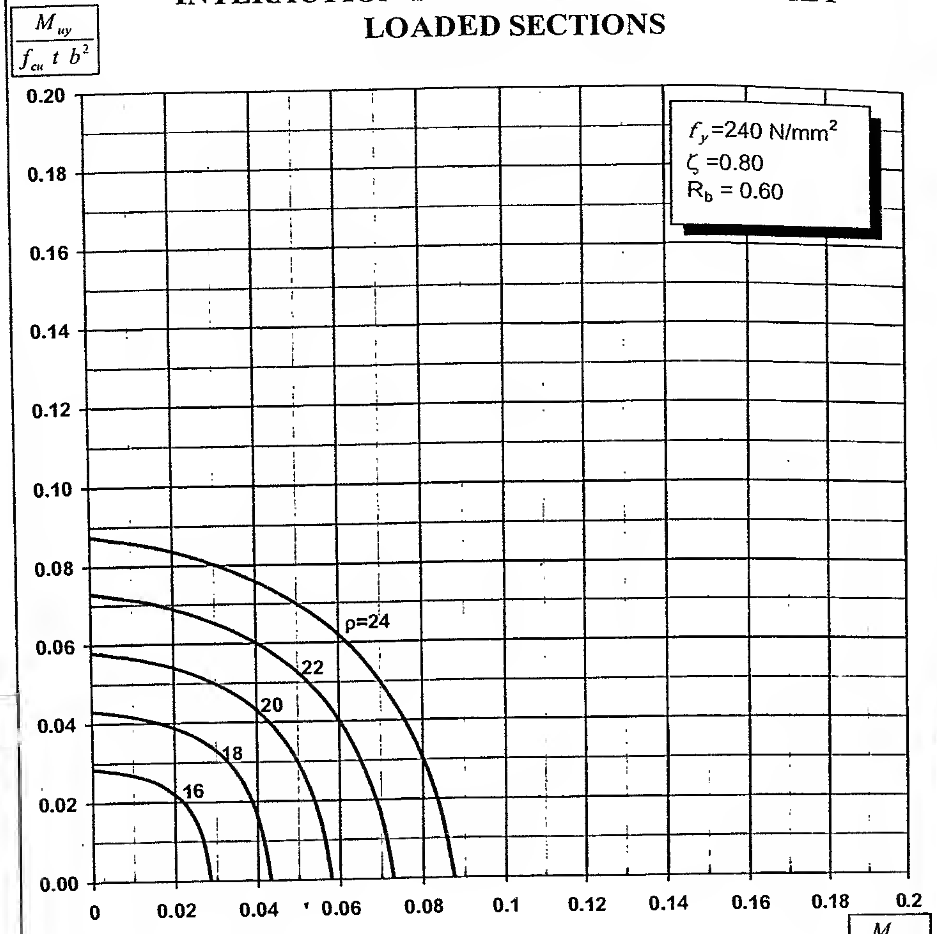


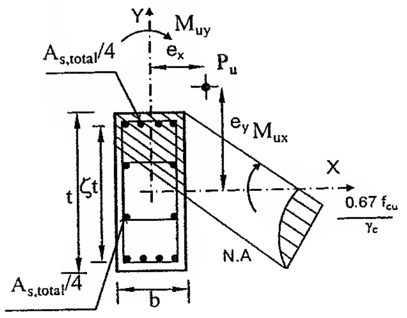
$$R_b = \frac{P_u}{f_{cu}\ b\ t}$$

$$\mu = \rho\ f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\ b\ t$$

INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



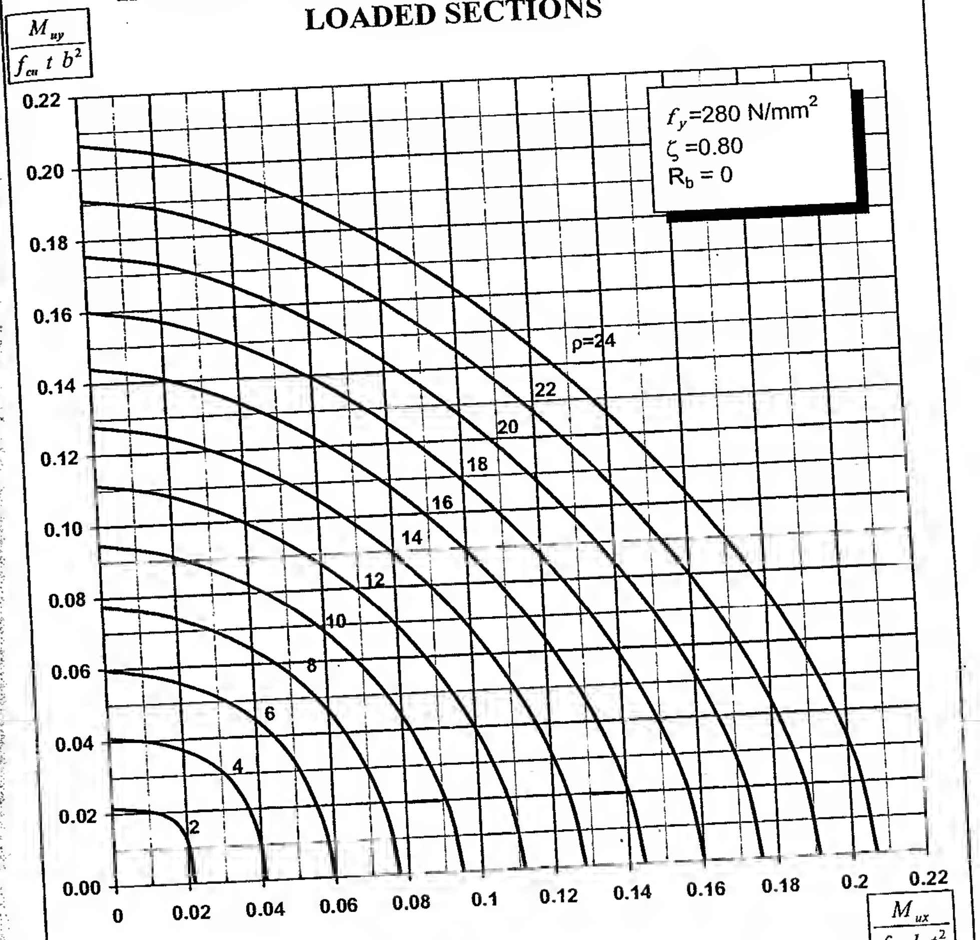


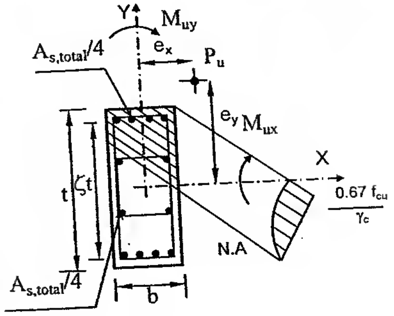
$$R_b = \frac{P_u}{f_{cu}\, b\, t}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\, b\, t$$

INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



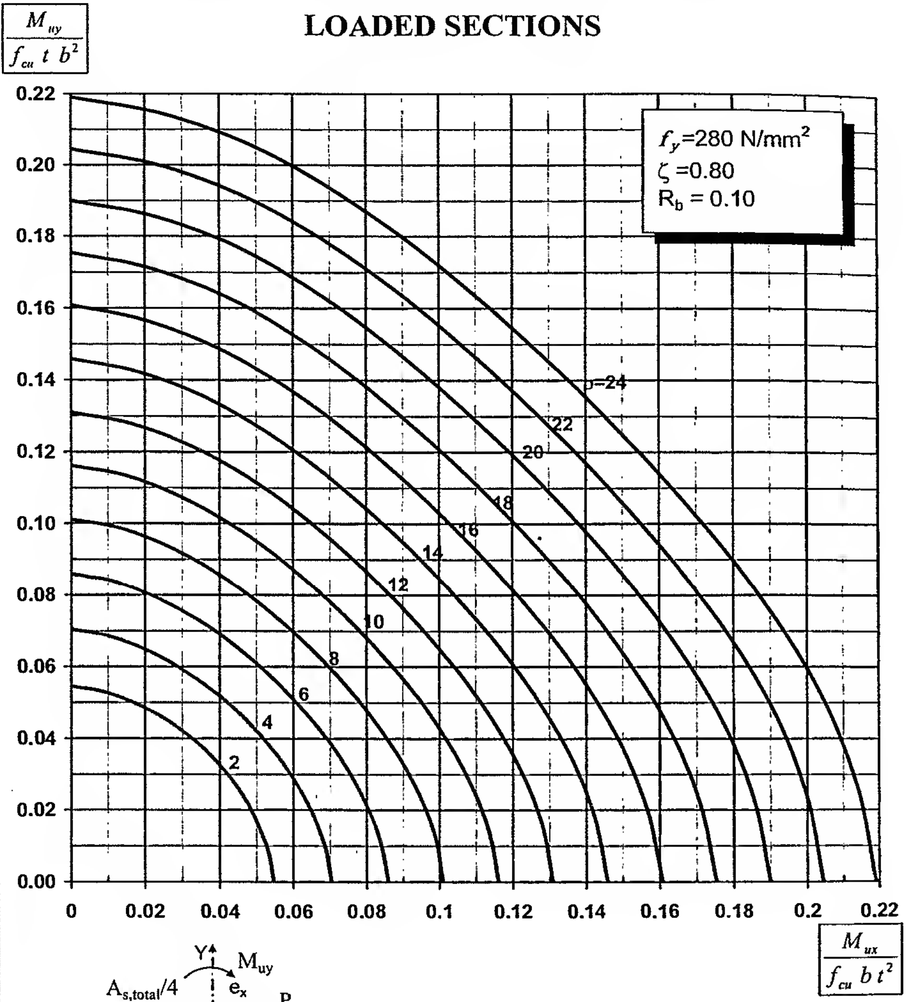


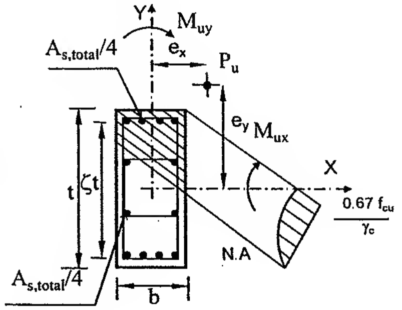
$$R_b = \frac{P_u}{f_{cu}\, b\, t}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\, b\, t$$

INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



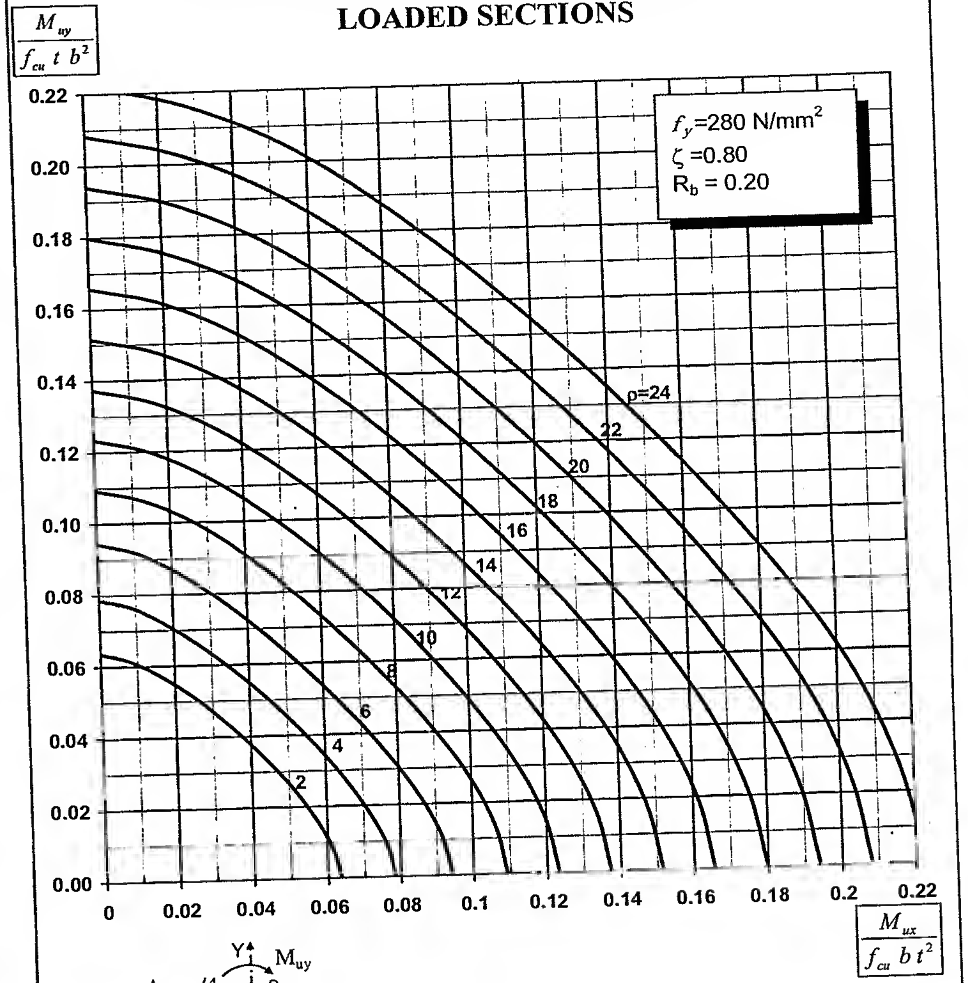


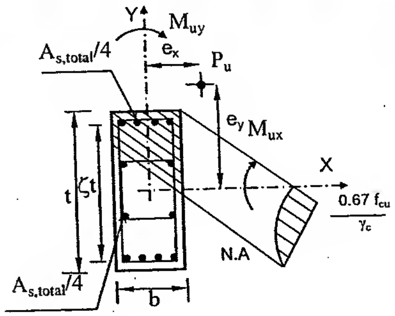
$$R_b = \frac{P_u}{f_{cu}\, b\, t}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\, b\, t$$

INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS





$$R_b = \frac{P_u}{f_{cu}\, b\, t}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

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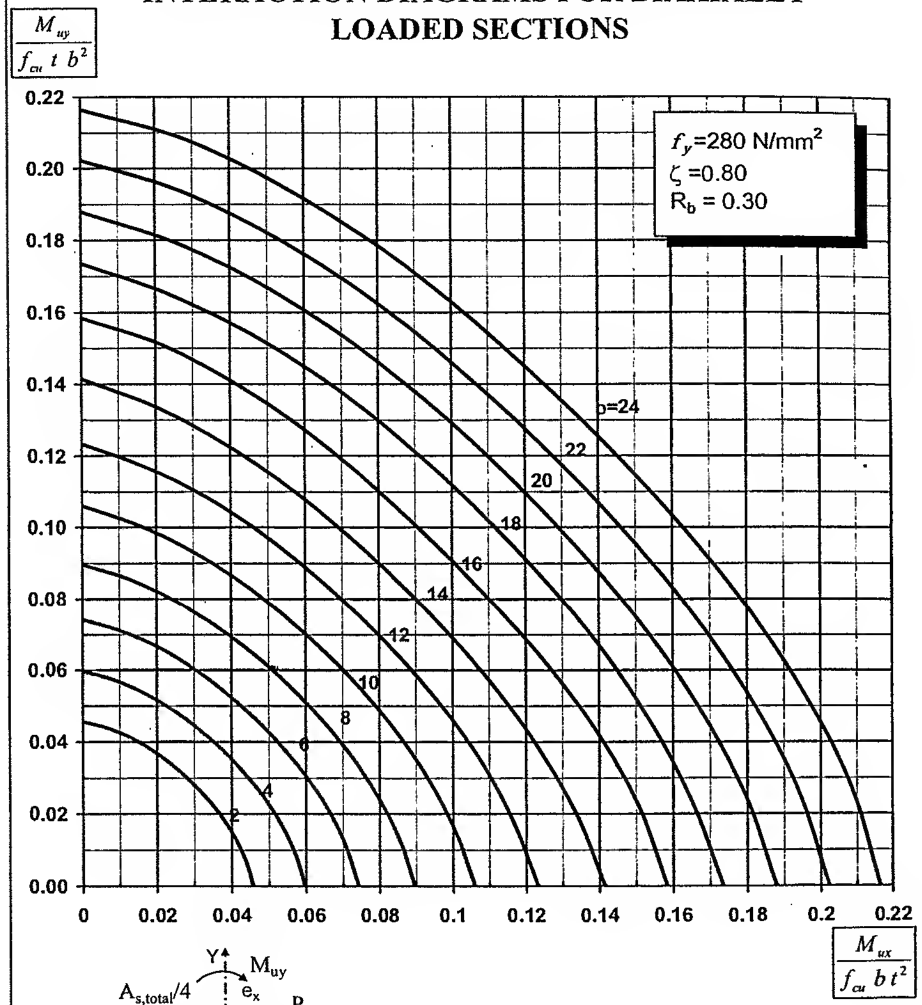
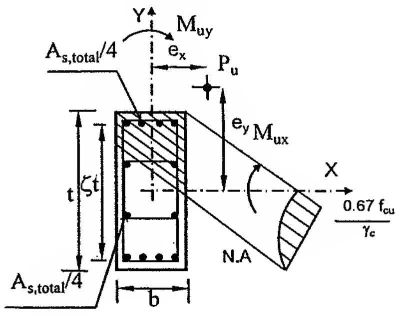
INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



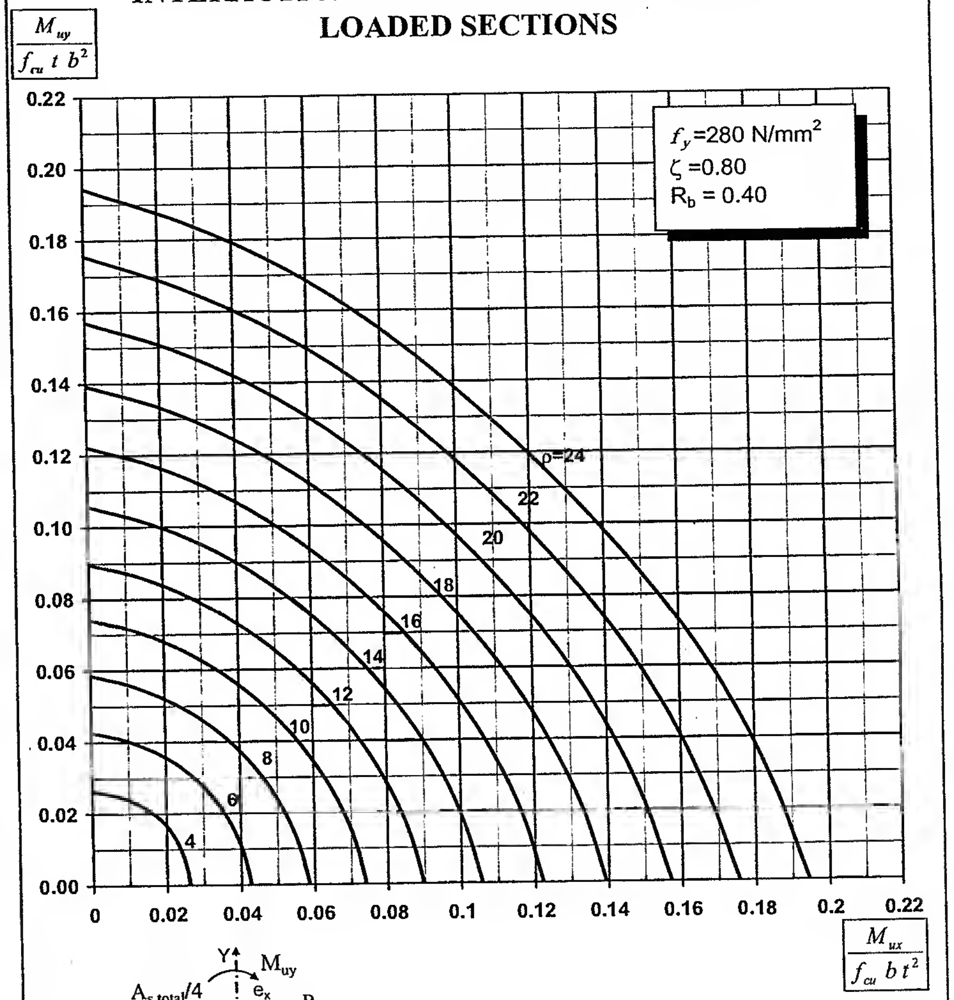


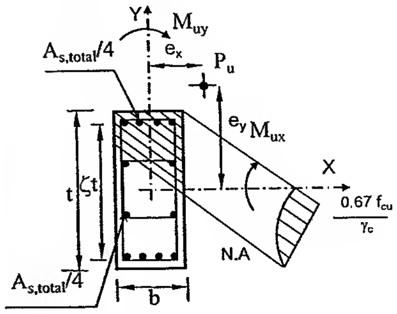
$$R_b = \frac{P_u}{f_{cu}\, b\, t}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\, b\, t$$

INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS



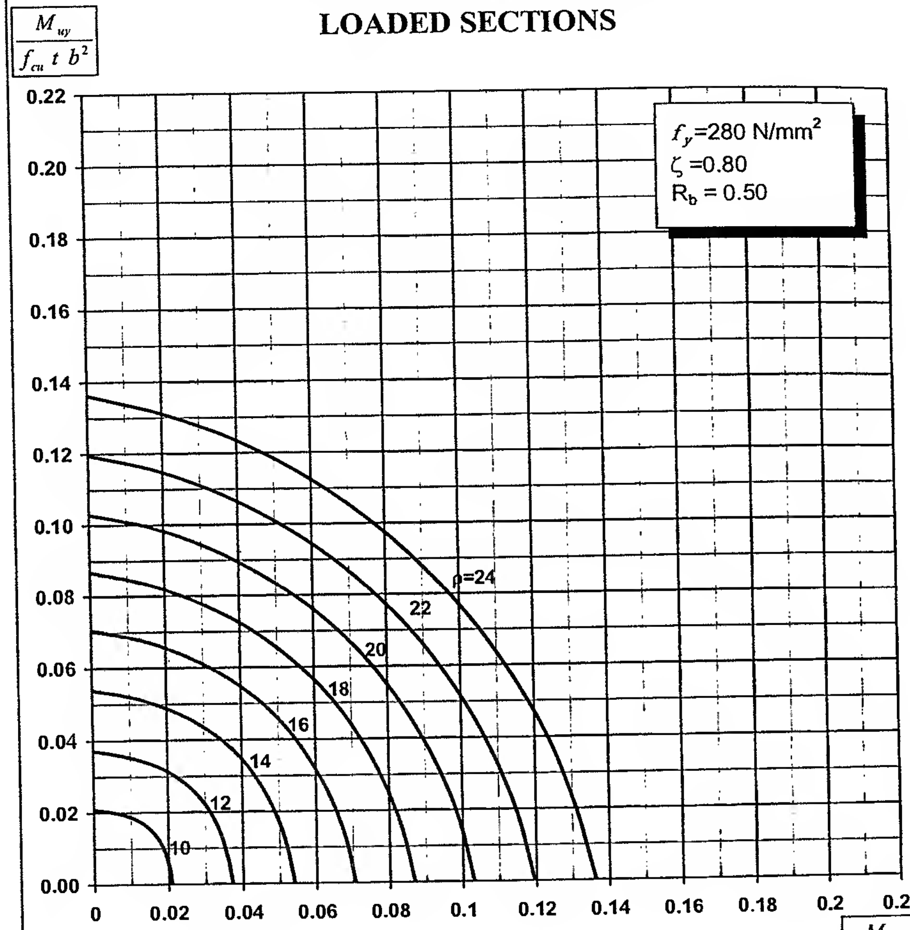


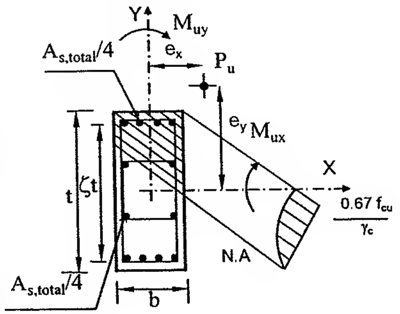
$$R_b = \frac{P_u}{f_{cu}\, b\, t}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\, b\, t$$

INTERACTION DIAGRAMS FOR BIAXIALLY LOADED SECTIONS





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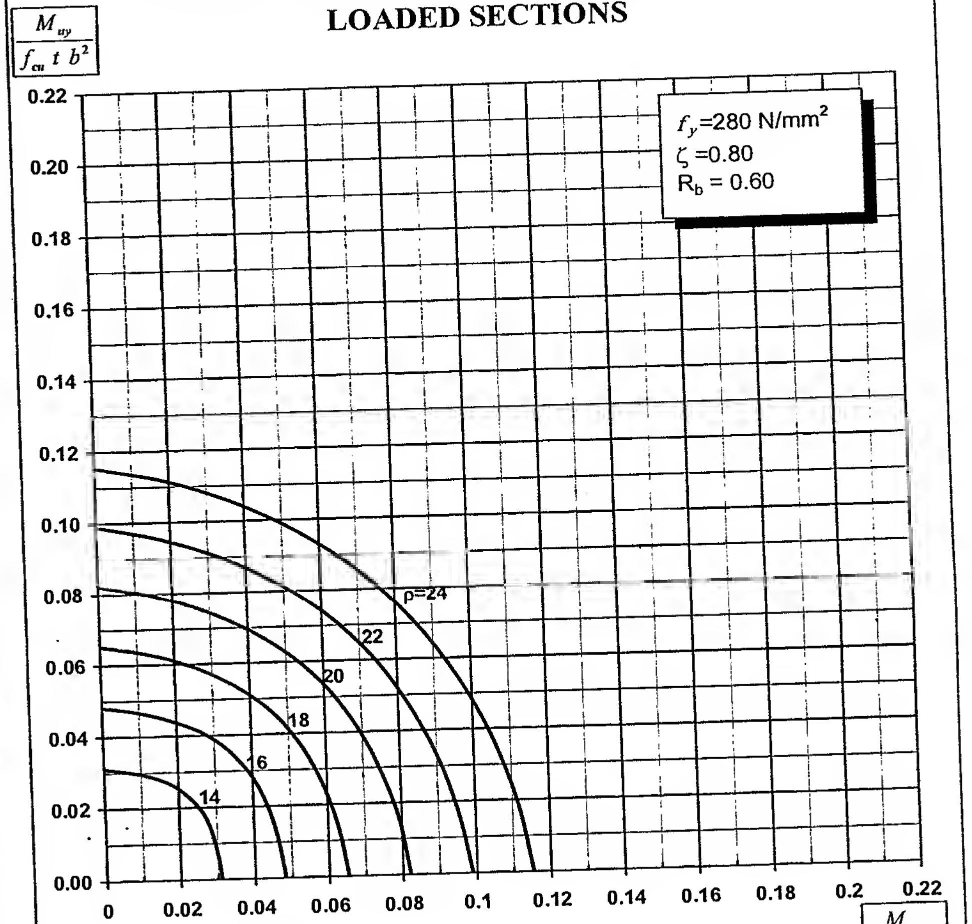

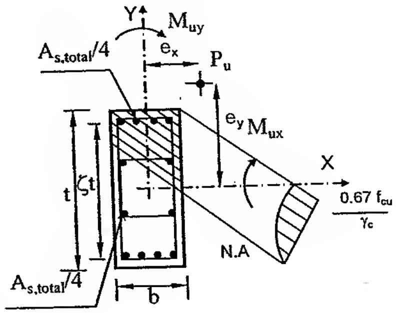

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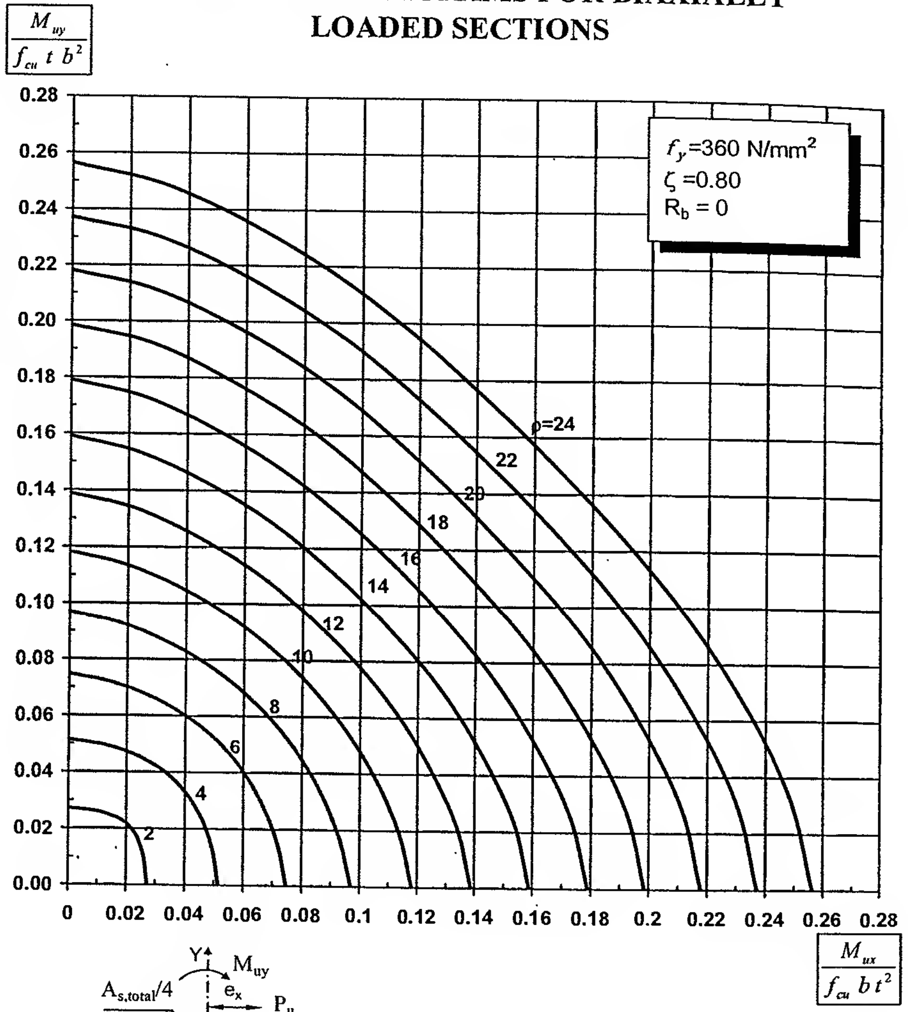

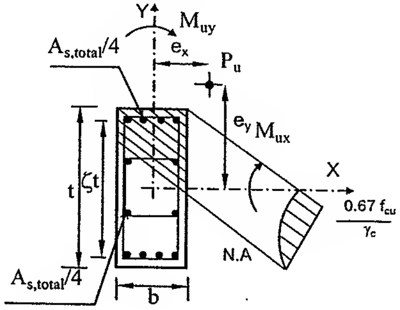

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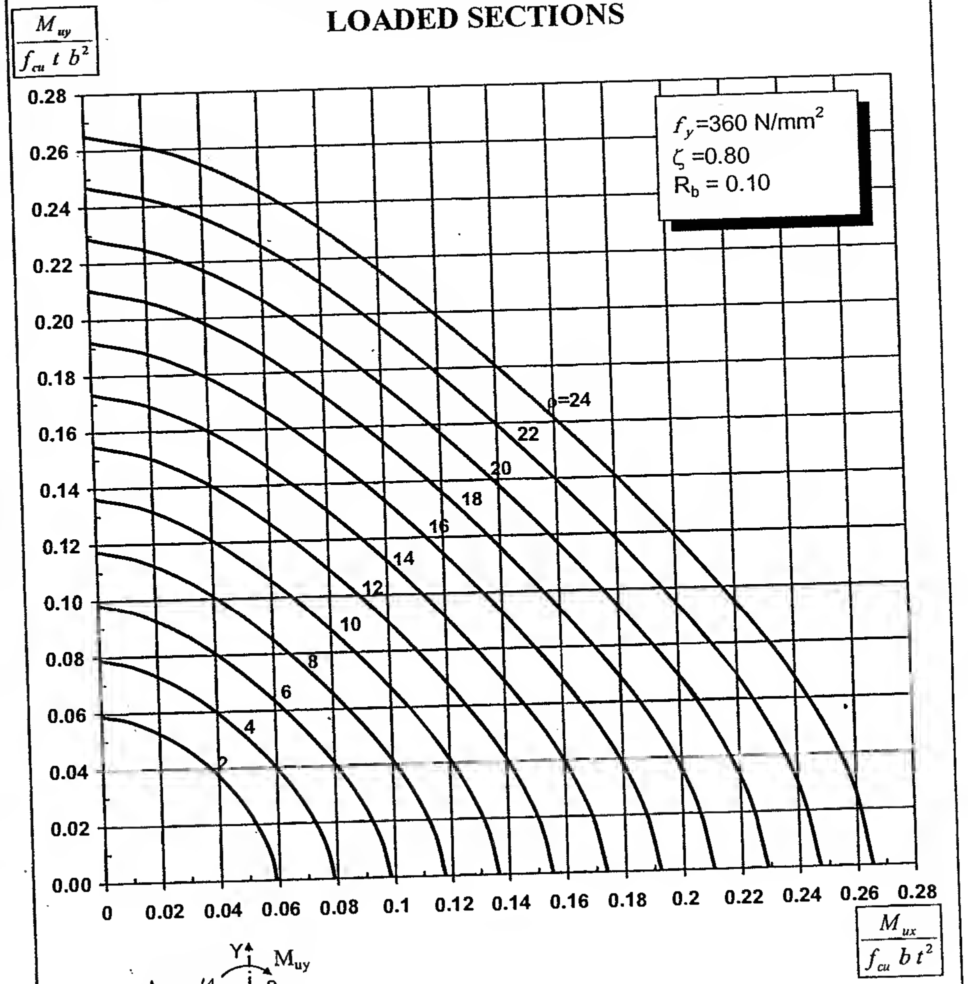

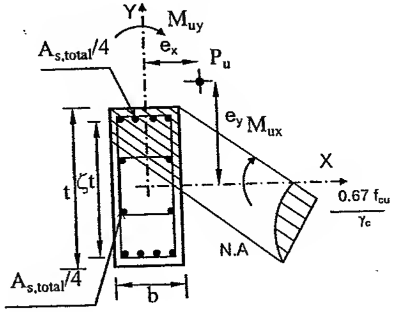

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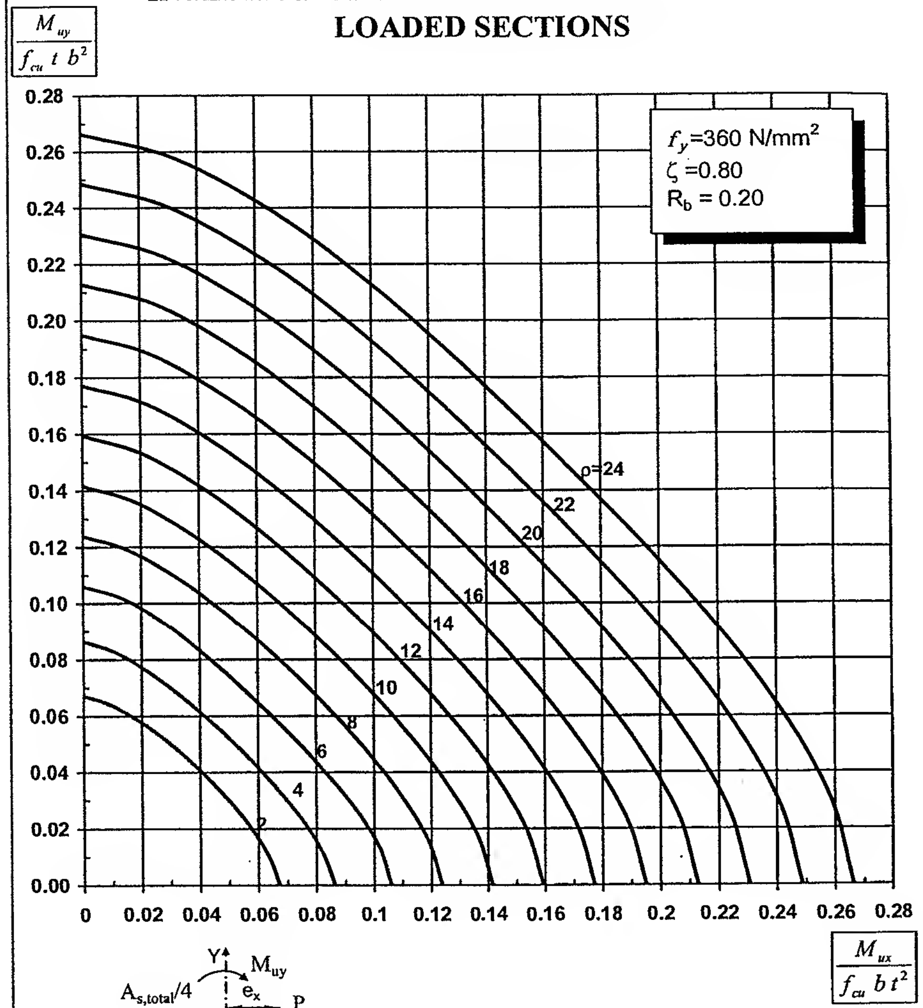

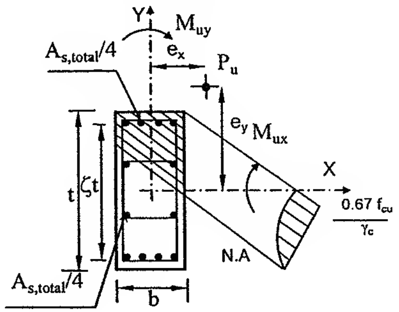

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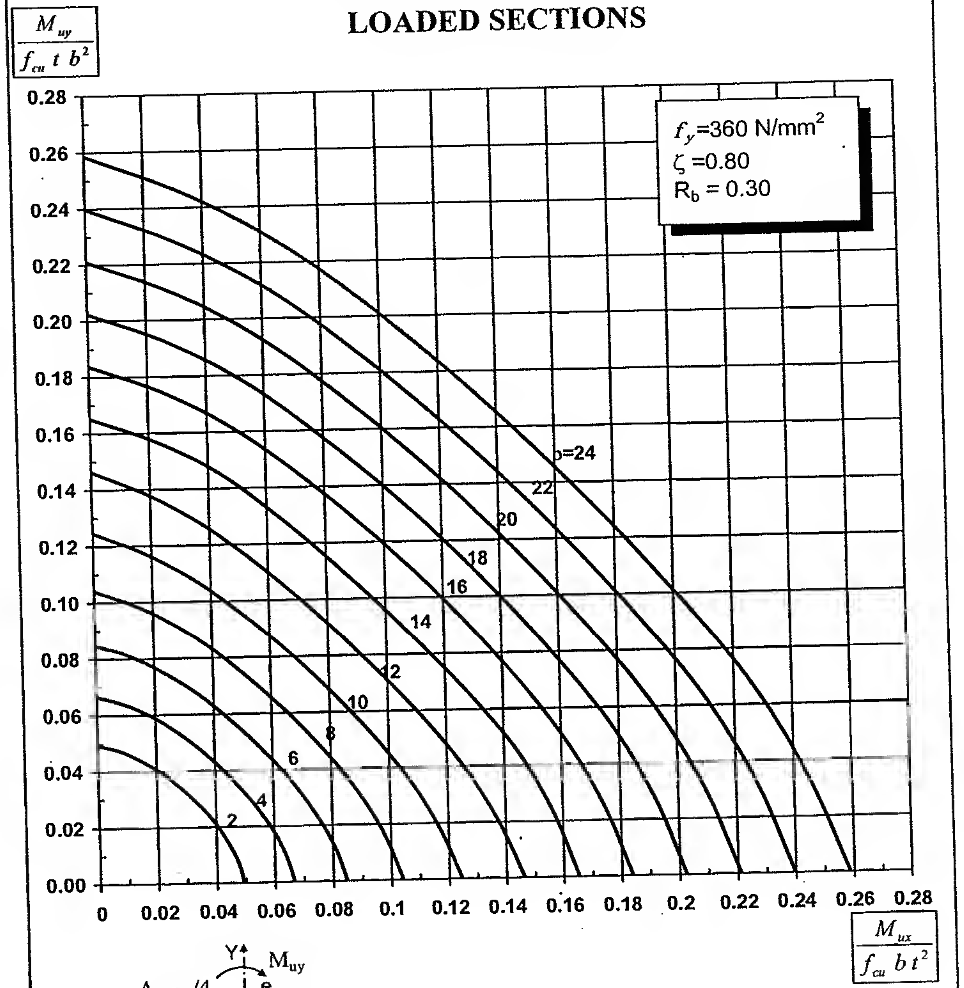

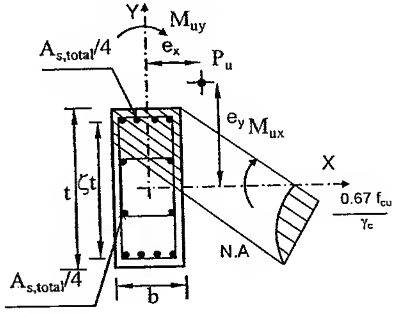

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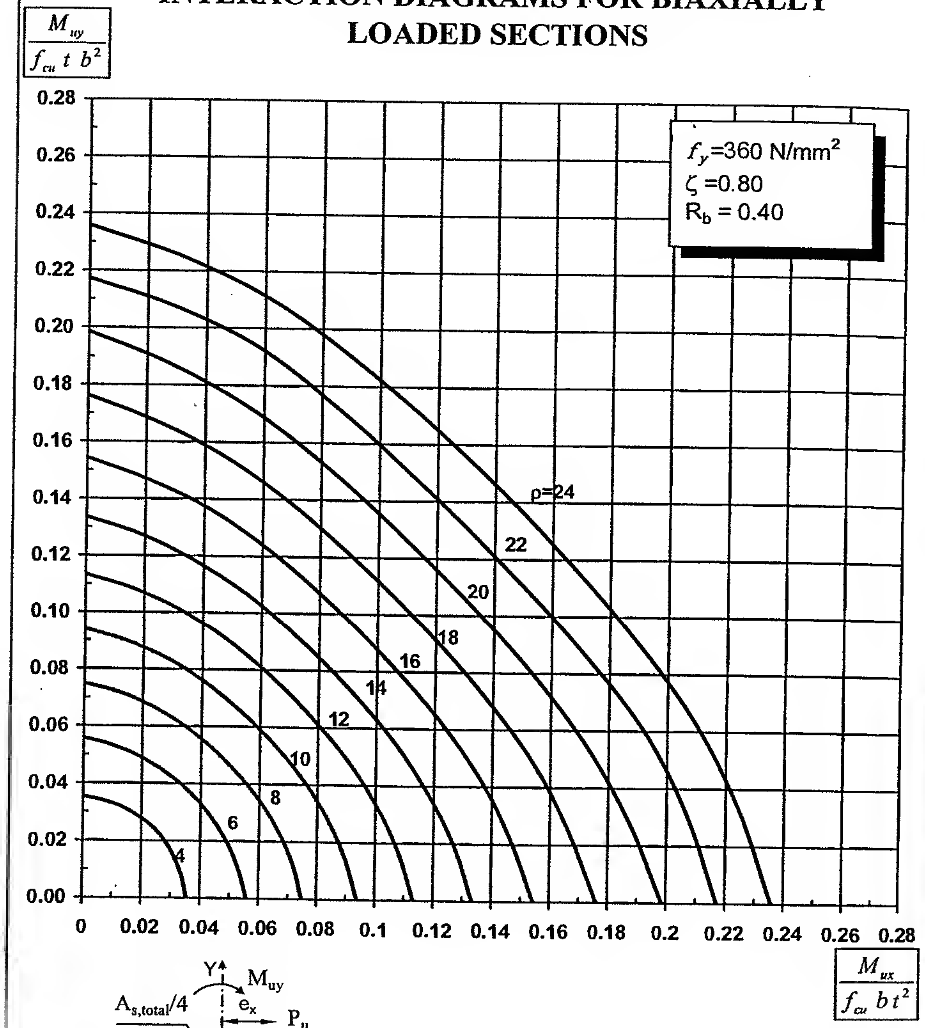

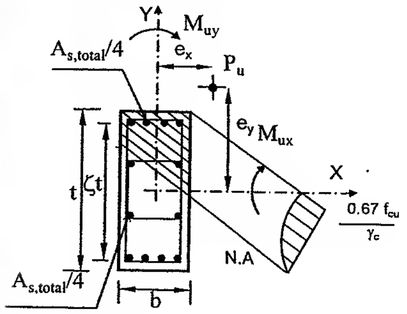

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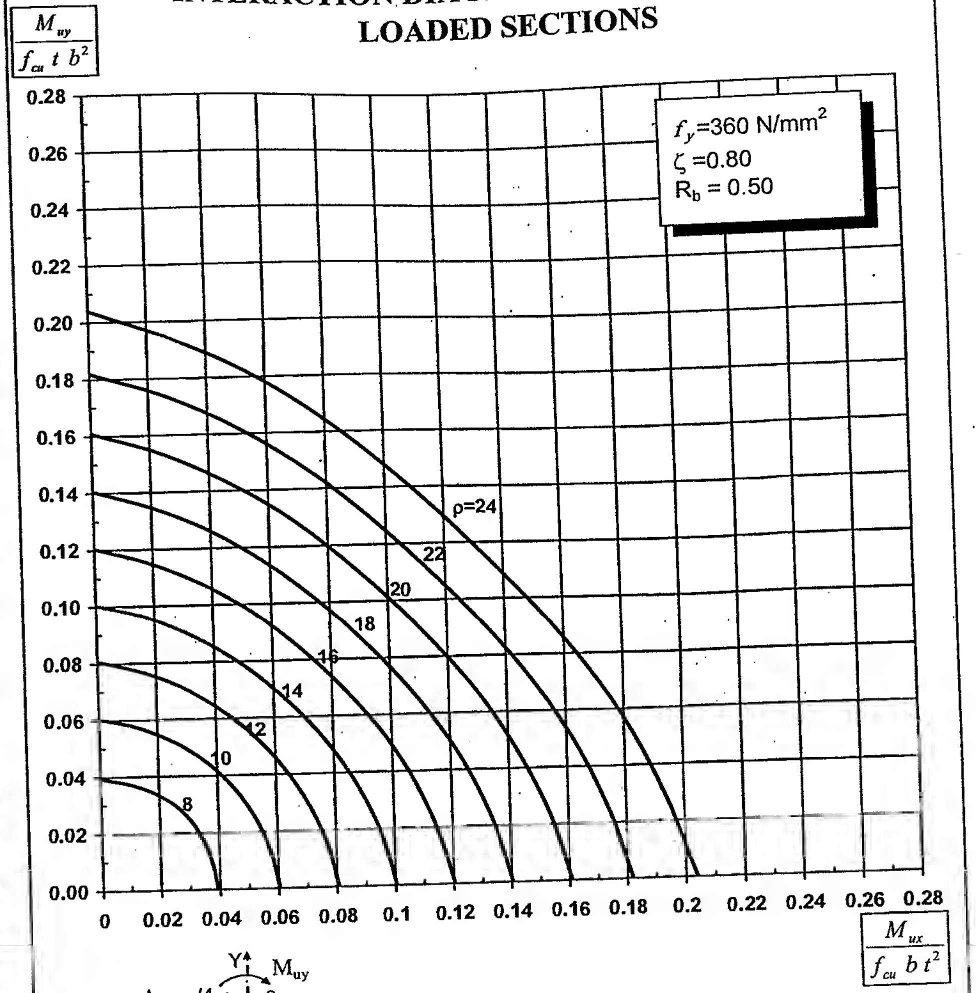

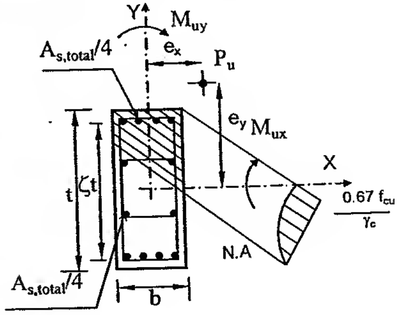

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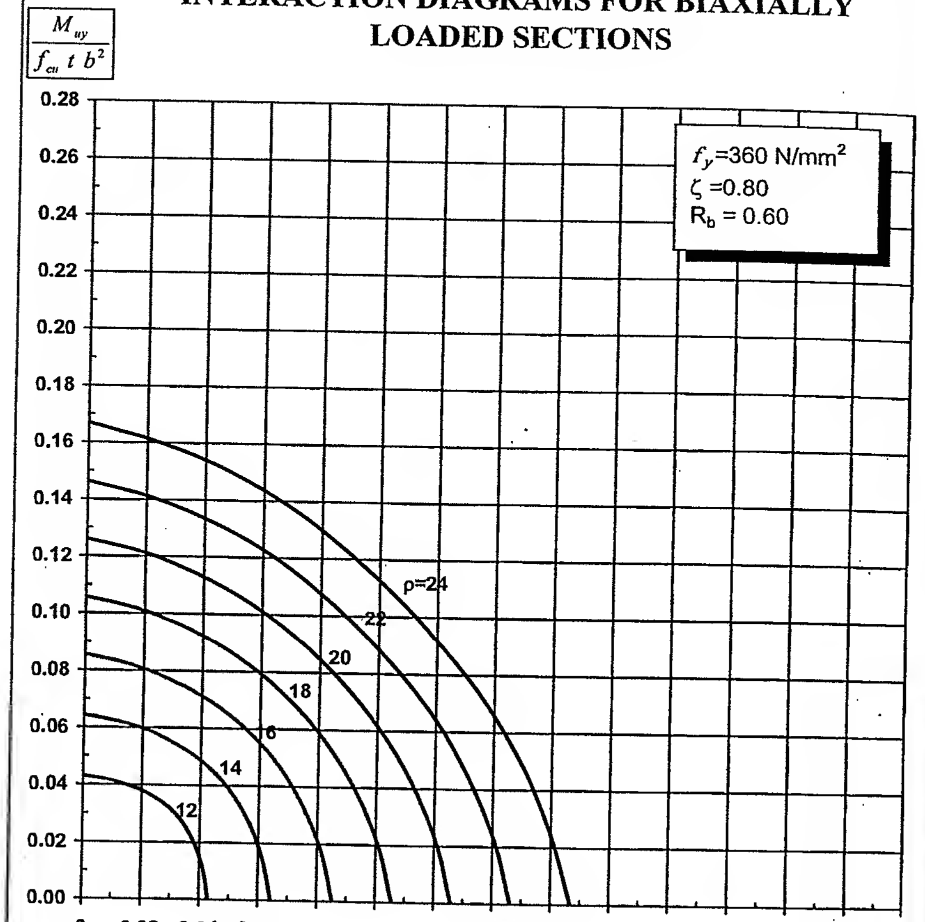

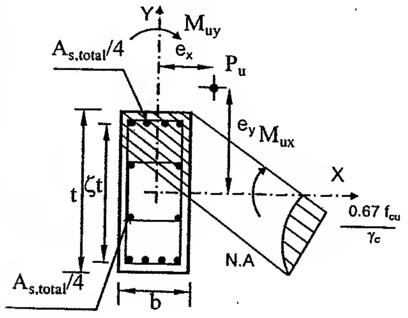

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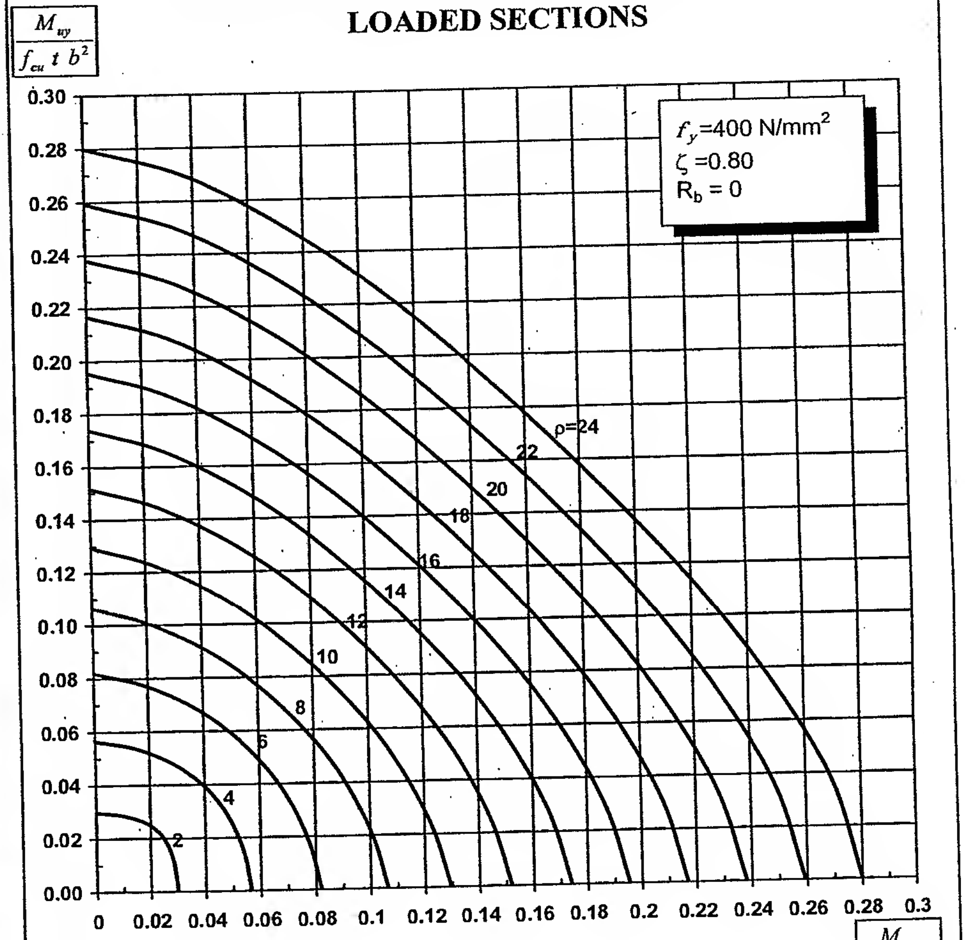

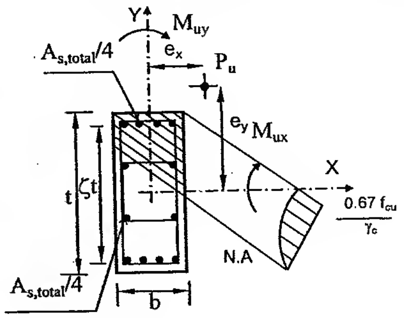

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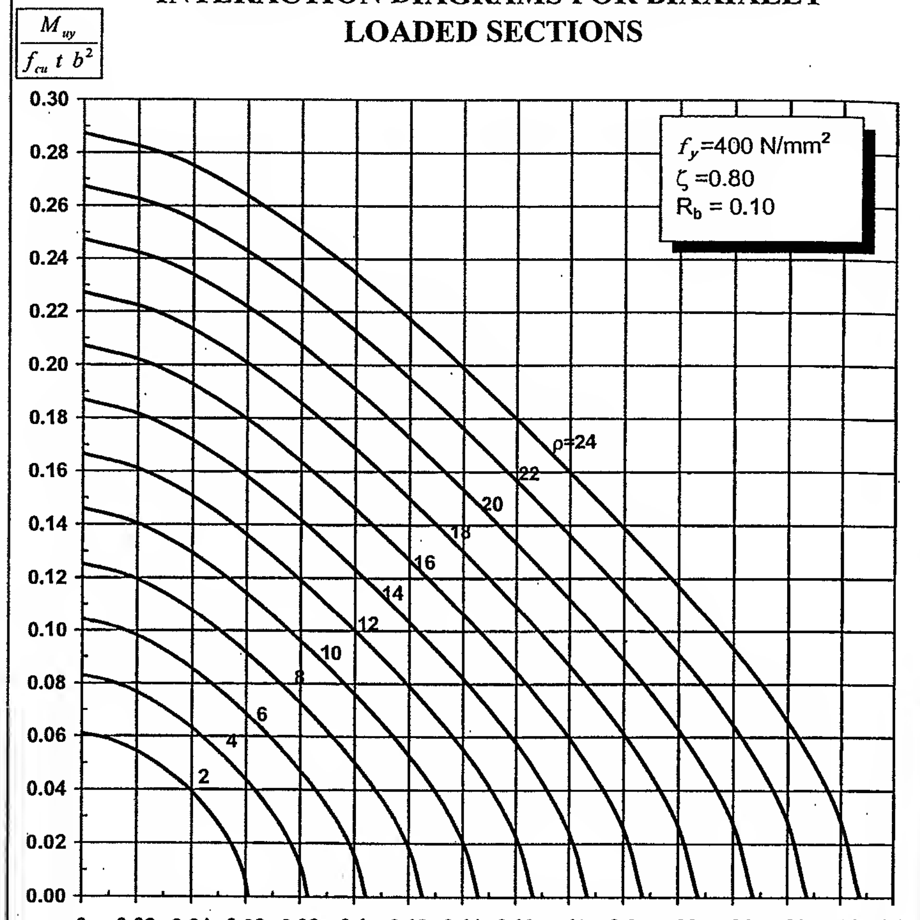

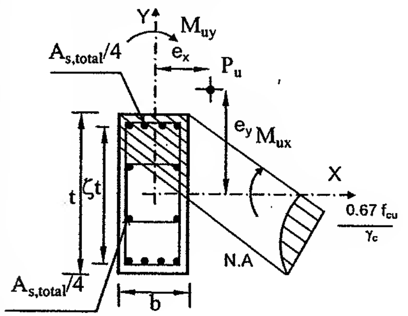

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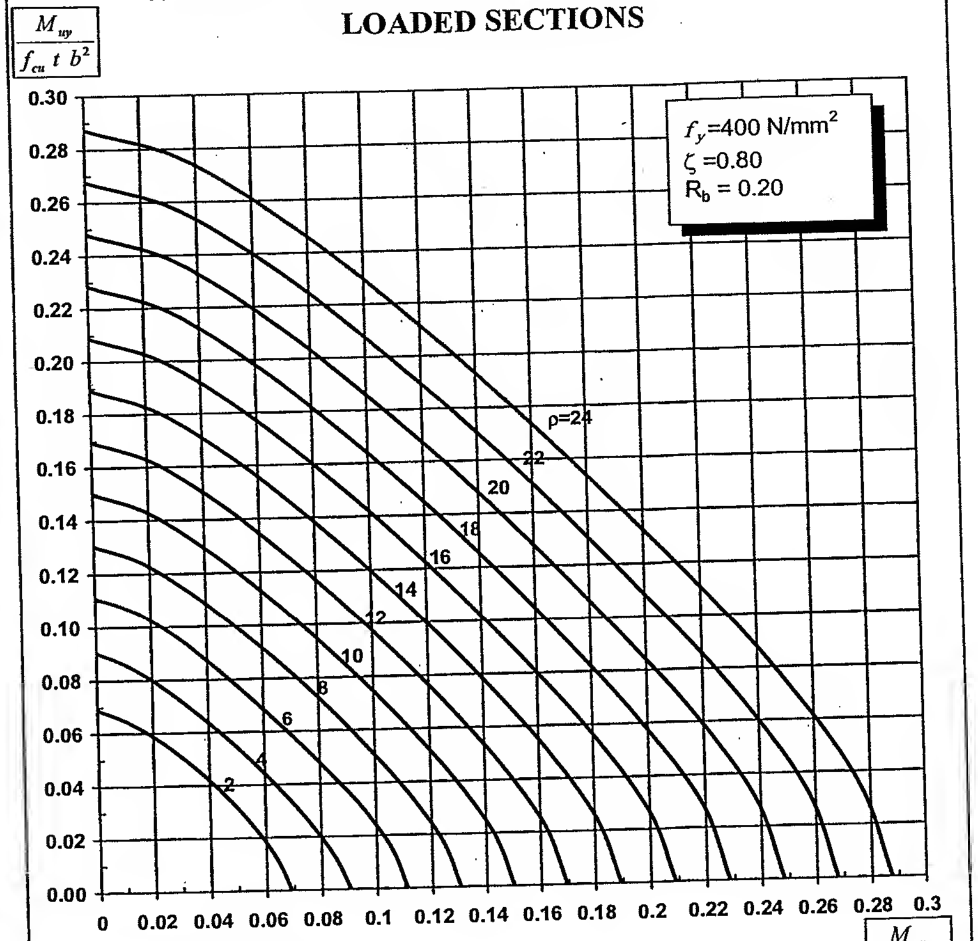

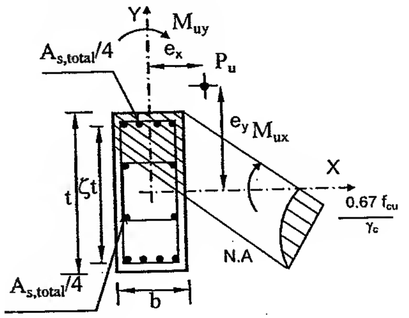

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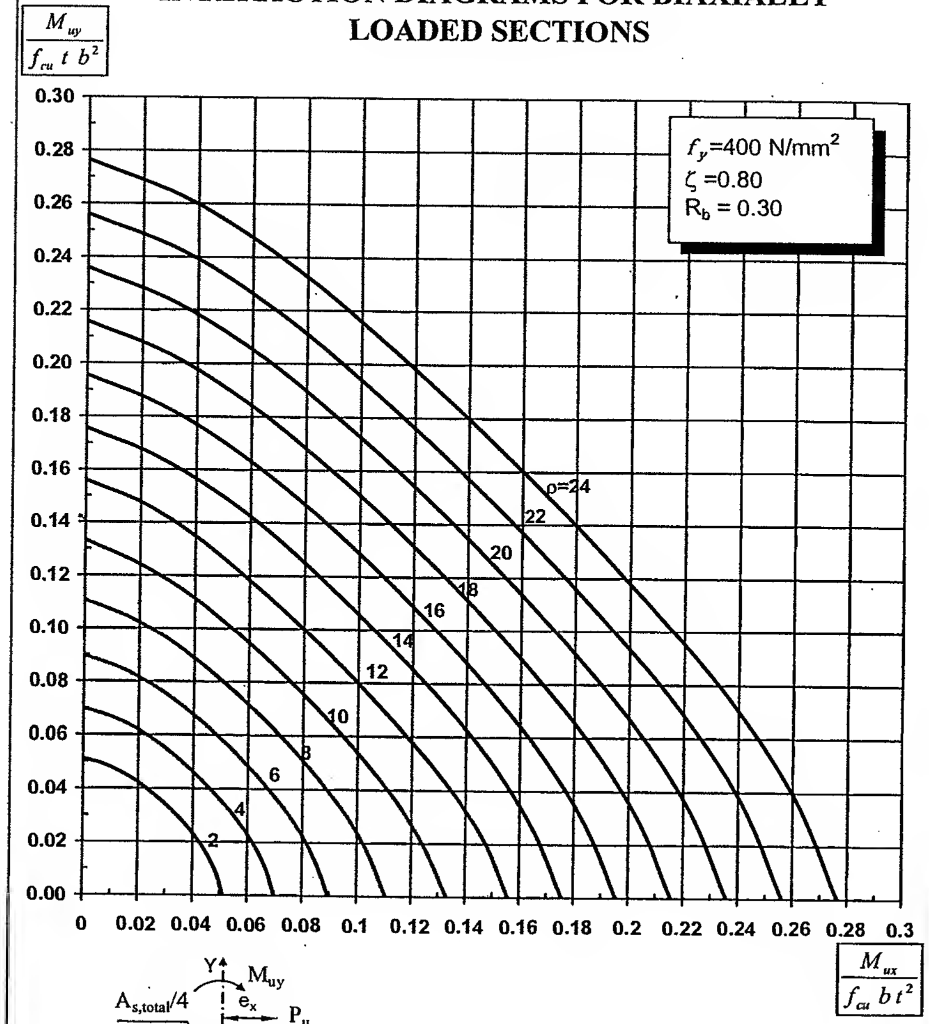

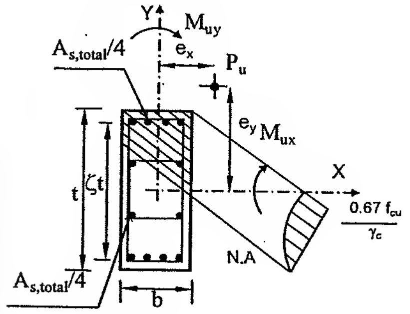

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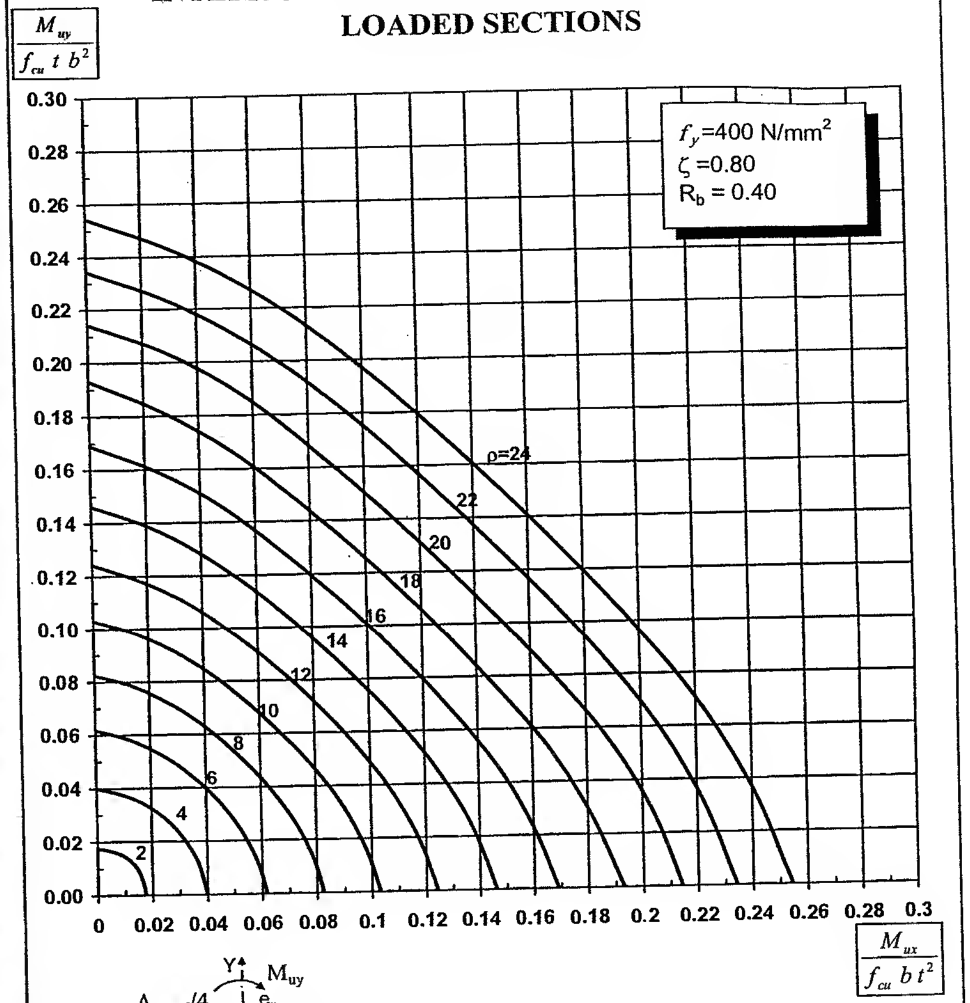

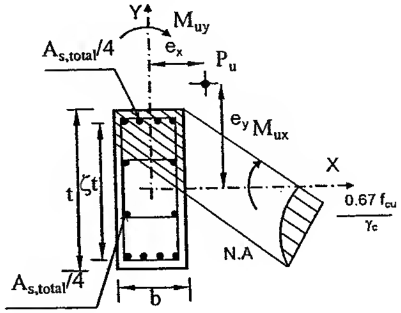

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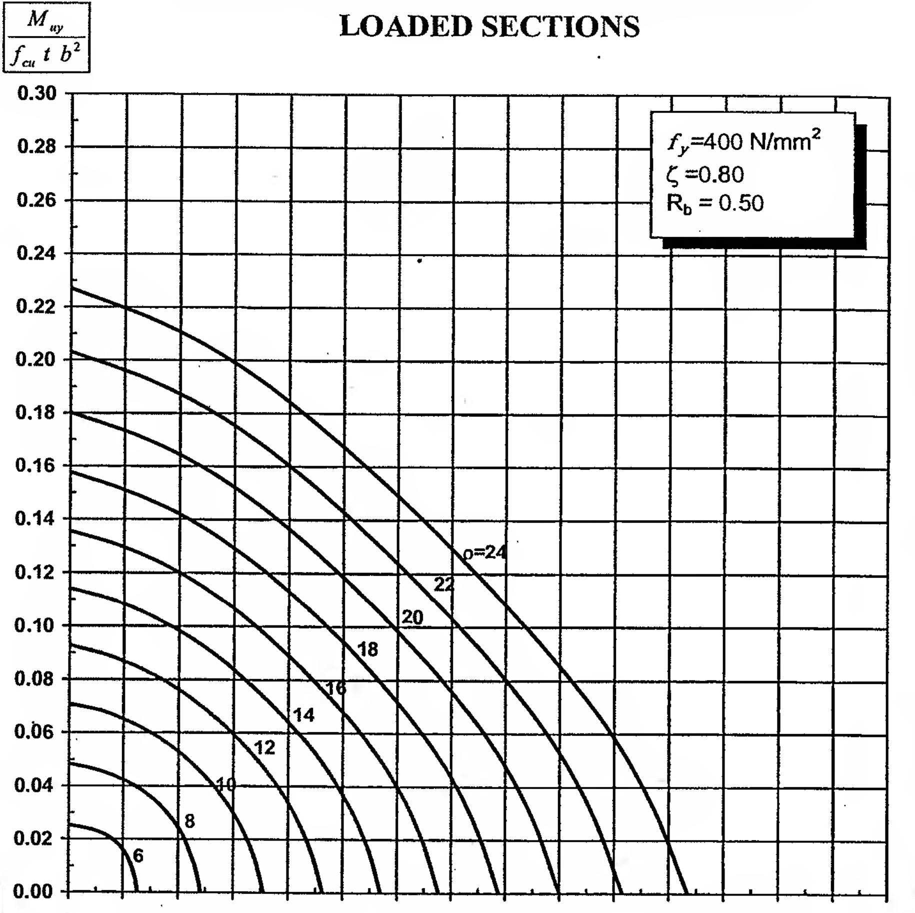

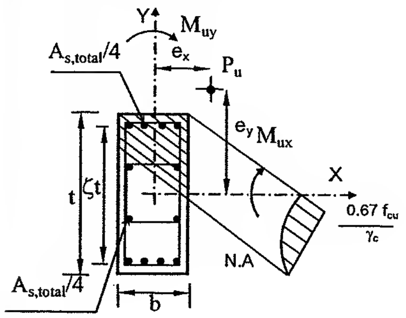

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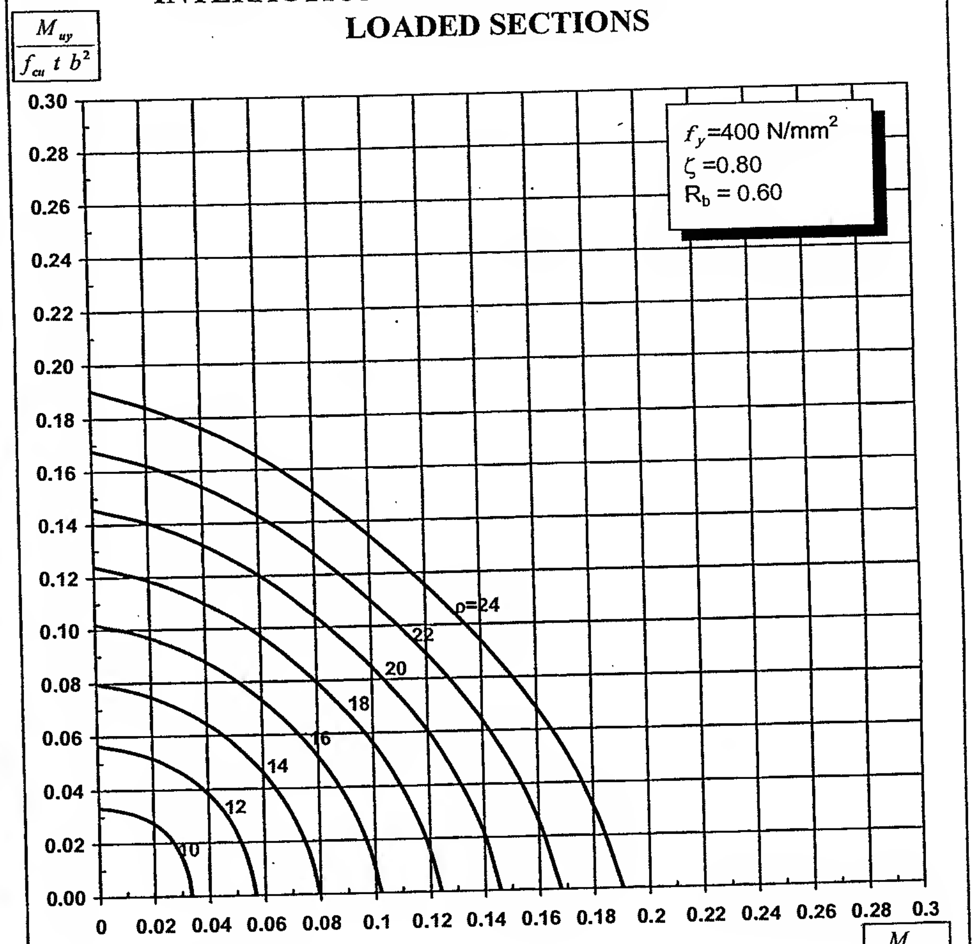

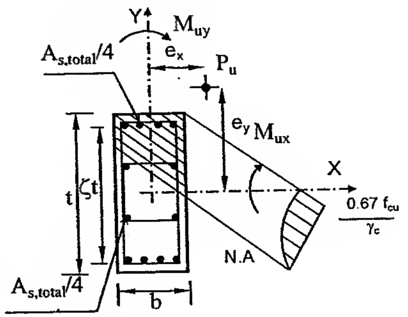

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$$\mu = \rho\, f_{cu} \times 10^{-4}$$

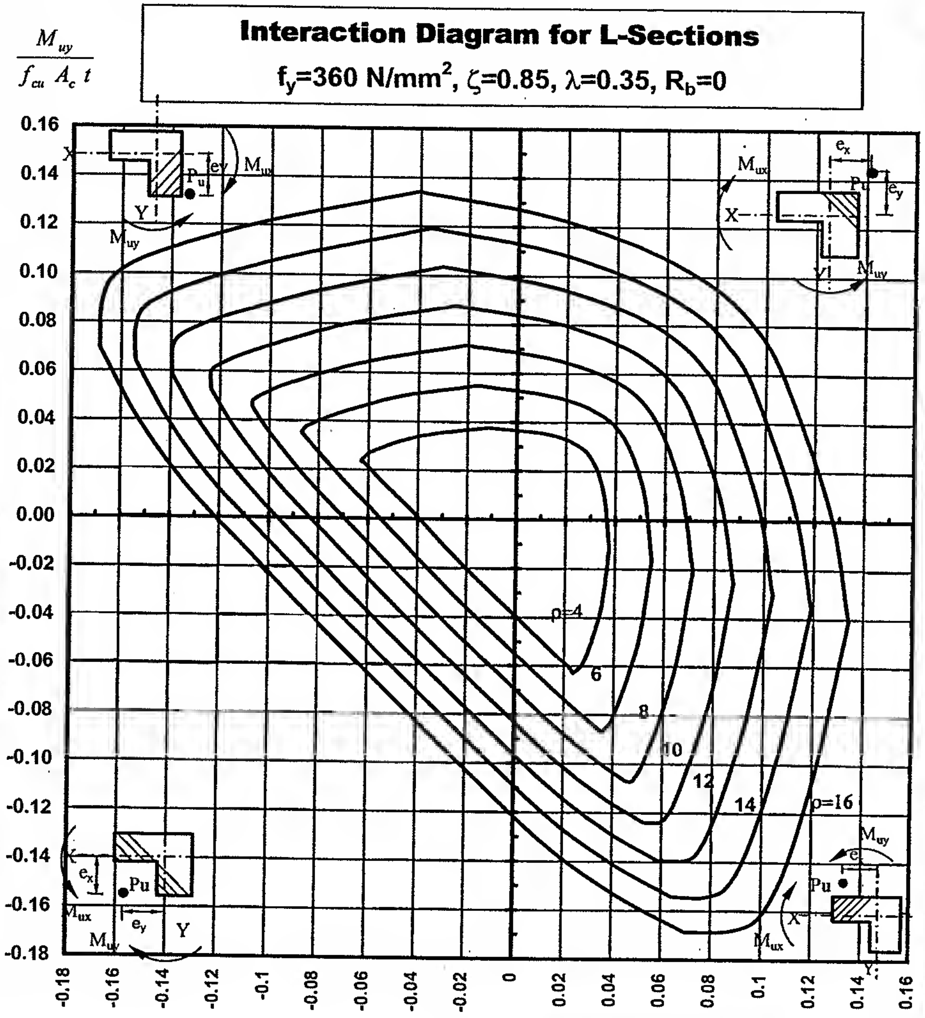
$$A_{s,total} = \mu\, b\, t$$

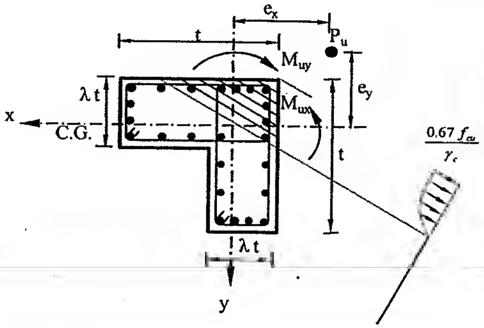
APPENDIX I

Interaction Diagrams for L-Sections

Interaction Diagram for L-Sections

f_y=360 N/mm^2, ζ=0.85, λ=0.35, R_b=0





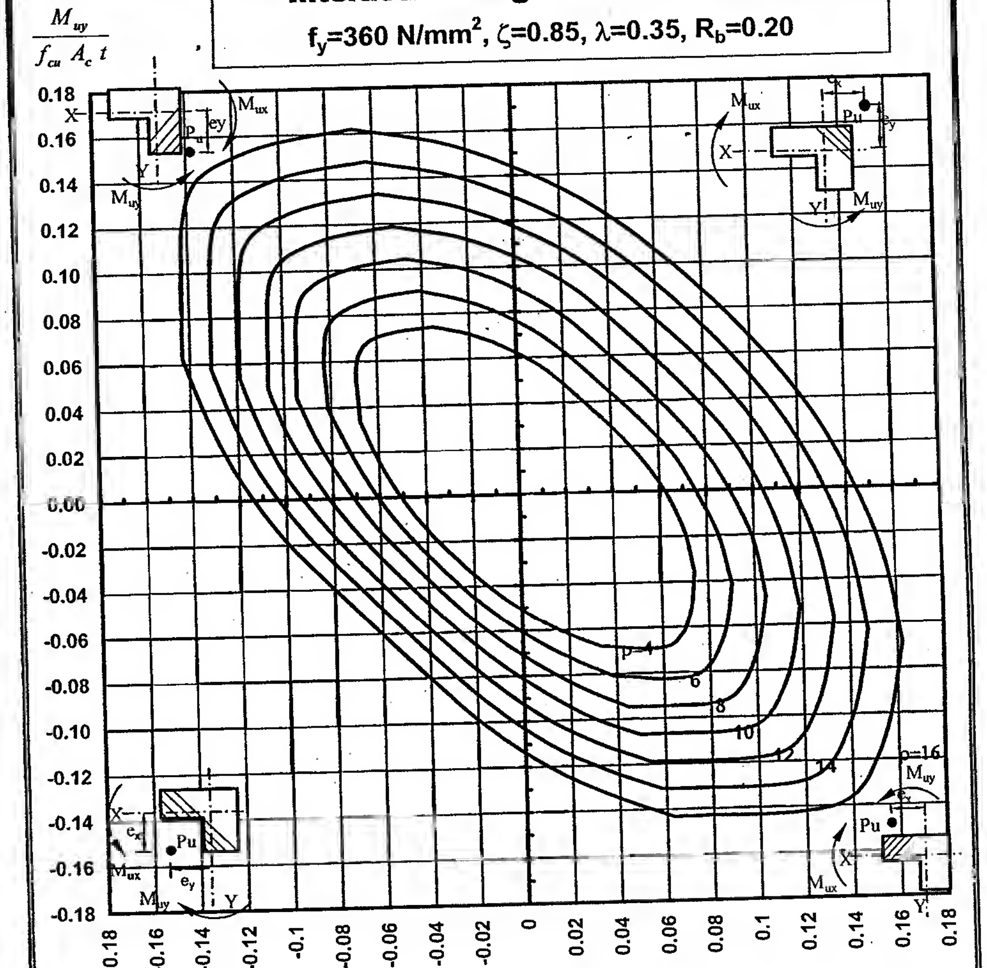
$$R_b = \frac{P_u}{f_{cu}\ A_c}$$

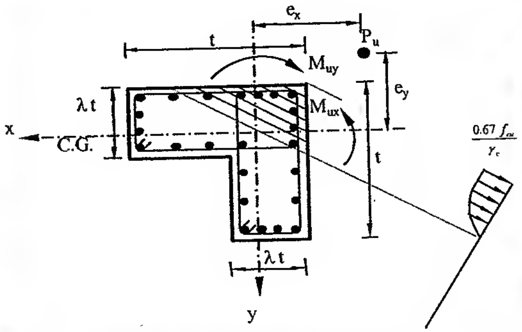
$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\ A_c$$

Interaction Diagram for L-Sections

f_y=360 N/mm^2, ζ=0.85, λ=0.35, R_b=0.20





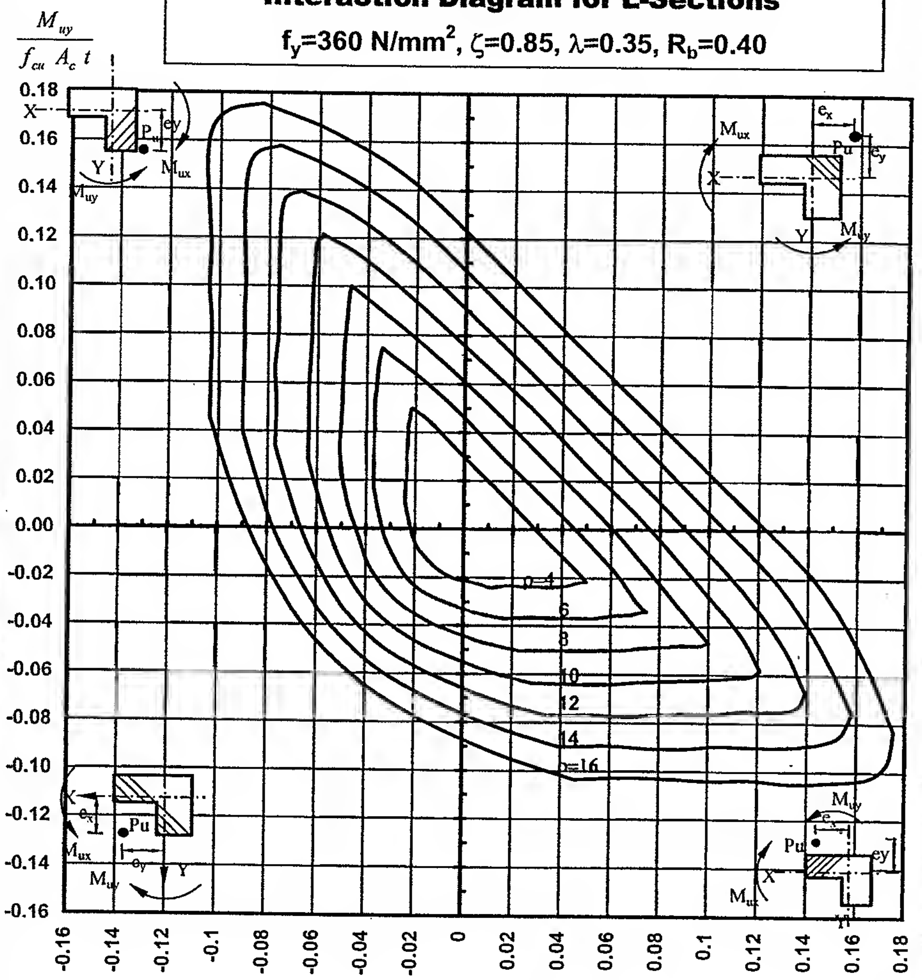
$$R_b = \frac{P_u}{f_{cu}\ A_c}$$

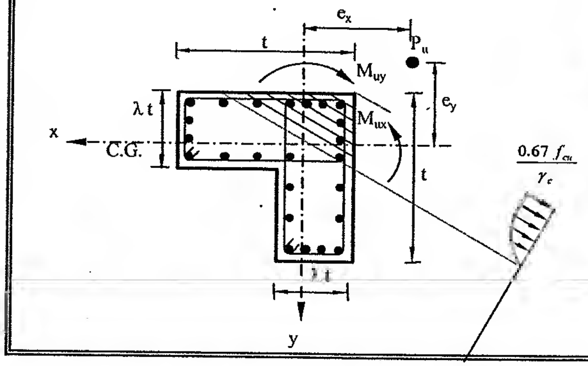
$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\ A_c$$

Interaction Diagram for L-Sections

f_y=360 N/mm^2, ζ=0.85, λ=0.35, R_b=0.40





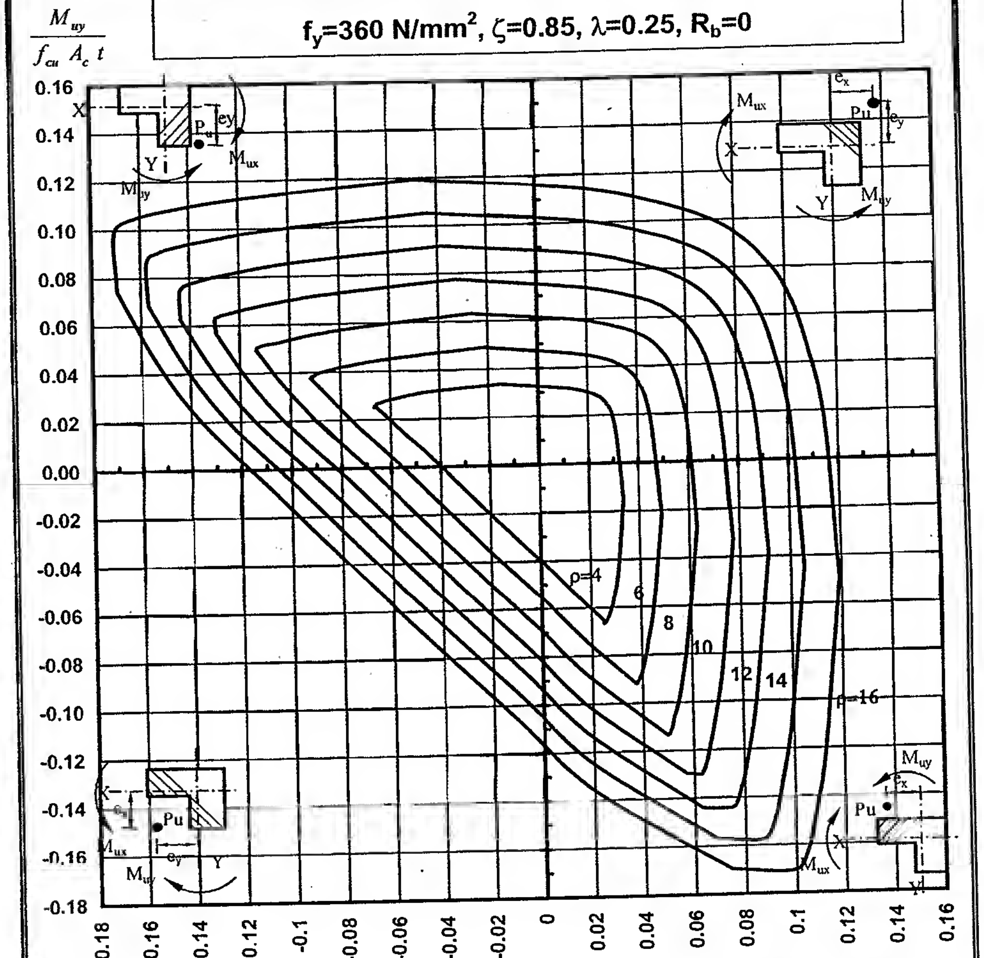
$$R_b = \frac{P_u}{f_{cu}\ A_c}$$

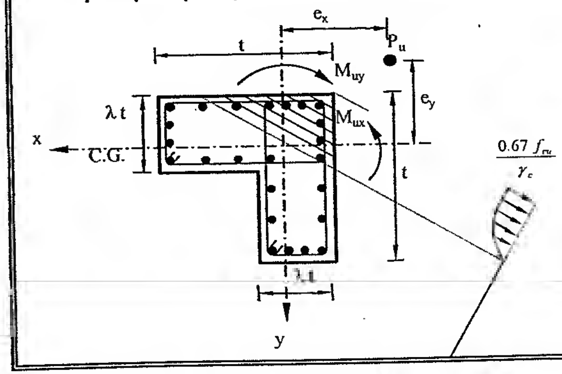
$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\, A_c$$

Interaction Diagram for L-Sections

f_y=360 N/mm^2, ζ=0.85, λ=0.25, R_b=0





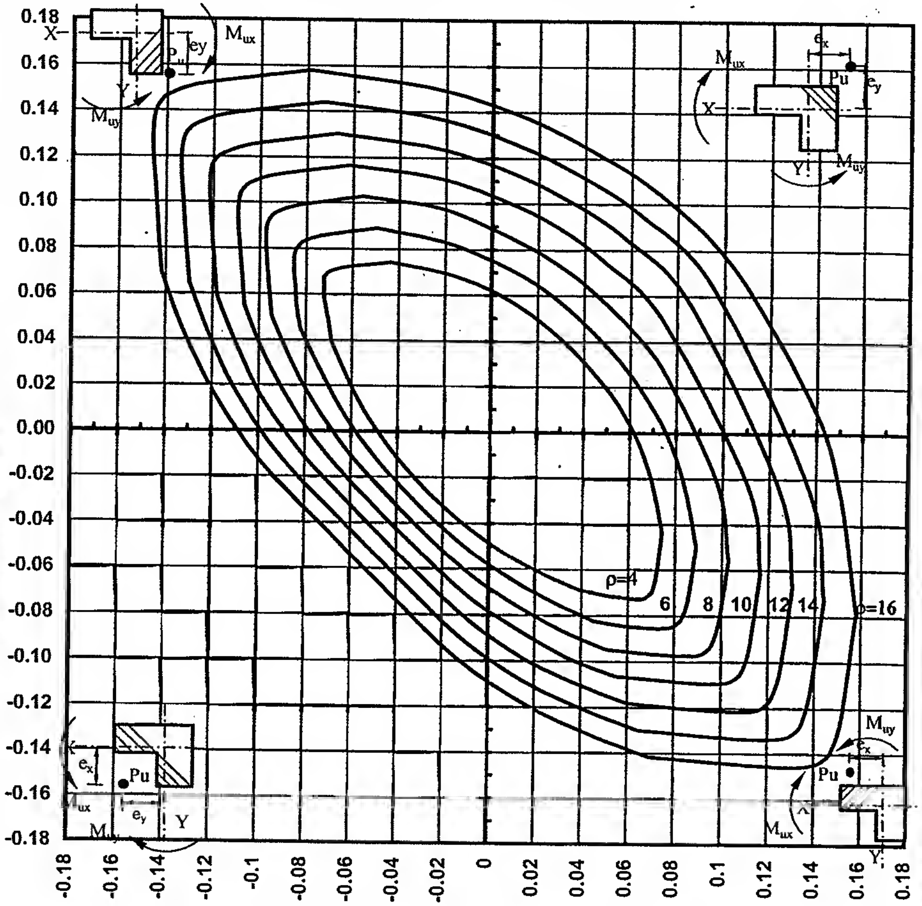
$$R_b = \frac{P_u}{f_{cu}\ A_c}$$

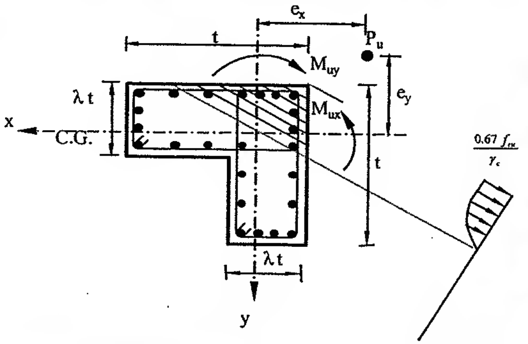
$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\, A_c$$

Interaction Diagram for L-Sections

f_y=360 N/mm^2, ζ=0.85, λ=0.25, R_b=0.20





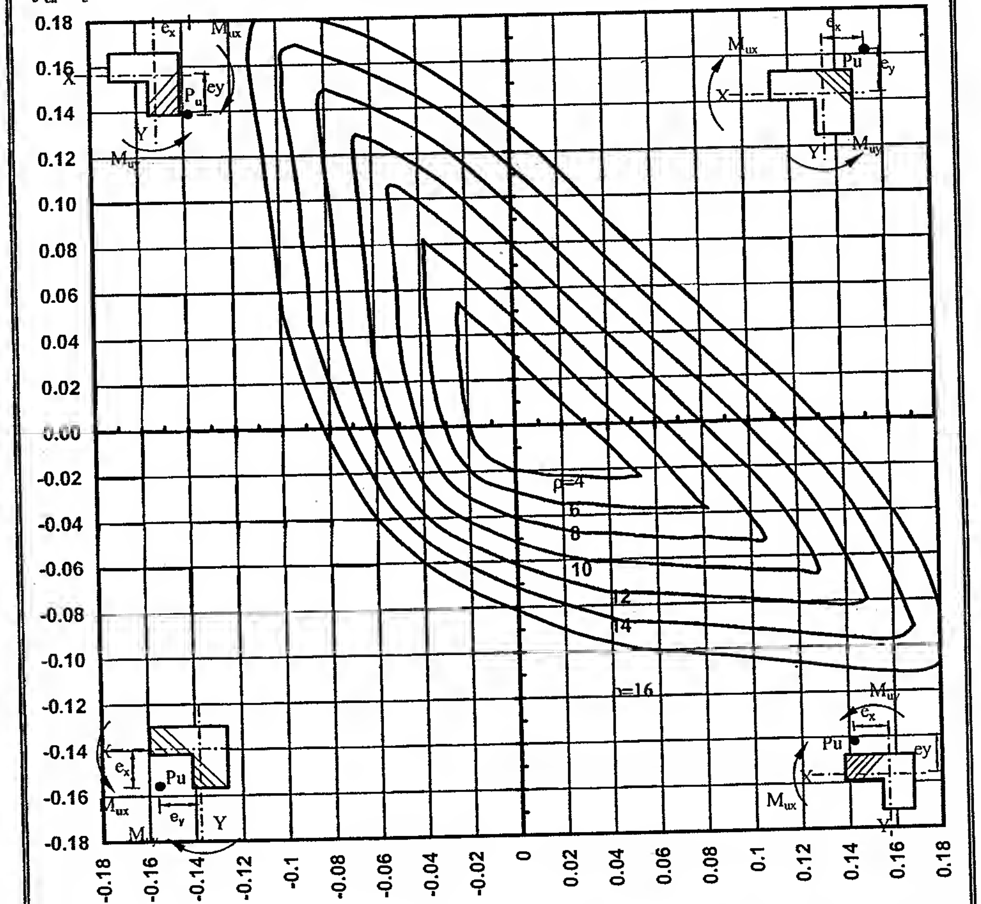
$$R_b = \frac{P_u}{f_{cu}\ A_c}$$

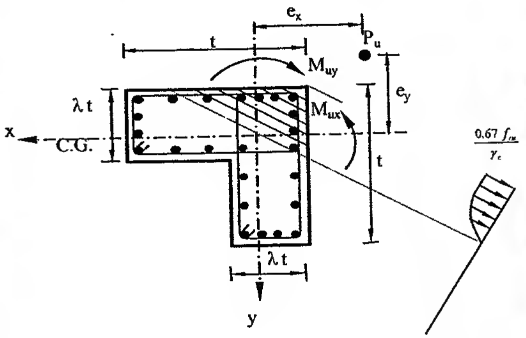
$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\ A_c$$

Interaction Diagram for L-Sections

f_y=360 N/mm^2, ζ=0.85, λ=0.25, R_b=0.4





$$R_b = \frac{P_u}{f_{cu}\ A_c}$$

$$\mu = \rho\, f_{cu} \times 10^{-4}$$

$$A_{s,total} = \mu\ A_c$$

Units Conversion Table

To transform from SI-units	To French -units	Multiply by factor
Concentrated loads		
1N	kg	0.1
1 kN	kg	100
1 kN	ton	0.1
Linear Loads /m′		
1 kN/m′	t/m′	0.1
Uniform Loads /m^2		
kN/m^2	t/m^2	0.1
N/m^2	kg/m^2	0.1
kN/m^2	kg/m^2	100
Stress		
N/mm^2 (=1 MPa)	kg/cm^2	10
kN/m^2	kg/cm^2	0.01
kN/m^2	ton/m^2	0.1
Density		
N/m^3	kg/m^3	0.1
kN/m^3	ton/m^3	0.1
kN/m^3	kg/m^3	100
Moment		
kN.m	ton.m	0.1
N.mm	kg.cm	0.01
Area		
m^2	cm^2	10000
mm^2	cm^2	0.01

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